Performance of equations to estimate the hourly actual vapor pressure

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Abstract: The objective of the present study was to analyze the performance of equations to estimate the hourly actual vapor pressure (e_a) in the regions of Paraná State, Southern Brazil, and in the main Brazilian climate types. Four equations were tested, being considered the equation that uses the relative humidity (*RH*) as standard. Hourly data series from automatic meteorological stations of the Brazilian National Institute of Meteorology (INMET) were used in the analyses, being 25 from the Paraná State (data measured between December 1, 2016 and November 8, 2018) and 8 representing the main Brazilians climate types (*Af, Am, Aw, Bsh, Cfa, Cfb, Cwa* and *Cwb*; data measured between December 12, 2018 and December 11, 2019). The association between standard and alternative equations was verified using linear regression analysis, correlation coefficient (*r*), index of agreement (*d*) and root mean square error (*RMSE*). The alternative equations did not differ from the standard equation in the locations of Paraná State (d = 1.0 and r = 1.0; $RMSE \leq 0.02$ kPa) and Brazilian climate types (d = 1.0; $r \geq 0.99$; $RMSE \leq$ 0.02 kPa). The equation to be used must be made considering the quality and availability of the necessary input data in each equation.

Keywords: water vapor, mathematical model, agrometeorology, Brazilian climates.

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1 Introduction

The water vapor derives from evaporation, which is the transition from the liquid phase to the vapor phase that occurs at a temperature lower than the boiling point. The pressure exerted by vapor on the liquid mass is denominated as vapor pressure (Gooch, 2011; Marshall, 2014; Speight, 2020). The atmospheric pressure is not exerted only by water vapor, there is a mixture of gases that compose it. According to Dalton's law of partial pressure, moist air behaves almost like an ideal gas (Webb et al., 1980; Callahan et al., 2019).

The evaporation process occurs when the liquid molecules overcome the force of attraction between each other and escape from the water layer, passing into vapor form. The process occurs until the air becomes saturated with water vapor. For each temperature, the equilibrium occurs at a specific vapor pressure, denominated saturation vapor pressure or maximum vapor pressure. The difference between the pressure exerted by the amount of water vapor in the air at a given time and temperature (e_a; actual vapor pressure) and the maximum pressure that can be reached under these conditions (e_s; saturation vapor pressure) is called a saturation vapor pressure deficit of the air (Δe). The Δe is directly related to the evaporation processes (air evaporative capacity), as it depends on the vapor

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pressure gradient between the evaporating surface and the air (Allen et al., 1998; Marek and Straub, 2001; Monteith and Unsworth, 2013).

Among other variables, the reference evapotranspiration (*ETo*) depends predominantly on Δe, and as the air temperature decreases, there is an increase in relative humidity and a decrease in reference evapotranspiration. Thus, studies involving air humidity are important to realize accurate *ETo* estimates. In humid climates of tropical regions, high relative humidity reduces the *ETo*, as the air is always close to saturation. Therefore, the humidity and air temperature are determinants in the vapor pressure accounting, which is an indicator of the evaporative capacity of the air (Bouzenada et al., 2017; Islam et al., 2019; Wang et al., 2020). The e_a is one of the most sensitive variables to estimate *ETo* (Hosseini et al., 2013; Islam et al., 2019).

The ASCE manual (ASCE-EWRI, 2005) presents some equations to calculate the e_a that considers different input data, such as relative humidity, dew point temperature and dry and wet bulb temperatures. However, there are no aspects in the literature showing the performance of these equations for different environmental conditions, such as climatic variation.

Due to the importance of vapor pressure, mainly considering its impact on more accurate *ETo* estimates, particularly in e_a, the objective of this study was to analyze the performance of equations to estimate hourly actual vapor pressure (e_a) in the regions of Paraná State, Southern Brazil, and in main Brazilian climate types.

2 Materials and methods

2.1 Equations to estimate the actual vapor pressure (e_a)

The literature commonly reports Equations 1 to 5 (ASCE-EWRI, 2005) to estimate the actual vapor pressure (e_a):

– e_a value calculated with relative humidity (Allen et al.,1998; Equation 1):

$$e_a = \frac{RH}{100} \cdot e^o(T) \tag{1}$$

– e_a value calculated with Tetens equation (Tetens, 1930; Equation 2):

$$e_a = 0.6108 \cdot exp\left[\frac{17.27 \cdot T_{dew}}{T_{dew}+237.3}\right] \tag{2}$$

– e_a value calculated with the average between the Tetens (1930) equation and relative humidity (Equation 3):

$$e_a = \frac{0.6108 \cdot exp\left[\frac{17.27 \cdot T_{dew}}{T_{dew}+237.3}\right] + \frac{RH}{100} \cdot e^o(T)}{2} \tag{3}$$

– e_a value calculated with average between temperature and relative humidity (Allen et al.,1998; Equation 4):

$$e_a = \frac{0.6108 \cdot exp\left(\frac{17.27 \cdot T_{min}}{T_{min} + 237.3}\right) \cdot \left(\frac{RH_{max}}{100}\right) + 0.6108 \cdot exp\left(\frac{17.27 \cdot T_{max}}{T_{max} + 237.3}\right) \cdot \left(\frac{RH_{min}}{100}\right)}{2} \tag{4}$$

Where: e_a – actual vapor pressure (kPa); *RH* – mean relative humidity (%); $e^o(T)$ – function of saturation vapor pressure (kPa); T_{dew} – dew point temperature (^oC); T – air temperature, which can be minimum, maximum or average (^oC).

In addition to the tests with Equations 1 to 4, due to the difficulty in obtaining the necessary data, only one case study was carried out with the equation that has the temperature of dry and wet bulbs as input (Equation 5):

– e_a value calculated with psychrometric data (ASCE-EWRI, 2005; Equation 5):

$$e_a = e^o(T_{dew}) - \gamma_{psy} \cdot (T_{dry} - T_{wet}) \tag{5}$$

Where: e_a – actual vapor pressure (kPa); $e^o(T_{dew})$ – function of saturation vapor pressure, considering dew point temperature (kPa); γ_{psy} – psychrometric constant (kPa $^oC^{-1}$); T_{dry} – dry bulb temperature (^oC); T_{wet} – wet bulb temperature (^oC).

The FAO method (Equation 1) was adopted as a standard (Allen et al., 1998) to verify the equation that best estimates the e_a. The Equation 1 uses relative humidity (*RH*) as an input. Hourly analyses were carried out associating the $e_{a\_standard}$ (Equation 1) versus $e_{a\_alternative}$ (Equations 2 to 5). All equations were inserted in an electronic spreadsheet, previously developed for this purpose.

2.2 Climates and data used

Most of the analyses in this study were carried out for Paraná State, located in the Southern region of Brazil, with predominant *Cfa* and *Cfb* climate types (Figure 1; Table 1; Maack, 2012). Data series from 25 automatic meteorological stations, obtained from the Brazilian National Institute of Meteorology (INMET) between December 1, 2016 and November 8, 2018, were used. The data used were: maximum and minimum air temperature (T; °C); maximum and minimum dew point temperature (T_{dew}; °C) of the air; and, maximum and minimum relative humidity (RH; %). Hourly periods that failed in some input variable in the analyzed equations were excluded. This criterion resulted in 2113992 effective data per equation, out of a total of 8455968 data from the series used.

The climatological normals of the Brazilian National Department of Meteorology (DNMET, 1992) served as a parameter to verify if the weather data used in the analysis well represents the regional climate trend in the Paraná State. The climatological normals are monthly averages of thirty years (1961-1990), calculated according to the criteria recommended by the World Meteorological Organization (WMO).

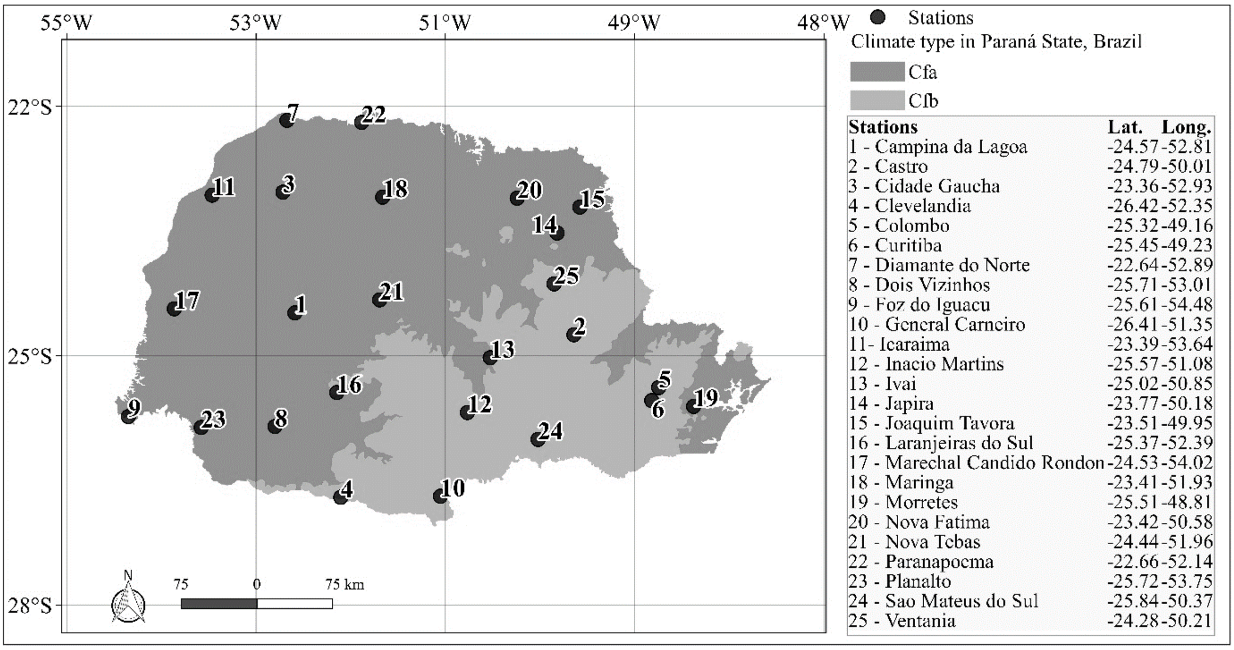


Figure 1 Paraná State containing the predominant climate classification and position of the automatic meteorological stations analyzed

Analyses of Brazilian climate types were performed using hourly data from eight automatic stations of the INMET between November 12, 2018 and November 11, 2019, each station being located in a different climate type (*Af*, Manaus-AM, latitude -3.11^{o}S and longitude -62.01^{o}W; *Am*, Macapá-AP, latitude 0.03^{o}N and longitude -52.01^{o}W; *Aw*, Cristalina-GO, latitude -16.78^{o}S and longitude -47.61^{o}W; *BSh*, Petrolina-PE, latitude -9.39^{o}S and longitude -40.52^{o}W; *Cfa*, Porto Alegre-RS, latitude -30.05^{o}S and longitude -51.17^{o}W; *Cfb*, Curitiba-PR, latitude -25.45^{o}S and longitude -49.23^{o}W; *Cwa*, Barbacena-MG, latitude -21.22^{o}S and longitude -43.77^{o}W; *Cwb*, Uberaba-MG, latitude -19.71^{o}S and longitude -47.96^{o}W).

The selected climate types (Table 1) are the most representative in Brazil, according to the Köppen's (1936) climate classification (Alvares et al., 2013). Hourly data of maximum and minimum air temperature (T; °C), maximum and minimum dew point temperature (T_{dew}; °C) of the air, and maximum and minimum relative humidity (RH; %) were obtained in each weather station. The data series of the eight stations had 210240 readings. However, when any of these variables was not available, it was decided to exclude the respective hour. Thus, the final composition of the database was 193902 hours reading, resulting from the exclusion of 7.8% of the hours analyzed.

The case study used data series from the automatic meteorological stations in Castro, Curitiba, Ivaí and Maringá (Figure 1), belonging to the INMET, between December 1, 2016 and December 1, 2017. The analyzed period was chosen due to the least number of failures in the necessary data, considering the four weather stations analyzed. Only these four meteorological stations provide, for Paraná State, the wet bulb data needed for the analyses carried out.

Table 1 Characterization of the Brazilian climates analyzed.

Climate	Description of each climate
Af	Tropical without dry season, with the average temperature of the warmest month exceeding 18°C. The total rainfall in the driest month is over 60 mm, with the highest rainfall from March to August, exceeding 1500 mm per year;
Am	Tropical monsoon, with annual precipitation above 3300 mm and annual average temperature of 27.6°C, seasonally varying between 25.8°C and 29.0°C;
Aw	Tropical with dry winter season, precipitation between 1600 and 1900 mm per year and annual average temperature between 19°C and 20°C;
BSh	Dry semi-arid, low latitude and altitude, characterized by shortage and irregular rainfall distribution, which are 250 to 750 mm per year, and annual average temperature of 27°C;
Cfa	Humid subtropical with hot summer, with good rainfall distribution throughout the year, on average 1500 mm, and annual average temperature of 19°C;
Cfb	Humid subtropical with temperate summer, with well-distributed rainfall throughout the year, exceeding 1200 mm, and mild summers with an annual average temperature of 17°C;
Cwa	Humid subtropical with dry winter and hot summer, annual precipitation below 700 mm, with January being the warmest month and July the coldest month, with averages temperatures of 23.5°C and 17.5°C, respectively;
Cwb	Humid subtropical with dry winter and temperate summer, with annual average rainfall of 700 mm and annual average temperature of 19.3°C.

Source: Adapted from Alvares et al. (2013).

The variables used in the case study on a daily periodicity were: average air temperature at dew point (T_{dew}); and, dry (T_{dry}) and wet (T_{wet}) bulb temperatures. In which T_{dry} and T_{wet} were collected with three (0, 12 and 18 h) or two (0 and 12 h) measurements per day, being averaged to obtain daily values. The e_a values calculated with the Equation 5 were compared with the e_a daily values, obtained with the average of 24 h of e_a, calculated with Equations 1 to 4.

2.3 Statistical analysis of the analyzed equations

The results obtained with the actual vapor pressure (e_a) estimation equations, including the case study, were compared and verified in linear regression analysis, as well as with the main errors, indexes and coefficients recommended in the literature (Equations 6 to 8; Jacovides and Kontoyiannis, 1995).

$$RMSE = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} \left(Y_{p_i} - Y_{a_i}\right)^2} \quad (6)$$

$$r = \frac{\sum_{i=1}^{n}[(Y_{p_i}-\bar{Y}_p)\cdot(Y_{a_i}-\bar{Y}_a)]}{\sqrt{\sum_{i=1}^{n}(Y_{p_i}-\bar{Y}_p)^2 \cdot \sum_{i=1}^{n}(Y_{a_i}-\bar{Y}_a)^2}} \quad (7)$$

$$d = 1 - \frac{\sum_{i=1}^{n}\left(Y_{a_i}-Y_{p_i}\right)^2}{\sqrt{\sum_{i=1}^{n}\left(\left|Y_{a_i}-\bar{Y}_p\right|\cdot\left|Y_{p_i}-\bar{Y}_p\right|\right)^2}} \quad (8)$$

Where: RMSE – root mean square error (kPa) r – Pearson's correlation coefficient (dimensionless); d – index of agreement from Willmott (1982) (dimensionless); Y_{p_i} – e_a values obtained with the standard method at each i-hour (kPa); Y_{a_i} – e_a values obtained with the alternative equation at each i-hour (kPa); n – number of analyzed hours (dimensionless); $\bar{Y}_p$ – average of e_a values obtained with the standard method for all analyzed hours (kPa); $\bar{Y}_a$ – average of e_a values obtained with the alternative equations for all analyzed hours (kPa).

The linear regressions, indexes and errors were obtained with software R (version 3.1), packages "ggplot2" version 3.3.2 and "hydroGOF" version 0.04, respectively (Zambrano-Bigiarini, 2017; Wickham et al., 2020).

3 Results and discussion

3.1 Trends of the climate variables

The lowest air temperatures were observed between May and October in the two predominant climates in Paraná State (Figure 2). There was also a small variation in *RH* throughout the year in the *Cfa* and *Cfb* climates.

In the eight Brazilian climate types analyzed, the monthly average trend of air temperature (T, T_{dew}) indicated small variation in *Af*, *Am* and *BSh* climates throughout the year (Figure 3). The other climates (*Aw*, *Cwa*, *Cwb*, *Cfb* and *Cfa*) showed lower values between May and October. The *RH* showed the lowest values between August and October for *Cwb*, *Aw* and *Af* climates. The other climates (*Aw*, *Cwa*, *Cwb*, *Cfb* and *Cfa*) showed small variation in the monthly values of *RH* throughout the year.

The dry (T_{dry}) and wet (T_{wet}) bulb temperatures in the case study carried out in the Maringá (*Cfa* climate), Castro, Curitiba and Ivaí (*Cfb* climate) stations, showed a very similar trend in the considered climates. The lowest temperatures occurred between May and October. The relative humidity in the *Cfa* climate showed similar trend to the temperatures. However, it was observed in the *Cfb* climate that there was small variation in *RH* throughout the year (Figure 4).

The temperatures and relative humidity used in the analyses, from the locations of Paraná (Figure 2), Brazil (Figure 3) and case study (Figure 4), had similar trends to the climatological normals (period from 1961 to 1990) of the Brazilian National Department of Meteorology (DNMET, 1992), showing that the period used to carry out the analyses does not include an atypical period of climatological data.

3.2 Equations to estimate hourly actual vapor pressure (e_a) in Paraná State

The hourly trend of e_a estimated with the standard equation (Equation 1) for the *Cfa* and *Cfb* climates showed average values above 1.50 kPa, with the lowest values occurring in the *Cfb* climate (Figure 5). The associations between e_a values estimated with standard (Equation 1) and alternative (Equations 2 to 4) equations were very close (d = 1.0 and r = 1.0 for all stations) and showed very small errors in all 25 automatic weather stations in Paraná State (Table 2; $RMSE \leq 1.60$ kPa). The result indicated that it is possible to use any of the analyzed equations, without considerable changes in the e_a results. Therefore, the equation choice to calculate e_a should be based on the quality and availability of the input variables data required in Equations 1 to 4.

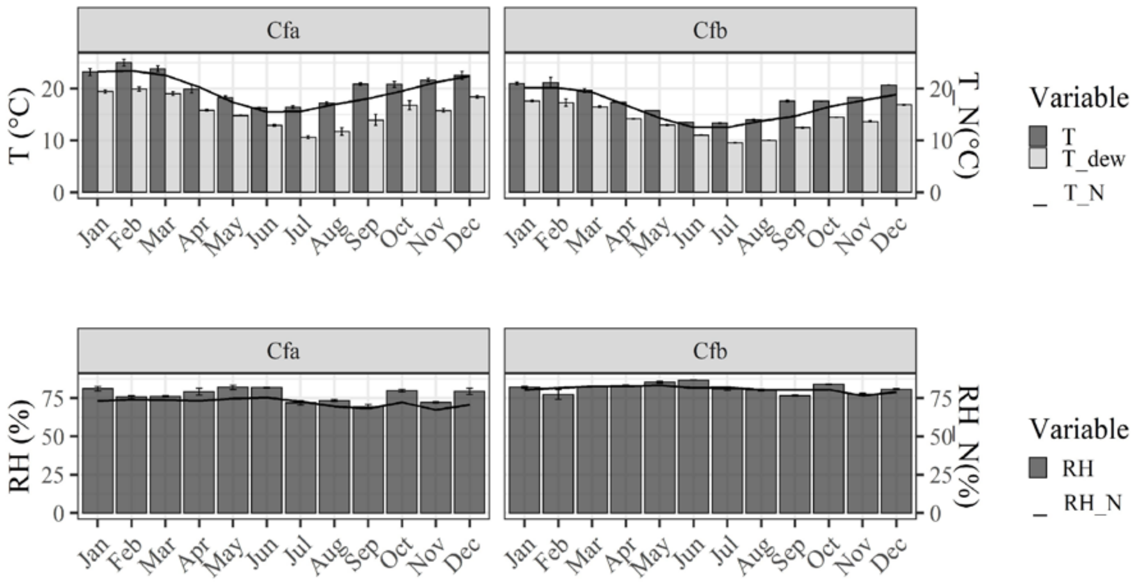


Figure 2 Monthly average and standard error of relative humidity (*RH*), air (*T*) and dew point (T_{dew}) temperatures in Paraná State, in *Cfa* and *Cfb* climates (between December 1, 2016 and November 8, 2018), having *RH* and *T* Normal (*RH_N* and *T_N*; 1961 to 1990) for comparison

By grouping the automatic stations of the locations analyzed in Paraná, according to the climate classification (Figure 6 and Table 2), there was an excellent linear association between the e_a values estimated with the standard and alternative equations. The regressions performed with the F test were significant at 5% probability, with p-value < 0.001 (Figure 6). Although Equation 2 provided the highest *RMSE* for the *Cfb* climate (0.02 kPa), it was still very low. All associations between standard and alternative equations showed an index of agreement d = 1.0 and a correlation coefficient r = 1.0. Therefore, any of the tested equations can be used to calculate the actual vapor pressure. Cai et al. (2007) comparing e_a values calculated with the Tetens equation (Equation 2) in four climates (arid, semiarid, semi-humid, humid) in China, observed an index of agreement $d > 0.93$ and $R^2 > 0.85$, with the less good estimations stations with arid climates. The best results were obtained for humid and semi-humid climates, as in humid conditions it is highly probable that the minimum air temperature is equal to the temperature at the dew point. The results of the present study agree with the Cai et al. (2007) considerations, since the *Cfa* and *Cfb* climates are humid subtropical.

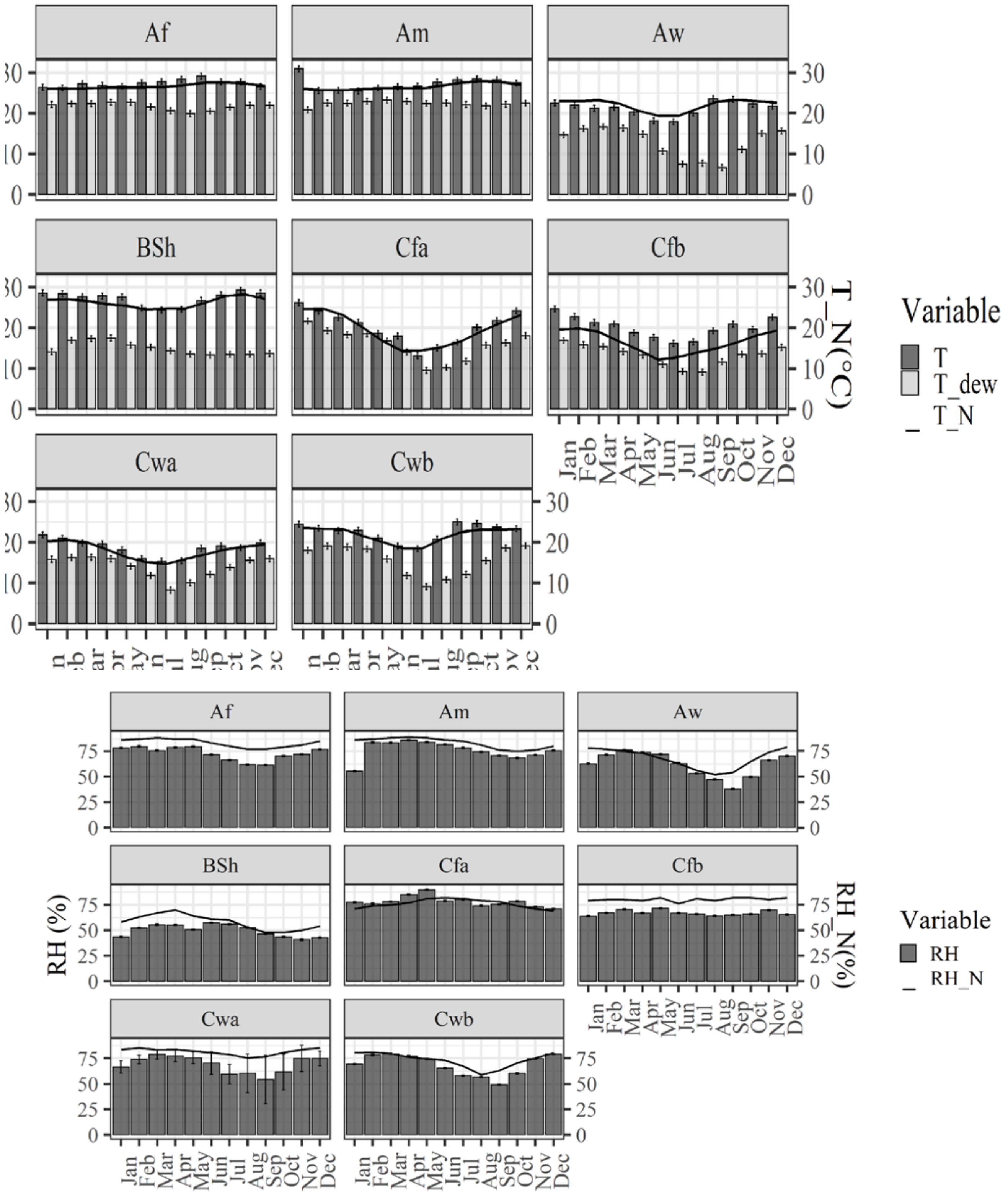


Figure 3 Monthly average and standard error of relative humidity (*RH*), air (*T*) and dew point (T_{dew}) temperatures, in the eight Brazilian climate types (*Af*, *Am*, *Aw*, *Bsh*, *Cfa*, *Cfb*, *Cwa* and *Cwb*; between December 12, 2018 and December 11, 2019), having *RH* and *T* Normal (*RH_N* and *T_N*; 1961 to 1990) for comparison.

The close association between the estimation actual vapor pressure equations (25 meteorological stations or average of *Cfa* and *Cfb* climates), verified in the indexes/errors (Table 2) and linear regression analysis (Figure 7), can be explained by analyzing the input variables of each equation.

Lawrence (2005) and Górnicki et al. (2017) consider that there is a dependence and direct relationship between the dew point temperature (T_{dew}) and relative humidity. The statement explains the close results verified in the e_a estimates with Equations 2, 3 and 4.

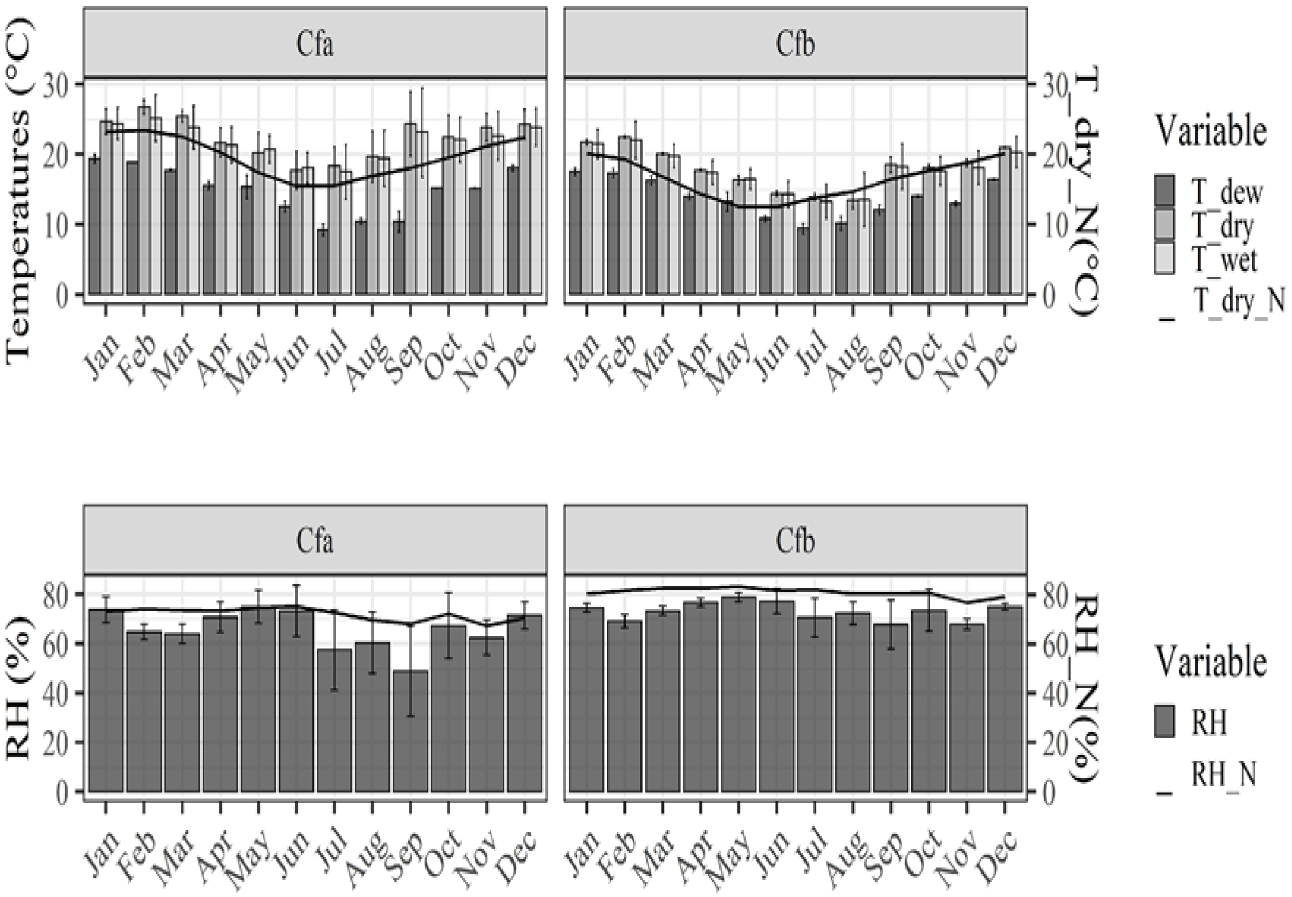


Figure 4 Monthly average and standard error of relative humidity (*RH*), dry (T_{dry}) and wet (T_{wet}) bulb temperatures and dew point (T_{dew}) temperatures in Maringá (*Cfa* climate) and Castro, Curitiba and Ivaí (*Cfb* climate) stations, between December 1, 2016 and December 1, 2017, having *RH* and T_{dry} Normal (*RH_N* and *T_dry_N*; 1961 to 1990) for comparison

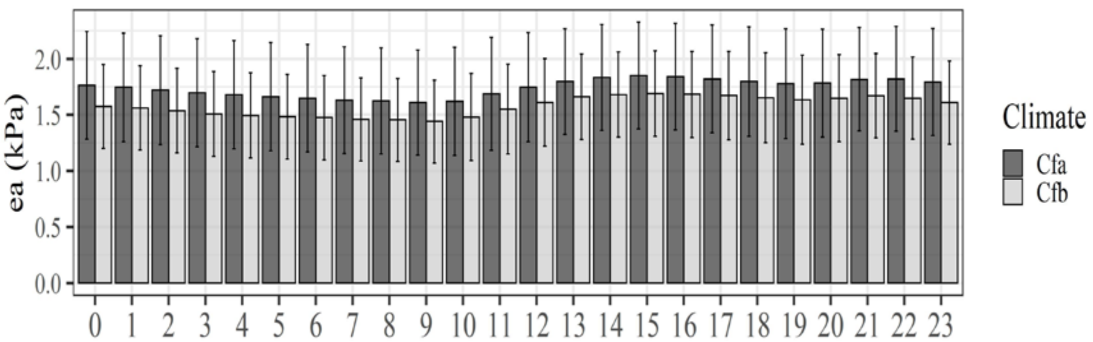
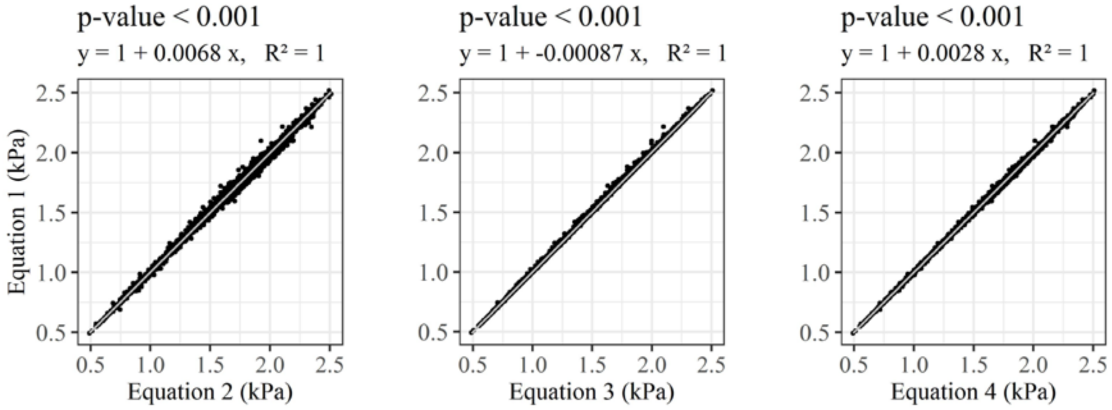


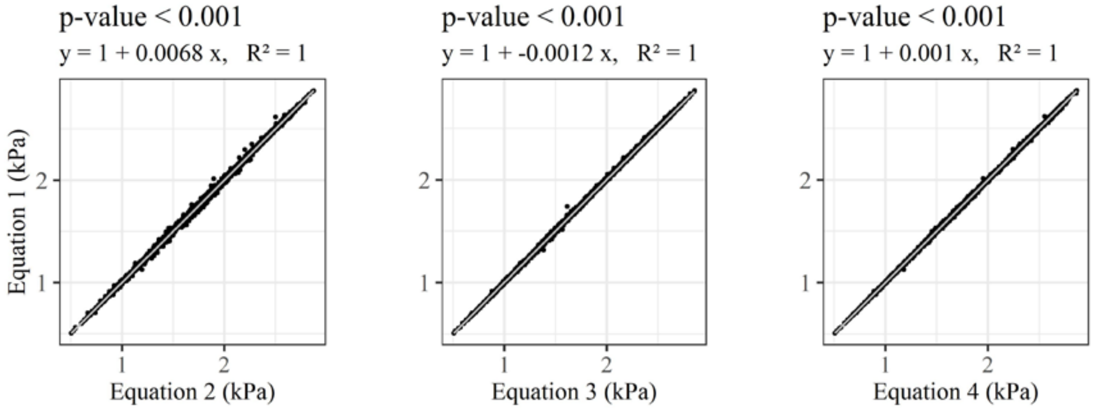
Figure 5 Hourly average and standard error of e_a in Paraná State, obtained with Equation 1 (standard), considering the stations or locations grouped in *Cfa* and *Cfb* climates (between December 1, 2016 and November 8, 2018)

The *RH* is defined as the relationship between the amount of water vapor retained in a sample of moist air, at a given temperature, and the maximum amount of water vapor that this same air could retain, at the same temperature (Bradley, 2015; Cai, 2019). Thus, as the *RH* depends on temperature, there is also the reason why there was no difference between Equations 1 and 4.

It is important to note that the input variables in Equations 1 to 4 were measured on different sensors and equipment for each weather station. Thus, it was observed that the measuring equipment of the analyzed stations are returning consistent environmental measures (temperatures and relative humidity), as data errors could impair the performance of associations between $e_{a\_standard}$ and $e_{a\_alternative}$, which was not observed.



a)



b)

Figure 6 Linear regression analysis between actual vapor pressure values (e_a) estimated with standard (Equation 1) and alternative (Equation 2 to 4) equations, in the predominant climate types in Paraná State: a) *Cfa*; b) *Cfb*.

Table 2 Root mean square error (RMSE; kPa), correlation coefficient (r; dimensionless) and index of agreement (d; dimensionless) obtained in the associations between $e_{a\_standard}$ (Equation 1) vs. $e_{a\_alternative}$ (Equations 2 to 4) in hourly periodicity, for Cfa and Cfb climate

Stations	--- Eq. 1 vs. Eq. 2 ---			--- Eq. 1 vs. Eq. 3 ---			--- Eq. 1 vs. Eq. 4 ---		
	RMSE	*r*	*d*	*RMSE*	*r*	*d*	*RMSE*	*r*	*d*
Campina da Lagoa	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Castro	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Cidade Gaúcha	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Clevelândia	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Colombo	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Curitiba	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Diamante do Norte	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Dois Vizinhos	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Foz do Iguaçu	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
General Carneiro	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Icaraíma	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Inácio Martins	0.01	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Ivaí	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Japirá	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Joaquim Távora	0.02	1.0	1.0	1.60	1.0	1.0	0.01	1.0	1.0
Laranjeiras do Sul	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Marechal Cândido R.	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Maringá	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0

Morretes	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Nova Fátima	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Nova Tebas	0.02	1.0	1.0	0.00	1.0	1.0	0.01	1.0	1.0
Paranapoema	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Planalto	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
São Mateus do Sul	0.02	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Ventania	0.01	1.0	1.0	0.01	1.0	1.0	0.01	1.0	1.0
Stations with Cfa climate	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
Stations with Cfb climate	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00

3.3 Equations to estimate hourly actual vapor pressure (e_a) in Brazilian climates

Equation 1 presented higher magnitudes of e_a values (close to 3.00 kPa; Figure 7) for *Am* and *Af* climates. The other climates (*Aw*, *Cfa*, *Cfb*, *Cwa*, *Cwb*, and *BSh*) had e_a values between 1.50 and 2.00 kPa.

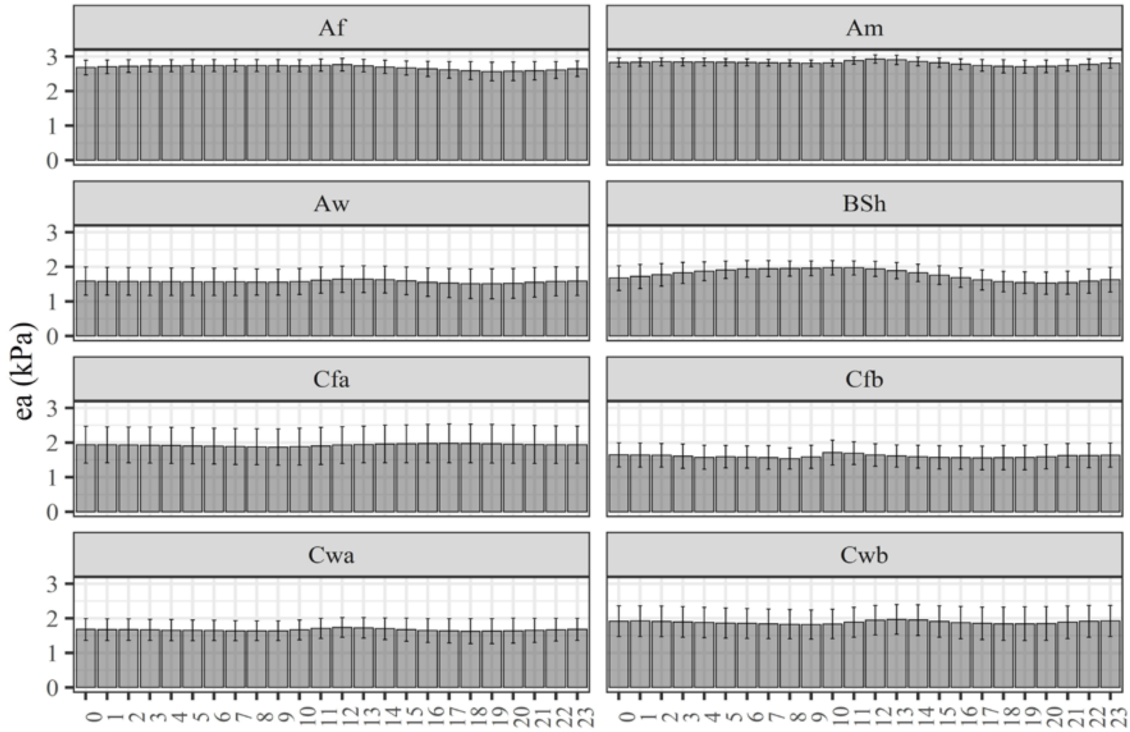


Figure 7 Hourly average and standard error of e_a in the Brazilian climate types (*Af*, *Am*, *Aw*, *BSh*, *Cfa*, *Cfb*, *Cwa* and *Cwb*), with Equation 1 (standard), between December 12, 2018 and December 11, 2019

The analyses for the eight Brazilian climate types also indicated that there were almost no differences between the values of $e_{a\_\text{standard (Equation 1)}}$ versus $e_{a\_alternative}$ (Equations 2 to 4). The associations were narrow, with indexes d = 1.0, correlations $r \geq 0.99$ and $RMSE \leq 0.02$ kPa (Table 3).

The results obtained for the Brazilian climate types (Table 3) indicated that, regardless of the climate, the alternative (Equations 2 to 4) and standard (Equation 1) equations showed very high correlation in estimating e_a. Allen et al. (1998) recommends in periods of one-week, ten-days or a month, values of e_a calculated using average measurements over the period. According to the authors, the procedure allows the failures in the input data be "diluted", without compromising the results of e_a. For this reason, Equation 1 is considered standard in estimating e_a. As Equations 2 to 4 showed a very good regression results in relation to Equation 1 (standard method), the results reinforce the quality in the database used.

Lyra et al. (2004) evaluating the influence of the vapor pressure deficit on the reference evapotranspiration, considered that the methods that used the average of *RH* to calculate the actual vapor pressure presented more narrow correlations. This aspect was not observed in the present study, since the tested equations presented close correlations and excellent indexes of agreement values (Tables 2 and 3) in the estimation of hourly actual vapor pressure (e_a), for Brazilian climate types.

Jerszurki et al. (2017) developing an alternative method to estimate *ETo*, obtained high variability in the saturation deficit (Δe) between Brazilian climate types, mainly between humid and arid climates. Despite the variation of Δe between climates, justified by the difference in temperatures and relative humidity (Figures 3, 4 and 5), the equations tested in the present study (Equations 1 to 4) were adequate in any of the eight climates analyzed. Therefore, the equation to be used to calculate the actual vapor pressure must be chosen according to the availability and quality of the climatic data series of the place to be analyzed.

Table 3 Root mean square error (*RMSE*; kPa), correlation coefficient (*r*; dimensionless) and index of agreement (*d*; dimensionless) obtained in the associations $e_{a\_standard}$ (Equation 1) vs. $e_{a\_alternative}$ (Equations 2 to 4) in hourly periodicity, for the Brazilian climate types

Climate	Locations	Standard equation vs. Alternative equation	*RMSE*	*d*	*r*
Af	Manaus-AM	Equation 1 vs. Equation 2	0.02	1.00	0.99
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
Am	Macapá-AP	Equation 1 vs. Equation 2	0.02	1.00	0.99
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
Aw	Cristalina-GO	Equation 1 vs. Equation 2	0.02	1.00	1.00
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
BSh	Petrolina-PE	Equation 1 vs. Equation 2	0.02	1.00	1.00
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
Cfa	Porto Alegre-RS	Equation 1 vs. Equation 2	0.02	1.00	1.00
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
Cfb	Curitiba-PR	Equation 1 vs. Equation 2	0.02	1.00	1.00
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
Cwa	Barbacena-MG	Equation 1 vs. Equation 2	0.02	1.00	1.00
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00
Cwb	Uberaba-MG	Equation 1 vs. Equation 2	0.02	1.00	1.00
		Equation 1 vs. Equation 3	0.01	1.00	1.00
		Equation 1 vs. Equation 4	0.01	1.00	1.00

3.4 Case study with the daily actual vapor pressure (e_a) estimation with Equation 5

In the case study with the Equation 5, wet bulb temperature data was needed. Due to the lack of hourly data, comparative analyses of e_a estimates in the locations of Castro, Curitiba, Ivaí and Maringá, Paraná State, were possible only for daily periodicity (Table 4).

Table 4 Root mean square error (*RMSE*; kPa), correlation coefficient (*r*; dimensionless) and index of agreement (*d*; dimensionless) obtained in the associations between $e_{a\_standard}$ (Equation 1) vs. $e_{a\_alternative}$ (Equations 2 to 5) in daily periodicity, in four locations

Associations	Stations	*RMSE*	*r*	*d*
Equation 1 vs. Equation 2	Castro	0.00	1.00	1.00
	Curitiba	0.01	1.00	1.00

	Ivaí	0.00	1.00	1.00
	Maringá	0.00	1.00	1.00
Equation 1 vs. Equation 3	Castro	0.00	1.00	1.00
	Curitiba	0.06	0.99	0.99
	Ivaí	0.23	0.89	0.92
	Maringá	0.00	1.00	1.00
Equation 1 vs. Equation 4	Castro	0.00	1.00	1.00
	Curitiba	0.00	1.00	1.00
	Ivaí	0.00	1.00	1.00
	Maringá	0.00	1.00	1.00
Equation 1 vs. Equation 5	Castro	0.11	0.97	0.98
	Curitiba	0.24	0.91	0.89
	Ivaí	0.19	0.97	0.95
	Maringá	0.35	0.96	0.89

There were close associations between standard (Equation 1) and alternatives (Equation 2 to 5) equations in Castro, Curitiba, Ivaí and Maringá, with $RMSE \leq 0.35$ kPa, index of agreement $d \geq 0.89$ and correlation $r \geq 0.89$, for daily periodicity. With the exception of Ivaí station, the other stations showed higher errors with Equation 5 for e_a estimates. However, the highest errors were not significant to impair the correlations and concordances observed in the four weather stations tested.

Although the statistical results of the daily e_a estimates (Table 4) were very good, with the exception of Ivaí station, the other locations had with Equation 5 the worst indexes and errors in the e_a estimation. Therefore, it is understood that more in-depth analysis would be important, encompassing a larger number of stations with wet bulb measurements, to be more representative. However, such analyzes will be possible only when the automatic weather stations have wet bulb temperature measurements. In this way, with the possible analyses, the results of the daily e_a estimates also indicated that any of the tested equations can be used to calculate e_a. It is believed that the use of Equation 5 is currently unfeasible on a large area scale. The reason is due to the difficulty in obtaining wet bulb data for a large number of locations (that is why only four stations were used in Paraná State), as well as the existing data are not in the hourly basis (reason why we chose to perform the analyses on a daily periodicity).

Based on the results and statistical indexes obtained in the present study, having reliable meteorological data, it is considered that the tested equations (Equations 2 to 4) can be used in locations with *Af*, *Am*, *Aw*, *BSh*, *Cfa*, *Cfb*, *Cwa* and *Cwb* climate types, without considerable changes in the e_a results.

4 Conclusions

The four alternative equations tested – Equations 2 (variable: T_{dew}), Equation 3 (variables: T_{dew}, RH and $e^o(T_{dew})$) and Equation 4 (variables: T_{min}, T_{max}, RH_{min} and RH_{max}) –, used to calculate the actual vapor pressure in hourly periodicity, did not differ statistically in the 25 climatological stations analyzed in Paraná State. The tested equations (Equations 2 to 4) also showed no difference for the eight Brazilian climate types: *Af* (Manaus-AM); *Am* (Macapá-AP); *Aw* (Cristalina-GO); *BSh* (Petrolina-PE); *Cfa* (Porto Alegre-RS); *Cfb* (Curitiba-PR); *Cwa* (Barbacena-MG) e *Cwb* (Uberaba-MG). The Equations 2 to 4 to be used to calculate the actual vapor pressure must be chosen according to the availability of the climatic data series;

The alternative equations tested (Equations 2 to 5), used to calculate the actual vapor pressure on a daily periodicity, did not show any difference in the climatological stations of Castro, Curitiba, Ivaí and

Maringá, located in Paraná State. The use of Equation 5 (variables: $e^o(T_{dew})$, T_{dry} and T_{wet}) is currently unfeasible on a large scale, due to the difficulty in obtaining wet bulb data on an hourly and daily basis for a large number of locations.

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