Energy consumption and mathematical modeling of microwave drying of potato slices

Hosain Darvishi

(Department of Mechanical Engineering, Shahre Ray Branch, Islamic Azad University, Tehran, Iran)

Abstract: In this research, drying characteristics and energy requirements for microwave drying of potato slices were reported at four microwave power densities, 5, 10, 15 and 20 W/g. During the experiments, potato slices were dried to the final moisture content of 0.08 from 2.294 kgH2O/kg dry matter. The experimental data were fitted to six drying models: Linear, Lewis, Henderson and Pabis, Wang and Singh, Page, and Midilli et al. models. The models were compared using the coefficient of determination, root mean square error and reduced chi-square. The Midilli et al. model best described the drying curve of potato slices. The effective moisture diffusivity was determined by using Fick’s second law and was observed to lie between $0.025 \times 10^{-8}$ and $3.05 \times 10^{-8}$ m$^2$/s for the potato samples. The minimum and the maximum energy requirements for drying of potato slices were also determined as 4.22 MJ/kgH2O and 10.56 MJ/kgH2O for 15 and 5 W/g, respectively.

Keywords: potato, drying, mathematical model, effective diffusivity, energy consumption


1 Introduction

Microwave heating is based on the transformation of alternating electromagnetic field energy into thermal energy by affecting the polar molecules of a material. Compared with hot air drying, microwave reduces the decline in quality, and provides rapid and effective heat distribution in the material as well (Diaz et al., 2003). Tippayawong et al. (2008) reported that the conventional practice results in low overall efficiency, approximately 30% and around 35%–45% of energy input is wasted as hot gas exhaust. In microwave drying, the quick absorption of energy by water molecules causes rapid evaporation of water, resulting in high drying rates of the food. The drying time can be greatly reduced by applying the microwave energy to the dried material.

Due to the concentrated energy of a microwave drying system, only 20%–35% of the floor space is required, as compared to conventional heating and drying equipment (Vadivambal and Jayas, 2007; Maskan, 2000). Also, it has also been suggested that microwave energy should be applied in the falling rate period for drying (Maskan, 2000). In the drying industry, the most important aim is to use lowest energy to extract the most moisture for obtaining optimum product storing conditions.

Akpinaret al. (2005) found that the potato slices are sufficiently dried in the ranges between 60 and 80°C and 20%–10% relative humidity at 1 and 1.5 m/s of drying air velocity during 10–12 h despite the exergy losses of 0–1.796 kJ/s. Bakal et al. (2011) investigated the effect of air temperature and two different shapes (cubical and cylindrical) with 3 aspect ratio of each shape on the drying kinetics of potato in a fluidized bed dryer. They reported that the Page model best described the drying behaviour of potatoes. Similar results were reported by Senadeera et al. (2003) for potatoes.

McMinn et al. (2003) carried out an extensive study of the effect of key process parameters on the drying
characteristics of potato samples dried using microwave and combined microwave–convective techniques. Reyes et al. (2007) found that the type of dryers and the drying temperature had a strong effect on drying rate and on the colour and the porosity of the dried potato slices, while the rehydration capacity and the maximum penetration force were not affected. Hatamipour et al. (2007) studied the effect of various pretreatments (tray dryer, with and without air circulation, and fluidized bed dryer) on the shrinkage and colour properties of six varieties of sweet potatoes. The effect of air conditions (air temperature, air humidity and air velocity) and characteristic sample size on drying kinetics of potatoes was examined during air drying by Krokida et al. (2003).

Leeratanarak et al. (2006) investigated drying of potato slices using both low-pressure superheated steam drying and hot air drying. Pimpaporn et al. (2007) studied the influence of various pretreatments and drying temperature on the low-pressure superheated steam drying kinetics and quality parameters of dried potato chips. Khraisheh et al. (2004) studied the quality and structural changes of potatoes during microwave and convective drying. They reported that air drying led to higher structural changes than in the case of microwave drying. Caixeta et al. (2002) found that the potato chips dried at higher steam temperatures and high convective heat transfer coefficients had less shrinkage, higher porosity, darker color, and lower vitamin C content. As little research has been performed on the effect of power density on energy consumption and drying efficiency in microwave drying method, the present research is focused on this issue. The aim of this research was (i) to determine the influence of microwave power density on the energy consumption and drying kinetics during microwave dehydration and (ii) to fit the experimental moisture data to six mathematical models.

2 Materials and methods

2.1 Materials

Potatoes were purchased from a local market, in Tehran, Iran. The samples were stored in a refrigerator at 4°C until used. The potatoes were washed with tap water, peeled and sliced into chips of 0.5±0.03 mm thick. Average initial moisture content of potato samples were determined by using a standard oven method at 105±2°C for 6 h (Aghbashlo et al., 2009) and were found to be 69.93±0.35% (w.b.).

2.2 Experimental set-up and methods

Figure 1 shows the diagram of the microwave drying system. The drying apparatus used was consisted of a laboratory microwave oven (M945, Samsung Electronics Ins) with features of 230 V, 50 Hz with a frequency of 2450 MHz and a digital balance (GF-600, A & D, Japan) with accuracy of ±0.01 g. The area on which microwave drying was carried out was 350×350×240 mm in size. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. In order to weigh the samples without taking them out of the oven, a weighing system was integrated to the oven. A sample tray in the microwave oven chamber was suspended on the balance with a nylon wire through a ventilation hole in the centre of the chamber ceiling. Moisture loss of the sample was recorded by means of a weighing system at 15 s intervals during drying using software for the balance. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying cavity was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The drying trial was carried out at four different microwave power densities being 5, 10, 15 and 20 W/g. Each drying process was applied until the initial moisture ratio was reduced to about 0.08 (kg water/kg dry matter).
2.3 Energy consumption and drying efficiency

Energy consumption by the microwave oven equals:

\[ Q = P \frac{t}{60} \]  

(1)

where, \( Q \) represents total energy consumption in each drying bout, kWh; \( P \) is the microwave power, kW; and \( t \) is drying time, min.

The microwave drying efficiency was calculated as the ratio of the heat energy utilized for evaporating water from the sample to the heat supplied by the microwave oven (Soysal, 2004; Yongsawatdigul and Gunasekaran, 1996).

\[ \eta = \frac{m \Delta w}{60 \times Pt} \times 100 \]  

(2)

where, \( \eta \) is the microwave drying efficiency, %; \( m \) is the mass of evaporated water, kg; and \( \Delta w \) is the latent heat of vaporization of water, kJ/kg. The latent heat of vaporization of water at the evaporating temperature of 100°C was taken as 2257, kJ/kg (Hayes, 1987).

The specific energy consumption was calculated as the energy needed to evaporate a unit mass of water (Mousa and Farid, 2002; Soysal et al., 2006).

\[ Q_s = \frac{60 \times 10^4 Pt}{m_v} \]  

(3)

where, \( Q_s \) is the specific energy consumption, MJ/kgH2O.

2.4 Moisture ratio and mathematical modeling

The moisture ratio (MR) was calculated using the Equation (4):

\[ MR = \frac{M_t - M_e}{M_0 - M_e} \]  

(4)

where, \( MR \) is the moisture ratio (dimensionless); \( M_t \) is the moisture content at \( t \), kgH2O/kg dry matter; \( M_e \) is the equilibrium moisture content, kgH2O/kg dry matter; and \( M_0 \) is the initial moisture content, kgH2O/kg dry matter.

The value of \( M_t \) is relatively small compared with \( M_e \) or \( M_0 \). Therefore, the moisture ratio (MR) was simplified to \( M_t/M_0 \).

Six semi-empirical models were applied to fit the experimental moisture data because they are widely used in drying agriculture products and they are equalities that explain the characteristic of the drying method in a safe way, as listed in Table 1. The terms used to evaluate the goodness of the fit of the tested models to the experimental data were the coefficient of determination (\( R^2 \)), root mean square error (RMSE) and the reduced chi-square (\( \chi^2 \)) between the experimental and predicted moisture ratio values. The statistical parameters were calculated using the following equations:

\[ R^2 = 1 - \frac{\sum_{i=1}^{N} (MR_{exp,i} - MR_{per,i})^2}{\sum_{i=1}^{N} (MR_{exp,i} - \bar{MR})^2} \]  

(5)

\[ \chi^2 = \frac{\sum_{i=1}^{N} (MR_{exp,i} - MR_{per,i})^2}{N - z} \]  

(6)

\[ RMSE = \left( \frac{\sum_{i=1}^{N} (MR_{exp,i} - MR_{per,i})^2}{N} \right)^{1/2} \]  

(7)

where, \( MR_{exp} \) is the experimental moisture ratio; \( MR_{per} \) is the predicted moisture ratio; \( Z \) and \( N \) are numbers of constants and observations, respectively. The best model describing the thin-layer drying characteristics of potato slices was chosen based on the higher value of \( R^2 \) and lower values of \( \chi^2 \) and RMSE.

<table>
<thead>
<tr>
<th>Model name</th>
<th>Model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( MR = 1 + bt )</td>
<td>-</td>
</tr>
<tr>
<td>Lewis</td>
<td>( MR = a e^{k t} )</td>
<td>Bruce (1985)</td>
</tr>
<tr>
<td>Henderson and Pabis</td>
<td>( MR = a e^{k t} )</td>
<td>Henderson and Pabis (1961)</td>
</tr>
<tr>
<td>Page</td>
<td>( MR = a e^{k t} )</td>
<td>Page (1949)</td>
</tr>
<tr>
<td>Midilli et al.</td>
<td>( MR = a e^{k t} )</td>
<td>Midilli et al. (2002)</td>
</tr>
<tr>
<td>Wang and Singh</td>
<td>( MR = 1 + \alpha t )</td>
<td>Wang and Singh (1978)</td>
</tr>
</tbody>
</table>

Note: \( k \), \( n \), \( a \), and \( b \) are the model constants.

2.5 Effective moisture diffusivity

Fick’s law of diffusion incorporated with drying experiments has been widely used to determine the moisture diffusivity of various fruits and other biological materials:

\[ \frac{\partial M}{\partial t} = D_{eff} \nabla^2 M \]  

(8)

The solution of Fick’s equation, with the assumption of moisture migration being by diffusion, negligible shrinkage, constant diffusion coefficients and temperature and for a slab:

\[ R = \frac{M_t}{M_0} = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp \left( -\frac{(2n+1)^2 D_{eff} \pi^2}{4L^2} t \right) \]  

(9)

where, \( D_{eff} \) is the effective moisture diffusivity, m²/s; \( t \) is the drying time, s; \( L \) is the half-thickness of a thin layer.
sample, m; and n is a positive integer.

For long drying time, only the first term of this series is significant, and then the solution becomes:

\[ MR = \frac{M_i}{M_0} = \frac{8}{\pi^2} \exp\left( -\frac{D_{eff} \pi^2}{4L^2} t \right) \]  

(10)

This could be further simplified to a straight-line equation as:

\[ \ln(MR) = \ln\left( \frac{8}{\pi^2} \right) - \left( \frac{D_{eff} \pi^2}{4L^2} \right) t \]  

(11)

The effective moisture diffusivity is determined by plotting the experimental drying data in terms of ln(MR) versus drying time with a slope of:

\[ \text{Slope} = \frac{D_{eff} \pi^2}{4L^2} \]  

(12)

2.6 Drying rate

The drying rate is expressed as the amount of the evaporated moisture over time. The drying rate of potato slices was calculated using the following equation:

\[ DR = \frac{1}{\Delta t} \frac{M_{i,u} - M_f}{M_i} \]  

(13)

where, \( DR \) is the drying rate, kgH2O/kgdry matter·min).

3 Results and discussion

3.1 Drying kinetics of potato slices

The potato slices were dried as a single layer with thickness of 5 mm at the drying microwave power densities of 5, 10, 15 and 20 W/g in a microwave dryer. The variations in moisture ratio of the potato slices as a function of drying time at different power densities are presented in Figure 2. It can be seen that the moisture content of the potato slices decreased with the increase in drying time. It only took 23, 5.75, 3.25 and 2.5 min to dry potato samples from an initial moisture ratio (MR) of 1 to a final moisture ratio (MR) of 0.08 at 5, 10, 15 and 20 W/g of drying power densities, respectively. It indicated that increasing the drying power density decreases the drying time. The decrease in drying time with an increase in the drying microwave power density has been reported for many foodstuffs, such as carrots (Sumnu et al., 2005), nettle leaves (Alibas, 2007), peaches (Wang and Sheng, 2006), tomato pomace (Al-Harahsheh et al., 2009), onions (Arslan and Ozcan, 2010), and mint leaves (Ozbek and Dadali, 2007).

Figure 2 Drying curves of moisture ratio with drying time at different microwave power densities

The drying rates versus average moisture content and drying time curves of potato slices are illustrated in Figure 3 and Figure 4. The drying rates decreased continually with the decrease in moisture content and increased with the microwave power and thus decreasing drying time. The results indicated that mass transfer within the sample was more rapidly during higher microwave power heating because more heat was generated within the sample creating a large vapor pressure difference between the center and the surface of the product due to characteristic microwave volumetric heating. It was observed that the drying rates were higher at the beginning of the drying operation, when the product moisture content was higher. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying
progressed, the loss of moisture in the product caused a decrease in the absorption of microwave power and resulted in a fall in the drying rate. Higher drying rates were obtained at higher microwave output powers. Thus, the microwave output power had a crucial effect on the drying rate (Soysal, 2004). It was observed that there was a period with a constant drying rate at 5 W/g, and the drying processed at 10, 15 and 20 W/g represented a falling-rate drying period.

3.2 Evaluation of the models

Non-linear regression was used to obtain each parameter value of every model. The statistical results from the models are summarized in Table 2. In all cases, the statistical parameter estimations showed that $R^2$, $\chi^2$ and RMSE values were ranged from 0.810 to 1, 0.00004 to 0.05709, and 0.00637 to 0.22781, respectively. Based on higher $R^2$, and lower values of $\chi^2$ and RMSE, it can be concluded that Midilli et al. model gave best results than the other models. Thus, it was selected to represent the thin layer drying characteristics of potato slices.

<table>
<thead>
<tr>
<th>Model name</th>
<th>$P/W \cdot g^{-1}$</th>
<th>Model constants</th>
<th>$R^2$</th>
<th>$\chi^2$</th>
<th>RMSE</th>
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<tr>
<td>Linear</td>
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<td>$a = -0.3916$</td>
<td>0.979</td>
<td>0.00264</td>
<td>0.04897</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>$a = -0.3189$</td>
<td>0.989</td>
<td>0.00124</td>
<td>0.03390</td>
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<tr>
<td></td>
<td>10</td>
<td>$a = -0.1767$</td>
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<td>0.02791</td>
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<tr>
<td></td>
<td>5</td>
<td>$a = -0.0420$</td>
<td>0.993</td>
<td>0.00055</td>
<td>0.02329</td>
</tr>
<tr>
<td>Lewis</td>
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<td>$k = 0.9170$</td>
<td>0.810</td>
<td>0.03186</td>
<td>0.17020</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>$k = 0.7412$</td>
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<td>0.01606</td>
<td>0.12211</td>
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<tr>
<td></td>
<td>10</td>
<td>$k = 0.3879$</td>
<td>0.849</td>
<td>0.01461</td>
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<tr>
<td></td>
<td>5</td>
<td>$k = 0.0809$</td>
<td>0.894</td>
<td>0.00831</td>
<td>0.09068</td>
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<tr>
<td>Henderson and Pabis</td>
<td>20</td>
<td>$a = 1.655, k = 1.2048$</td>
<td>0.880</td>
<td>0.05709</td>
<td>0.22781</td>
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<td>0.02942</td>
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<td>0.02193</td>
<td>0.14497</td>
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<td></td>
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<td>$a = 1.34, k = 0.0999$</td>
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<td>20</td>
<td>$k = 0.417, n = 2.061$</td>
<td>0.997</td>
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<tr>
<td></td>
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<td>$k = 0.017, n = 1.539$</td>
<td>0.998</td>
<td>0.00016</td>
<td>0.01276</td>
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<tr>
<td>Midilli et al.</td>
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<td>$a = 1.013, b = -0.039, k = 0.378, n = 1.861$</td>
<td>0.999</td>
<td>0.00017</td>
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<tr>
<td></td>
<td>15</td>
<td>$a = 1.012, b = -0.041, k = 0.354, n = 1.381$</td>
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<td>0.00006</td>
<td>0.00774</td>
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<td>0.00007</td>
<td>0.00817</td>
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<td></td>
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<td>$a = 1.013, b = -0.006, k = 0.022, n = 1.344$</td>
<td>1</td>
<td>0.00004</td>
<td>0.00637</td>
</tr>
<tr>
<td>Wang and Singh</td>
<td>20</td>
<td>$a = -0.0317, b = -0.3295$</td>
<td>0.984</td>
<td>0.00202</td>
<td>0.04291</td>
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<td></td>
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<td></td>
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<td>0.02430</td>
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<td></td>
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<td>$a = 0.0002, b = -0.0452$</td>
<td>0.995</td>
<td>0.00043</td>
<td>0.02070</td>
</tr>
</tbody>
</table>

Figure 5 compares experimental data with those predicted with the Midilli et al. model for potato slices at 5, 10, 15 and 20 W/g. There was a very good agreement between the experimental and predicted moisture ratio
values, which closely banded around a 45° straight line. The Midilli et al. model has also been suggested by others to describe the infrared drying of tomatoes (Celma et al., 2009), fluidized bed drying of olive pomace (Meziane, 2011), sun, oven, and microwave oven drying of savory leaves (Arslan and Ozcan, 2011), thin layer drying of eggplants (Ertekin and Yaldiz, 2004), thin layer drying of potato, apple and pumpkin slices (Akpinar, 2006), and mint leaves (Doymaz, 2006).

3.3 Effective moisture diffusivity

The effective moisture diffusivity was calculated by using the method of slopes. According to the experimental data obtained at different drying power densities, the logarithm of moisture ratio values, ln(MR), were plotted against drying time (t). The linearity of the relationship between ln(MR) and the drying time is illustrated in Figure 6.

The values of effective moisture diffusivity are presented in Table 3, and obtaining values between 0.025×10⁻⁸ and 3.05×10⁻⁸ m²/s. The \( D_{eff} \) values reported herein are within the general range of 10⁻¹¹ to 10⁻⁹ m²/s for food materials (Madamba et al., 1996). As expected, the effective moisture diffusivity values increased greatly with the increase in drying power density because of increasing in the vapor pressure inside the samples. This might be explained by the increased heating energy, which would increase the activity of the water molecules leading to higher moisture diffusivity when samples were dried at higher microwave power density. Similar results are found to correspond well with those existing in the literature, such as 5.612×10⁻⁹ to 1.317×10⁻⁸ m²/s for fluidized bed drying of potatoes (Bakal et al., 2011), 4.606×10⁻⁶ to 7.065×10⁻⁶ m²/s freeze-drying of sweet potato cubes with far-infrared (Lin et al., 2005), 3.17×10⁻⁷ to 15.45×10⁻⁷ m²/s for thin-layer drying of potato slices in length of continuous band dryer (Aghbashlo et al., 2009), and 2.90×10⁻⁸ to 4.88×10⁻⁸ m²/s, 7.04×10⁻⁸ to 24.22×10⁻⁸ m²/s , and 3.15×10⁻⁸ to 5.36×10⁻⁸ m²/s for convective, microwave and combined drying of potato cylinders, respectively (McMinn et al., 2003). The differences between the results can be explained by effect of drying methods, types, composition, and tissue characteristics of the potatoes and the proposed model used for calculation.

<table>
<thead>
<tr>
<th>( P/W \cdot g^{-1} )</th>
<th>( D_{eff} \times 10^8/m² \cdot s^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.05</td>
</tr>
<tr>
<td>15</td>
<td>2.35</td>
</tr>
<tr>
<td>10</td>
<td>1.24</td>
</tr>
<tr>
<td>5</td>
<td>0.025</td>
</tr>
</tbody>
</table>

3.4 Energy consumption

Figure 7 shows the variations of drying efficiency values for potato samples drying under microwave heating. Results showed that the drying efficiency values decreased continuously with time and increased as the power density and moisture content were increased. The average drying efficiency, total energy requirement for a charge of the dryer and the energy needed for drying of potato slices are given Figure 8. As it is understood from these figures,
the minimum specific energy ($4.22 \times 10^3$ MJ/kgH$_2$O) and maximum drying efficiency (62.4%) was computed for drying potato slices at microwave power density of 15 W/g. The minimum drying efficiency (21.38%) and maximum specific energy ($10.56 \times 10^3$ MJ/kgH$_2$O) was computed at 5 W/g. Besides, the energy consumption decreasing with increasing drying microwave power output was more effective on energy requirement. The best result with regard to energy consumption and drying efficiency was obtained from 15 W/g power density level among all drying power density levels. The specific energy consumption obtained in the drying process using 15 W/g power density level was 2.5-fold lower than 5 W/g power density level. One of many reasons might be that the drying time is longer under lower power, hence results in a increase in energy consumption.

4 Conclusions

The drying kinetics of the potato slices was investigated in a microwave dryer as a single layer at the drying microwave power densities of 5, 10, 15 and 20 W/g. The entire drying took place in a falling rate period at 10, 15 and 20 W/g, and took place in a constant period at 5 W/g. The moisture content and drying rates were influenced by the drying power density. Increases in drying power density caused decreases in drying time and increases in the drying rate. The effective diffusivity increased with the increase in the drying power density. Based on non-linear regression analysis, the Midilli et al. model was considered adequate to describe the thin-layer drying behavior of potato slices. The effective diffusivity varied from $0.025 \times 10^{-8}$ to $3.05 \times 10^{-8}$ m$^2$/s over the microwave power densities ranged from 5 to 20 W/g. According to the results, it can be said that 15 W/g must be selected for drying potato slices. Specific energy consumption and drying efficiency at this level were 4.22 MJ/kg[H$_2$O] and 53.54%, respectively.

References


