Modeling and Experimental Validation of Total Solar Fraction for Even Span Greenhouses by Shadow Area Concept

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ABSTRACT

In this paper, an attempt has been made to evaluate total solar fraction (which is the ratio of the area of the shadow outside greenhouse to the total shadow area of the greenhouse) experimentally and theoretically for an even span greenhouse at New Delhi (28.58°N) by measuring the shadow length and then evaluating the shadow area. Greenhouse floor dimensions are 6m × 4m, wall height 2m and ridge height 3m. Experimental shadow data were obtained for a typical day of each month from 9 am to 3 pm for one year. It has also been validated theoretically (relation between geometrical shape of greenhouse, sun rays, and sun earth angles). There is fair agreement between experimental and theoretical results in both cases. Solar fraction obtained from shadow area is higher during winter than summer period. The value of solar fraction is higher at any particular time of the day, than that obtained earlier by Auto-CAD because the present study includes beam and diffuse radiations together, and it is the sum of solar fraction from all walls of greenhouse. The hourly variation of greenhouse room air temperature has also been predicted for typical day of winter and summer month respectively.

Key words: Solar fraction, Even Span Greenhouse, solar energy

1. INTRODUCTION

In order to predict the accurate performance of greenhouse in terms of room air temperature, it is mandatory to know the distribution of solar flux on different walls and floor in terms of solar fraction inside the greenhouse (Pucar, 2002; Ravi, 2004; Tiwari, 2003; and Tiwari et al. 2002). This is required to write the energy balance for each component of greenhouse for prediction of room air and plant temperatures. Recently, Tiwari et al.(2003) defined the concept of solar faction for the north wall of a greenhouse in terms of solar radiation falling on it or in terms of projected length of solar rays beyond the north wall. Total solar fraction can be defined as sum of transmitted solar radiation falling on each section of greenhouse at a given time to the total solar radiation transmitted to the inside of the greenhouse at the same time. It can also be defined in terms of the shadow area of the greenhouse which includes the effect of the direct as well as the diffuse radiation. Earlier, the solar fraction had been defined in terms of the projected rays as the ratio of the length of the projected ray beyond north wall to the total length of projected ray by considering only direct radiation (Tiwari et al. 2002).
In the present paper, the total solar fraction has been defined as the ratio of the area of the shadow outside the greenhouse at a particular time to the total area of the shadow of the

greenhouse at the same time. Both direct and diffuse radiation has been considered unlike earlier study (Diffuse radiation creates hidden shadow that what we see during the experiment), the diffuse radiation is also a part of global radiation that creates the shadow. The main reason for evaluating the solar fraction of the greenhouse is to know how much the percentage of losses of the solar radiation occurs through transparent greenhouse cover. A model has been developed to calculate total solar fraction.

2. METHOD OF CALCULATION

The method used for calculating the total solar fraction in terms of the shadow area of the greenhouse is as follows

$$\text{Total solar fraction} = \frac{\text{Area of shadow outside of the greenhouse at a particular time, m}^2}{\text{Total area of shadow of the greenhouse at the same time, m}^2}$$

To calculate the shadow area outside the greenhouse the only term which is measured is the shadow length of north wall L, for number of the days of the year n, at a particular time (Hour angle,ω) taking the solar time as solar noon. The sun – earth angles are calculated by using the following expressions (Duffe and Beckman, 1991; Tiwari, 2003):
Solar altitude angle (α_s) can be calculated as

$$\sin\alpha_s = \cos\varphi\cos\delta\cos\omega + \sin\varphi\sin\delta \tag{1}$$

where the declination angle, (δ) and the hour angle, (ω) are evaluated by

$$\delta = 23.45\sin\left(\frac{360(284+n)}{365}\right) \tag{2}$$

and

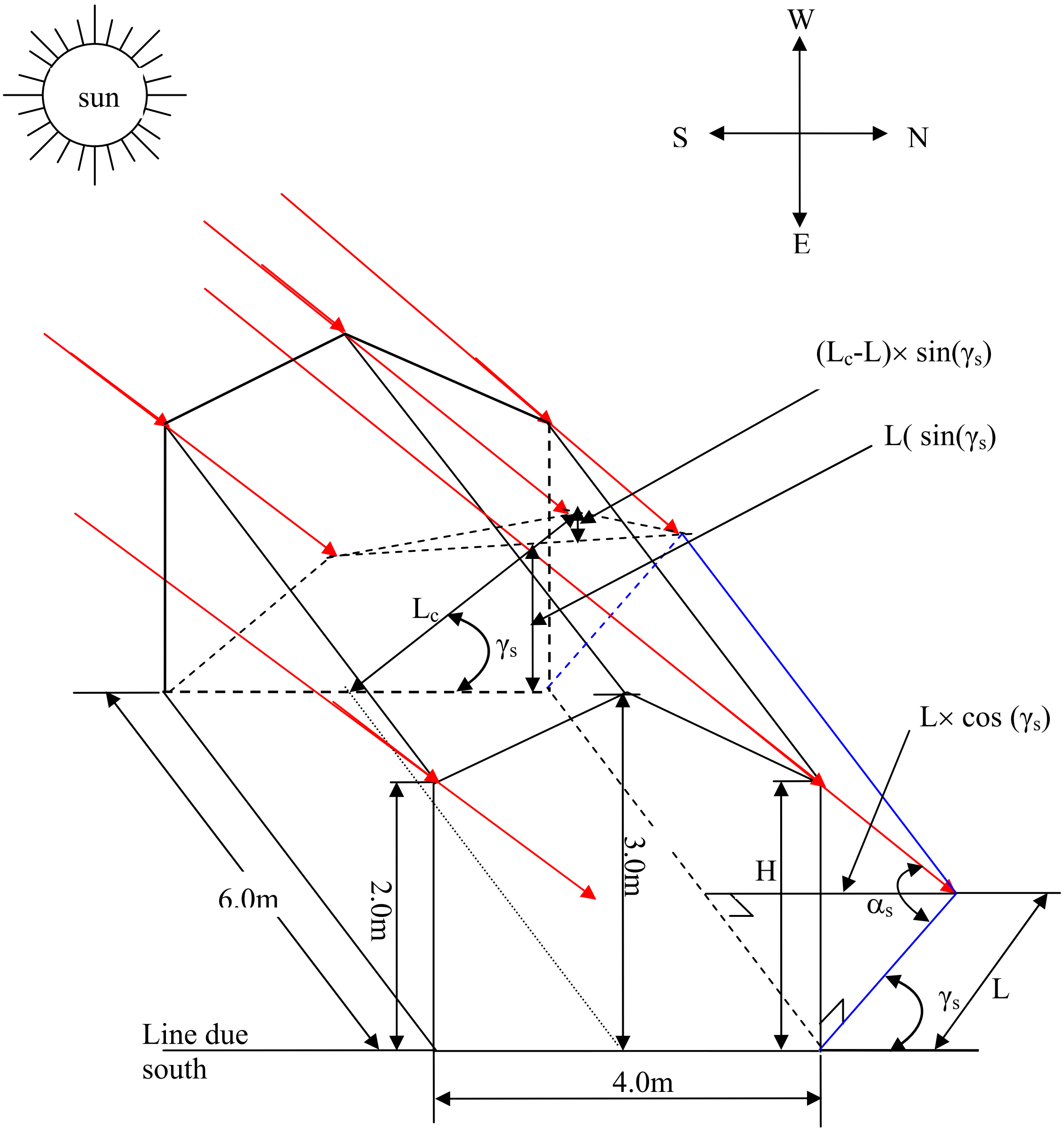
$$\omega = 15°(t_{solar} - 12.0) \tag{3}$$

The latitude angle (φ), in general $-90 \le \varphi \le 90$; north is positive, for New Delhi φ =28.58° N
The solar azimuth angle (γ_s) can be calculated as follows

$$\cos\gamma_s = \frac{\sin\alpha_s\sin\varphi - \sin\delta}{\cos\alpha_s\cos\varphi} \tag{4}$$

Since the solar azimuth angle γ_s is the angle between a line due south and the projection of the sun rays on the horizontal and in our case (a selected experimental greenhouse) the greenhouse is E-W oriented as shown by isometric shape in Figure1a. The shadow area of the north wall will be in the form of a parallelogram as shown in Figure1b, except at noon when it will be rectangular. It can be calculated by the following formula

$$A_n = 6 \times L \times Cos\gamma_s \tag{5}$$



For a wall of height = H,

$$\text{Shadow length of "H"} = L = L_{th} = \frac{H}{\tan(\alpha_s)} \tag{6}$$

Figure 1a. Isometric shape of an even span greenhouse, showing the solar rays and shadow of north and west walls.

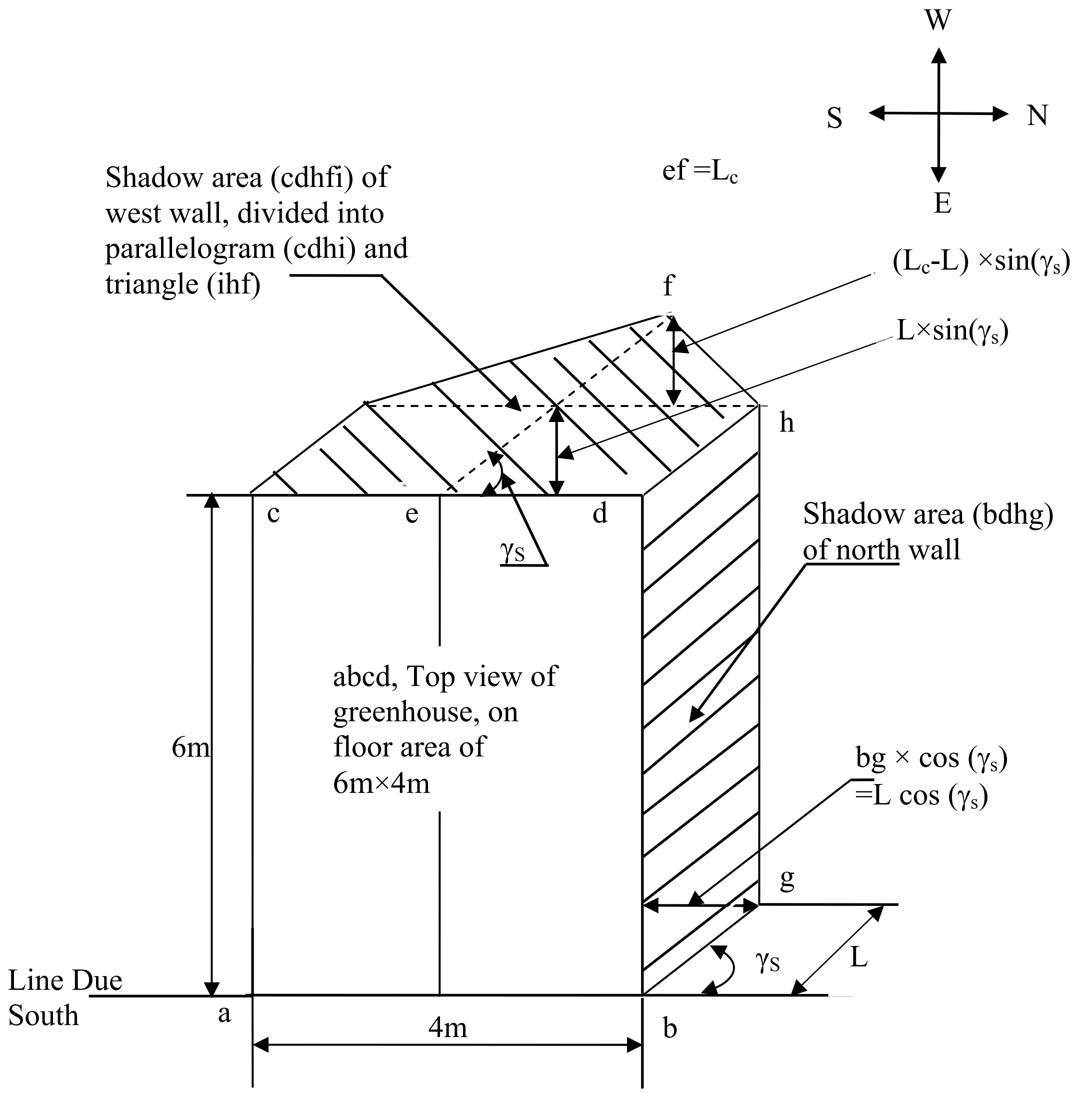


Figure 1b. Top view of greenhouse, showing the shadow area of north and west walls, and solar azimuth angle.

The shadow of the east and west walls is a polygon, with constant base line length and width that varies from morning to evening and is zero at noon time. It can be divided into a parallelogram (cdhi) and a triangle (ihf) as shown in Figure1b. The parallelogram base (cd) is 4m and sides (ci and dh) equal to the shadow length of the north wall L, the triangle part has a base (hi) of 4m also, and the opposite corner joined at the center (e) of the base (cd) by a line (ef) which is the projection of the tip point at hump (shadow length of greenhouse ridge). This line is also makes an angle equal to the solar azimuth angle (γ_s) with the base (cd).To evaluate these areas the ridge projection on the ground has to be decided.
The length of the shadow at the center of the greenhouse (ridge) (L_c) can be evaluated from the ratio with the length of shadow of the north wall (L), as follows:

$$L_c = \frac{ridge\ height}{wall\ height} \times L = \frac{3}{2} \times L \qquad (7)$$

where $\frac{3}{2}$ is the ratio of ridge height of greenhouse to the wall height .

If one draws a line perpendicular to the base of the greenhouse, Fig.1b, one gets the area of parallelogram equals to $4 \times L \times \sin \gamma_s$, and the area of the triangle part equals to

$\frac{1}{2} \times 4(L_c - L) \sin \gamma_s$

By adding the above two equations one gets the total area of east or west walls as,

$$A_p = 4 \times L \times \sin \gamma_s + \frac{1}{2} \times 4\ (L_c - L) \sin \gamma_s \qquad (7)$$

where A_p is polygon shadow area of east or west wall since they are symmetrical.

After simplification, one gets, $A_p = 2 \times (L_c + L) \times \sin \gamma_s$ (8)

The outside shadow area of greenhouse equals to the area of north wall and the area of west wall (morning) or area of east wall (evening), and hence

$$A_o = A_n + A_P = 6 \times L \times \cos \gamma_s + 2 \times (L_c + L) \times \sin \gamma_s \qquad (9)$$

The total shadow area of greenhouse equals the outside shadow and inside area of greenhouse

$$A_t = A_i + A_o = 24 + 6 \times L \times \cos \gamma_s + 2 \times (L_c + L) \times \sin \gamma_s \qquad (10)$$

As per definition, the total solar fraction can be evaluated by the following expression

$$F_t = \frac{A_o}{A_t} = \frac{6 \times L \times \cos \gamma_s + 2 \times (L_c + L) \times \sin \gamma_s}{24 + 6 \times L \times \cos \gamma_s + 2 \times (L_c + L) \times \sin \gamma_s} \qquad (11)$$

Using the values of total solar fraction in energy balance for each components of greenhouse given by Tiwari (2003), the hourly variation of room air temperature with and without brick north wall can be evaluated for a given design and climatic parameters. The solar fraction of brick north retained inside the greenhouse while in the greenhouse without brick north wall solar fraction is wasted from the glazing canopy cover.

Coefficient of correlation (r) of experimental and theoretical results of total solar fraction has been evaluated for each day by the following expression:

$$r = \frac{N' \sum X_i Y_i - \left(\sum X_i\right)\left(\sum Y_i\right)}{\sqrt{N \sum X_i^{\ 2} - (X_i)^2} \sqrt{N \sum Y_i^{\ 2} - (Y_i)^2}} \qquad (12)$$

From the results obtained it is seen that the coefficient of correlation mostly equals to one and in some cases it exactly one, which means matching between theoretical and experimental data. Root mean square deviation (e) has also been calculated for the same days, using the following expression:

$$e = \sqrt{\frac{(e_i)^2}{N'}} \qquad (13)$$

where, $e_i = \frac{X_i - Y_i}{X_i}$

Table 1. Sun earth angles for typical days of typical months of the year that used in thermal modeling. Latitude angle, (φ = 28.58° N)

Time (hr)	Hour angle, (ω)	No.of day of the year, n	Declination angle, (δ)	Solar altitude angle, (α_s)	Solar azimuth angle, (γ_s)
			15-Jan.		
09:00	-45	15	-21.27	23.898	-46.115
10:00	-30	15	-21.27	32.355	-33.475
11:00	-15	15	-21.27	38.091	-17.846
12:00	0	15	-21.27	40.151	0
13:00	15	15	-21.27	38.091	17.846
14:00	30	15	-21.27	32.355	33.475
15:00	45	15	-21.27	23.898	46.115
			15-Jun.		
09:00	-45	166	23.31	49.43	-93.25
10:00	-30	166	23.31	62.59	-85.9
11:00	-15	166	23.31	75.54	-72.09
12:00	0	166	23.31	84.73	0
13:00	15	166	23.31	75.54	72.09
14:00	30	166	23.31	62.59	85.9
15:00	45	166	23.31	49.43	93.25

3. EXPERIMENT

An experiment on an even span greenhouse has been conducted either on 15th date of each month or one day before or after 15th date of each month. The length of shadow of north wall for each day has been given in table 2a, and the theoretical shadow length for 15th day of each month is shown in table 2b. The measurement of shadow was done with the help of measuring tape at each hour of the day. Experimental solar fraction was evaluated by using the Equation (11). It has to be noted that the experiment has been conducted for each day w. e. f. 9:00am, because before that the greenhouse was covered by shadow of a nearby building.

Table 2a.Experimental shadow length (L, in m) of north wall of greenhouse for a typical day of typical months of the year

Time, (hr)	Jan. L, (m)	Jun. L, (m)
09:00	4.5	1.75
10:00	3.2	1
11:00	2.7	0.5
12:00	2.4	0.15
13:00	2.7	0.5
14:00	3.2	1
15:00	4.5	1.75

4. THERMAL MODELING

Thermal modeling of different components of greenhouses has been carried out under the following assumptions:
i- Analysis based on quasi-steady state conditions
ii- There is no radiative exchange between the walls and roofs of greenhouse due to small temperature differences
iii- The heat transfer from floor to the ground is one-dimensional

4.1 Greenhouse without Brick North Wall

Energy balance equations for different components of an empty even span greenhouse can be written as follows:

Floor:

$$\alpha_f(1-F_t)\sum_{i=1}^{6}(\tau Ii A'i) = h_{fr}A_f(T_f - T_r) + h_\infty A_f(T_f - T_\infty) \tag{14}$$

Room Air:

$$(1-\alpha_f)(1-F_t)\sum_{i=1}^{6}(\tau Ii A'i) + \rho F_t\sum_{i=1}^{6}(\tau Ii A'i) + h_{fr}A_f(T_f - T_r) = 0.33NV(T_r - T_a) + \sum_{i=1}^{6}(Ui A'i)(T_r - T_a) + M_a C_a \frac{dT_r}{dt} \tag{15}$$

Eliminating T_f from Eq. 14 and substituting for T_f in Eq. 15.

The final solution will be in the form of first order differential equation as follows:

$$\frac{dT_r}{dt} + aT_r = B(t) \tag{16}$$

where $a = \frac{(UA)_{eff}}{M_a C_a}$ and $B(t) = \frac{F(t) + (UA)_{eff} T_a}{M_a C_a}$

$$\left[(1-\alpha_f)(1-F_t) + \rho F_t + F_1\alpha_f(1-F_t)\right]\sum_{i=1}^{6}(\tau Ii A'i) = F(t) \text{ \&}$$

$$\left(U_f A_f + 0.33NV + \sum_{i=1}^{6}(Ui A'i)\right) = (UA)_{eff} \quad \text{and} \quad \frac{h_{fr}}{h_{fr} + h_\infty} = F_1$$

The analytical solution of equation (16) can be written as

$$T_r = \frac{\overline{B}(t)}{a}\left(1 - e^{-at}\right) + T_{ro}e^{-at} \tag{17}$$

Where, T_{ro} is greenhouse room temperature at t = 0 and $\overline{B}(t)$ is the average of $B(t)$ for the time interval between 0 and t and a is constant during the time.

4.2 Greenhouse with Brick North Wall

By assuming the heat conducted through the north wall equals to the heat transfer by convection to the atmosphere

North wall:

$$\alpha_n F_n \sum_{i=1}^{5}\left(\tau I_i A'_i\right) = h_{nr} A_{nw}\left(T_n - T_{rn}\right) + h_{na} A_{nw}\left(T_n - T_a\right) \tag{18}$$

Floor:

$$\alpha_f \left(1 - F_t\right)\sum_{i=1}^{6}\left(\tau Ii A'i\right) = h_{fr} A_f\left(T_f - T_{rn}\right) + h_{\infty} A_f\left(T_f - T_{\infty}\right) \tag{19}$$

Room Air:

$$\begin{aligned}&\left(1-\alpha_n\right)F_n \sum_{i=1}^{5}\left(\tau I_i A'_i\right) + \left(1-\alpha_f\right)\left(1-F_t\right)\sum_{i=1}^{5}\left(\tau Ii A'i\right) + h_{nr} A_{nw}\left(T_n - T_{rn}\right) \\ &+ h_{fr} A_f\left(T_f - T_{rn}\right) = 0.33NV\left(T_{rn} - T_a\right) + \sum_{i=1}^{5}\left(Ui A'i\right)\left(T_{rn} - T_a\right) + M_a C_a \frac{dT_{rn}}{dt}\end{aligned} \tag{20}$$

By eliminating T_n and T_f from Eqs. 18 and 19 respectively and substituting for them in Eq.20 the final solution will be in the form of first order differential equation as shown below:

$$\frac{dT_{rn}}{dt} + a' T_{rn} = B'\left(t\right) \tag{21}$$

where $a' = \dfrac{(UA)'_{eff}}{M_a C_a}$ and $B'\left(t\right) = \dfrac{F'\left(t\right) + (UA)'_{eff} T_a}{M_a C_a}$

$$\left[\left(1.-\alpha_n\right)F_n + \left(1-\alpha_f\right)\left(1-F_t\right) + \alpha_f\left(1-F_t\right)F_1 + \alpha_n F_n F_2\right]\sum_{1=1}^{5}\left(\tau I_i A'_i\right) = F'\left(t\right)$$

$$(UA)'_{eff} = \left[U_n A_{nw} + U_f A_f + 0.33NV + \sum_{i=1}^{5}\left(U_i A'_i\right)\right] \text{ and } F_2 = \frac{h_{nr}}{h_{nr} + h_{na}}$$

The analytical solution of equation (21) can be written as following

$$T_{rn} = \frac{\overline{B}'\left(t\right)}{a'}\left(1 - e^{-a't}\right) + T_{rno} e^{-a't} \tag{22}$$

where, T_{rno} is greenhouse room temperature at t = 0 and $\overline{B}'\left(t\right)$ is the average of $B'\left(t\right)$ for the time interval between 0 and t and a is constant during the time.

In the above thermal modeling to distinguish between the total solar fraction for greenhouse with and without brick north wall, we use different notation for the total solar fraction of each greenhouse, because it carries different values in each greenhouse due to presence and absence of brick north wall.

5. RESULTS AND DISCUSSION

Equation (11) has been used to evaluate the total solar fraction (F_t) for a typical day (15th day of month) of all months of the year. The different angles for a typical day of the winter and summer months have been given in table 1. These angles have been used to calculate the various parameters namely area of north (A_n), east or west (A_p) walls, and the theoretical shadow length (L_{th}) table 2b., etc.

Table 2b. Theoretical shadow length (L_{th}, in m) of north wall of greenhouse for a typical day of typical months of the year (Eq. 6)

Time (hr)	Jan. L_{th}, (m)	Jun. L_{th}, (m)
09:00	4.51	1.71
10:00	3.16	1.04
11:00	2.55	0.52
12:00	2.37	0.18
13:00	2.55	0.52
14:00	3.16	1.04
15:00	4.51	1.71

The hourly variations of total solar fraction (F_t) for typical winter and summer months of the year are shown in (figs. 3a-b).
Experimental solar fractions by using the data of table 2a.is also shown in the same figure.
However, it is obtained that the values at 9:00am, 10:00am and 11:00am are numerically similar to that at 3:00pm, 2:00 pm and 1:00pm respectively as expected (figs. 3a - 3b)
From (figs. 3a-b) it is clear that the total solar fraction is maximum during early morning and late evening due to low altitude of the sun position in winter (fig. 3a) as well as in summer (fig. 3b) condition. However, it is also important to notice that the total solar fraction is significantly reduced in summer due to highest altitude of the sun position.

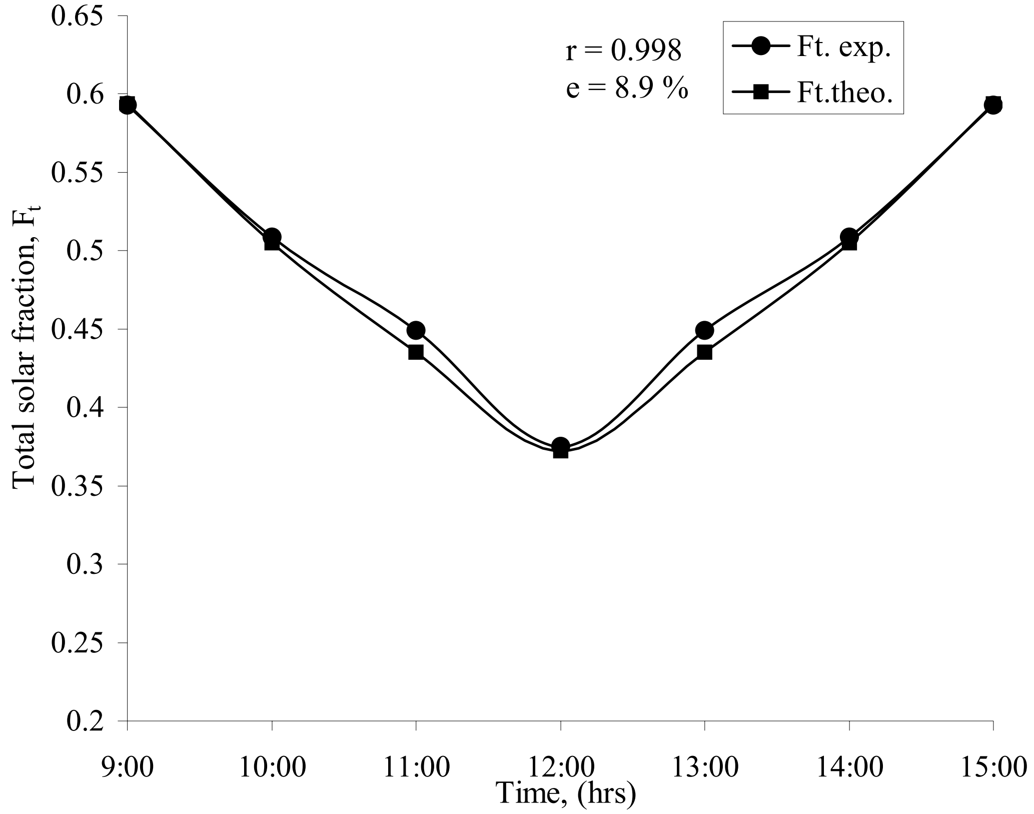


Figure 3a. Hourly variation of total solar fraction with time on 15th Jan. 2004 experimentally and theoretically at New Delhi

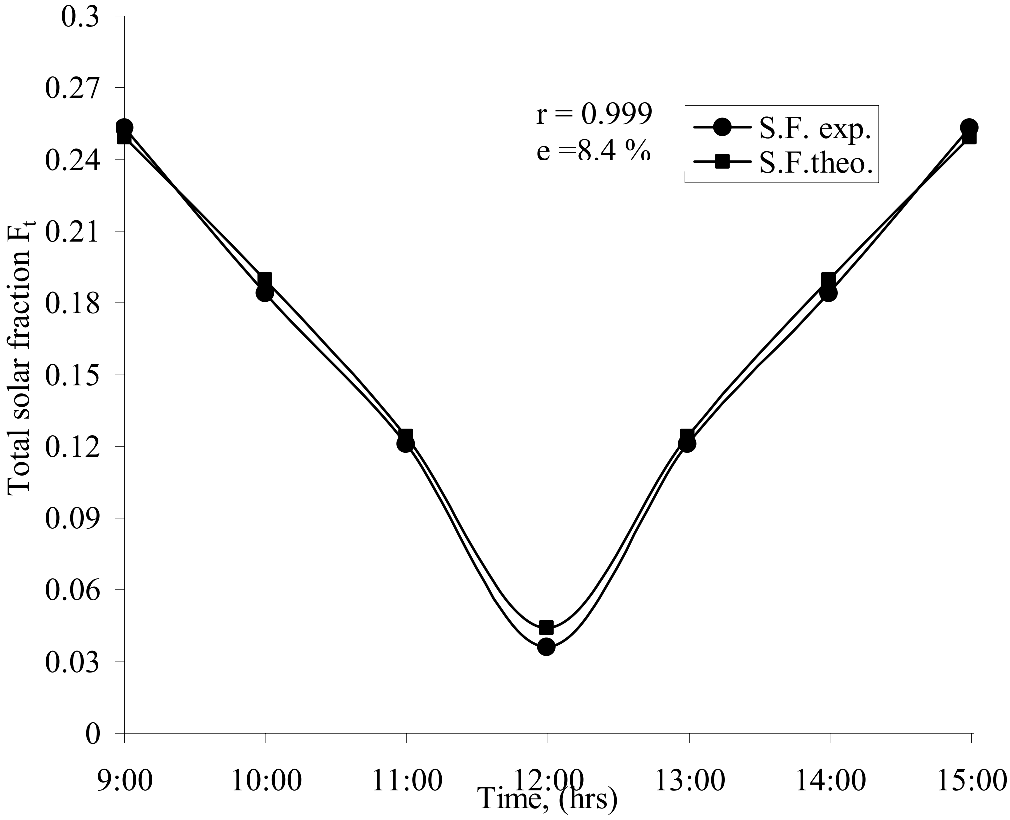


Figure 3b. Hourly variation of total solar fraction with time on 15th Jun. 2004 experimentally and theoretically at New Delhi

It can be observed that there is a fair agreement between experimental and theoretical results. This is also clear from coefficient of correlation values and square mean root deviation, which are calculated by using Eqs.(12-13) and shown for each curve. The root mean square deviation (e) varies between 8-9 % between experimental and theoretical values, which indicates the validation of the present model to evaluate total solar fraction. The total solar fraction varies throughout the day in all cases. This is in accordance with the results reported by Tiwari et al.(2003)

The variation of monthly average total solar fraction for each month has been shown in Figure 4.

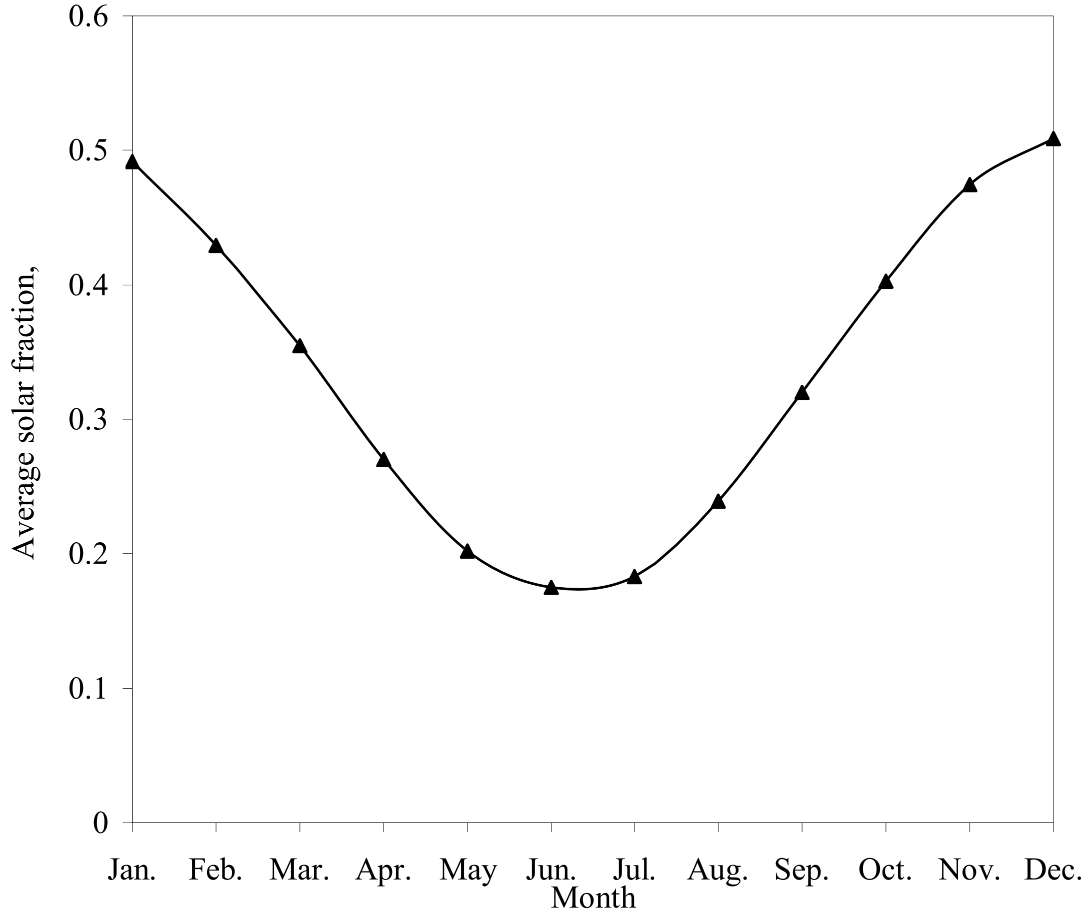


Figure 4. Monthly variation of average total solar fraction for New Delhi in the year 2004.

It is observed that the total solar fraction is minimum during summer period due to motion of sun overhead and maximum during winter period due to lower altitude angle of the sun. This also validates an earlier results obtained by Tiwari et al.(2003).
The obtained values of total solar fraction have been used 'as per thermal modeling section' to compute hourly variation of room air temperature for a typical day of winter and summer month using equations as per (Ghosal et al., 2004, Tiwari, 2003). The results are shown in (figs. 5).
It can be seen that there is a significant increase in room air temperature by 5°C in winter (Fig.5a) and 12°C in summer (Fig.5b).
It is due to more heat loss in winter in comparison with summer due to lower value of ambient air temperature and due to condensation on the greenhouse cover which takes place during winter period. Also the radiative and the convective heat exchange between the floor and other walls inside the greenhouse is negligible in winter due to low temperature inside the greenhouse compared to summer.

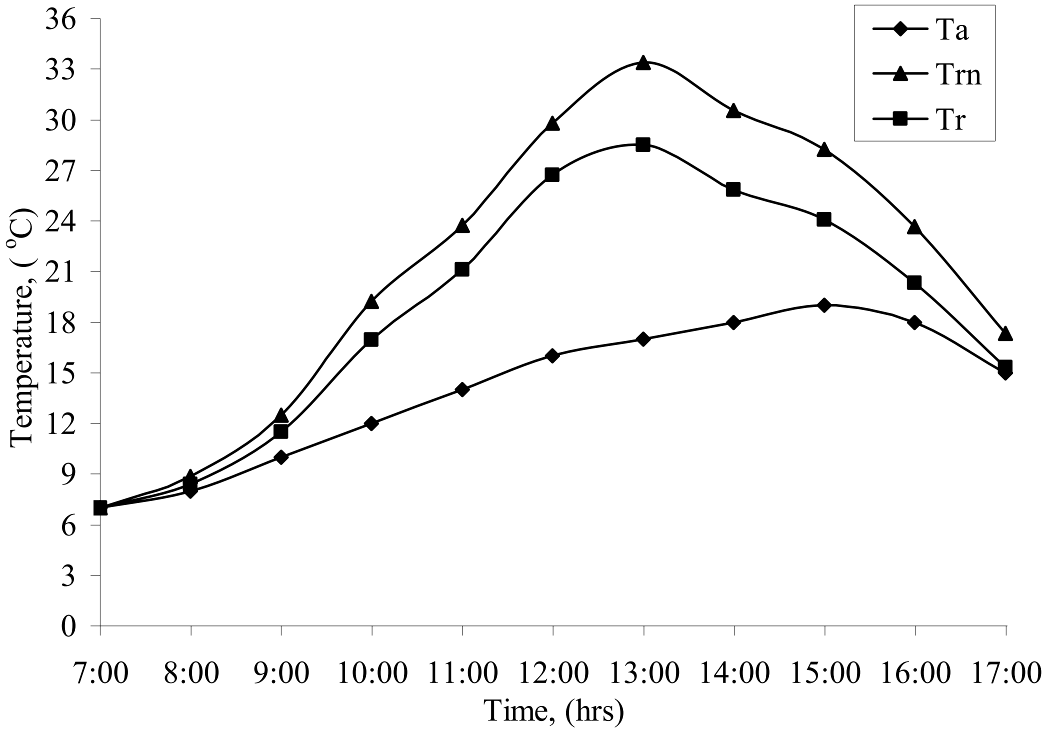


Figure 5a. Hourly Variation of ambient temperature (Ta) and room air temperatures in greenhouses without (Tr) and with brick north wall (Trn) during winter period at New Delhi (Jan.-2004)

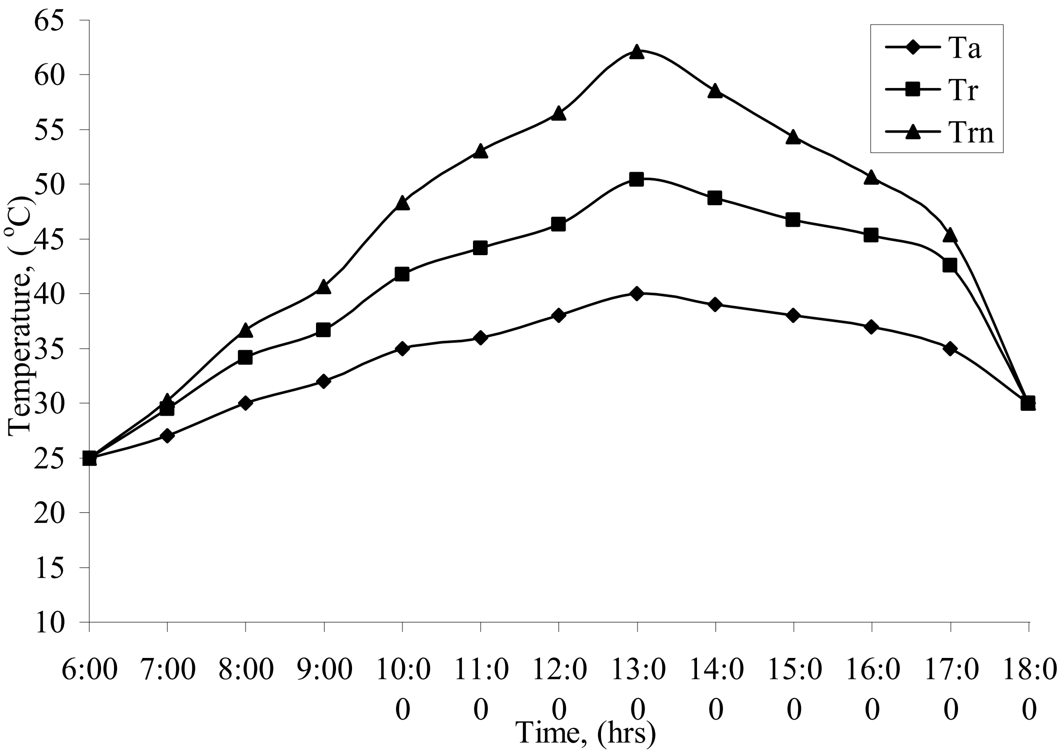


Figure 5b. Hourly Variation of ambient temperature (Ta) and room temperature in greenhouse without (Tr) and with brick north wall (Trn) during summer period at New Delhi (Jun.-2004)

6. CONCLUSION

On the basis of the results reported the following conclusions have been made:

(i) The total solar fraction for a specified day is high in the morning and evening and minimum at noon

(ii) The total solar fraction is maximum during winter period and minimum during summer period

(iii) The theoretical results of total solar fraction validates the experimental results, and

(iv) The room air temperature with retained solar fraction (north wall) is higher than that with wasted solar fraction (without north wall).

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8. Nomenclature

A_f area of floor of the greenhouse, m^2

A'_i area of ith section of greenhouse, m^2

A_i inside shadow area of greenhouse, m^2, ($A_i = A_f$)

A_n shadow area of north wall, m^2

A_{nw} area of north wall, m^2

A_o total outside shadow area of greenhouse, m^2

A_p shadow area of west wall (morning) or east wall (evening), m^2

A_t total (inside +outside) shadow area of greenhouse, m^2

C_a specific heat of air (J/kg°C)

e square mean deviation coefficient
F_n solar fraction due to north wall, (decimal)
F_t total solar fraction of greenhouse, (decimal), ($F_t = F_{to}$)
h_{fr} heat transfer coefficient from floor to the room air of the greenhouse (W/m^2°C)
h_{na} heat transfer coefficient from the brick north wall to the ambient (W/m^2°C)
h_{nr} heat transfer coefficient from brick north wall to the greenhouse air (W/m^2°C)
$h\infty$ heat transfer coefficient from floor to larger depth of the ground (W/m^2°C)
Ii solar radiation falling on inclined surfaces of greenhouse (W/m^2)
L length of shadow of north wall, m (measured with tape)
L_c length of shadow of plane passing through the center, m
L_{th} theoretical shadow length of north wall, m
M_a total mass of air in greenhouse enclosure (kg)
N Number of air change per hour
N' number of data
n number of day of the year
r coefficient of correlation
t_{solar} solar time,
T_a ambient temperature, °C
T_f temperature of the floor of greenhouse, °C
T_n temperature of the brick north wall surface, °C
T_r room air temperature of greenhouse without brick north wall, °C
T_{rn} room air temperature of greenhouse with brick north wall, °C
T_∞ temperature of the ground at large depth (assumed equal to Ta), °C
U_f overall heat transfer coefficient from greenhouse air to floor (W/m^2°C)
Ui overall heat transfer coefficient from greenhouse air to the ambient through greenhouse cover (W/m^2°C)
U_n overall heat transfer coefficient from greenhouse air to the ambient air through north wall (W/m^2°C
V Volume of greenhouse, m^3
X_i experimental values of total solar fraction
Y_i theoretical values of total solar fraction

8.1 Greek Symbols

α_f absorptivity of the floor
α_n absorptivity of the north wall
α_s solar altitude angle,
γ_s solar azimuth angle
δ declination angle
φ latitude angle
ρ reflectivity of the greenhouse canopy covers
τ transmitivity of the greenhouse canopy cover
ω hour angle

Abbreviation
exp. experimentally
theo. theoretically