Quick Method for Determination of Random Distributions Parameters

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ABSTRACT

In this article the method is offered and some results of research of the law of probability distribution of the random variables in agrotechnological processes is proposed. The purpose of this research was the description of the processes that occur in the field under the influence of agrotechnological processes and soil compaction particularly. The goal was to acquaint a broad audience of researchers with the results of theoretical research in which an analysis of the law of distribution of random variables has been made. For determining the rational parameters related to precision farming requirements, our work is based on principles developed by St.-Petersburg´s school. The most informative indexes of estimation and the approach of various scientists to the problem have shown that not enough attention has been paid to the laws of distribution of the above-stated variables. An approximation by the normalised gamma-distribution was made to describe the distributions of records of the suitable results concerning agrotechnological processes. The relation to bulk density of the soil is shown. Finally, the basic diagnostic parameter for the estimation of soil compaction is presented.
Keywords: Bulk density, soil compaction, agrotechnological process, random process, normalization, gamma-distribution

1. INTRODUCTION

We are living in a random world, but for us this is quite a lucky state that in most cases we can consider it as randomized. In some cases we can predict and partially control random processes. Usually we are saying that agriculture is a complex of reproduction processes with limited controllability. If this is true, then the general function of scientific agricultural research is to increase these limits of controllability. However, agricultural management is engaged at various levels and each level has its own group of interest, and each group tries to control the agriculture. The result is a clash of interests between producers and consumers, i.e. all taxpayers. Unfortunately, now the Estonian scientific research is undercapitalized; therefore, it is impossible to carry out a transition to a new paradigm which is supported by the theory of probability.

Most methods of field experiments are based on the presumption that the measurable parameters conform to normal distribution. This is convenient for all scientists because normal distribution enables application of standard methods and analysis. However, application of the additional technique of the gamma-distribution allows one to carry out the above mentioned distributions as stochastic processes. The low distribution may not have a normal distribution (Ventsel, 2000; 2002, Feller, 1971) because the factors that influence yield (e.g. soil compaction) have at least one physical limit. Concerning assessment of soil compaction of Estonian soils (Nugis et al., 2004) before compaction, the expected physical limit of bulk density may be within the range 0.87…0.9 Mg m^{-3} and after compaction it increased up to 1.70…1.89 Mg m^{-3}. For crop growing conditions the optimum bulk density remains within the range of 1.15...1.45 Mg m^{-3}.

In our research, we emphasize the agroecological aspects of the problem which, we feel, have not been investigated sufficiently. Therefore, to evaluate this problem more deeply we have applied the methods of mathematical statistics (Lurie, 1970; Lurie *et al*., 1970; Lurie, Grombchevski, 1970; Lurie, Liubimov, 1970).

A thorough literature survey revealed other publications containing the data assessed by mathematical statistics (Fadel, 2004; Li and Lin, 2004; Kumar and Dewangan, 2004).

The results of introducing this model have been presented as manuscripts earlier in Germany (4ECPA/1ECPLF Conference, 2003) Australia in ISTRO 16th Triennial Conference, 2003 (Nugis *et al*., 2003) and in two International Conferences at St.- Petersburg (Müüripeal *et al*., 2002; 2005).

2. MATERIAL AND METHODS

The objective of this investigation was to study agrotechnical processes related to changes of soil physical properties as a result of soil compaction and its assessment by methods of mathematical statistics involving gamma-distribution.

Field experiments in Central Estonia at Adavere, 58°43´N, 25°54´E (Jõgeva County) and in Western Estonia (Pärnu County) at Reiu (58°19´N, 24°38´E) were carried out (co-author Rein Lehtveer). Soil samples were taken by Litvinov cylinders ($50 cm^3$) from the layers (0-15 cm, 15-25 cm and 25-35 cm) of soil and after drying, the bulk density was determined (Mg m^{-3}). The above mentioned method was improved to some extent: energy consumption (GJ ha^{-1}) by Proctor apparatus - PST and soil penetration resistance (kN cm^{-2}) by means of the Alexeiev penetrometer (Alex) were measured. At the same time we have specified the soil moisture content and depth of the humus horizon. These data have been measured by steps of 5 m across the path of the tractor. Practically we did not know the previous history of this field as related to soil compaction. A strict look at the above mentioned soil sample showed that it was not altogether random. However, these samples were considered to approach a random distribution. This methodological approach is in accordance with findings reported in literature (Fadel, 2004).
Also we collected data from two parallel observation points in which the total sample size of the database was n=255.

In addition to the sample statistics of different measured properties, we have observed the shape parameter ***k*** of gamma distribution and also the coefficient of variation which depends on the volume of the sample.

3. THE THEORY

3.1. Criterion of Optimization

For assessment of the work of machines, a top priority is the effectiveness of their functions. For example for drilling machines the main function is providing suitable field distribution of seeds, at the same time for ploughs it is to provide the necessary distribution of ploughing depth, whereby the variation should be within the range of ±1 cm. The impact of soil compaction on the bulk density of soil may be distributed between limits, perhaps ±0.15 Mg m^{-3}. All the above-mentioned distributions can be described by a three-dimensional distribution which are more or less regularized. The regularity can be estimated by variation. It is interesting for us that soil bulk density also has an average value which can vary in separate parts of a field. In our work the coefficient of variation, root mean square deviation or standard deviation, dispersion and arithmetic mean are estimated by generally known equations.

If the sample size ***n*** is large enough, then the mathematical expectation approaches an arithmetic mean that would appear as:

$$M[X] = m_x = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (1)$$

3.2. How Representative are the Samples?

If we choose the variation coefficient as the criterion of optimization, then we know that it depends on the sample size. According to Standards (ISO 7256/1, ISO 7256/2), the total sample size is big enough if it is within the range of 100…300 measurements. The first author, M. Müüripeal, during the field experiments in 1993, found that the coefficient of variation of the interval between the seeds was stabilised when the number of measurements of the seed intervals was within the range of 200…250. Therefore, when we started to study soil compaction, we selected the number of measurements as 250. With the increase of the sample size, soil bulk density variation coefficient became even less and was stabilized around the theoretical trend line (Figure 1).

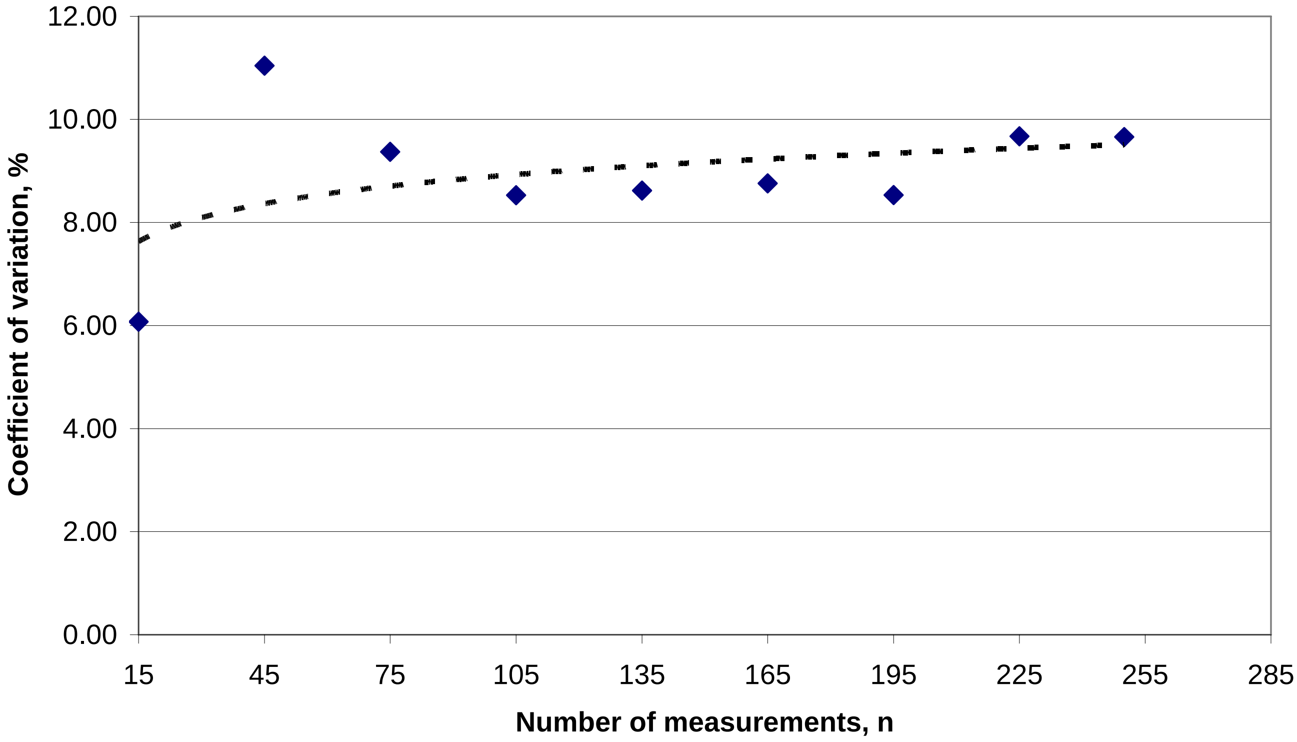

Figure 1. Stabilization of variation coefficient of soil bulk density (measured with an interval 5 m) depending on the number of measurements, n

3.3. Approximation of Field Data by the Theoretical Exponential Distribution

During previous investigations (Müüripeal, 1993; 1991), it was found that the field distribution of seeds was approximated by exponential distributions. It should be noted that in reproduction processes our “Mother Nature” uses exponential distribution, too.

For simplified formalization of probabilistic agrotechnological processes some authors (Lurie, 1970; Ventsel, 1991; 2000; 2002) have found that some ordinate stochastic events can be represented as an ordinary, stationary stream with no aftereffects which in the theory of probability is called Poisson´s stream. Thus, in the case of Poisson stream of events, a probability is specified that on a random length of any timespan defined by **τ,** precisely **k** quantity of events falls and it depends on the length of timespan. This can be described by the following formula (Ventsel, 1991):

$$P\{X(t,\tau)\} = a^k e^{-a} / k!\,,\qquad (k=0, 1, 2, \ldots), \tag{2}$$

where ***t*** is time and ***τ*** is any site of time; **a = λτ** is the mathematical expectancy and **λ**-intensity of stream.

Having performed the appropriate replacement, we can write the equation of probabilty of the events at a respective timespan:

$$P_k(\tau) = \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau} \tag{3}$$

The probability that in the chosen interval of time no events take place, the probability can be expressed in a simplified form of the stream:

$$P_0(\tau) = e^{-\lambda\tau}\,. \tag{4}$$

Distributive function of intervals of time between consecutive random events is expressed as:

$$F(t) = 1 - e^{-\lambda t} \tag{5}$$

At differentiation in time, we shall obtain the distributive density of random events at exponential distribution:

$$f(t) = dF(t)/dt = \lambda e^{-\lambda t} \tag{6}$$

The mathematical expectancy of intervals of time of consecutive events at exponential distribution is expressed as follows:

$$m_t = \int_0^\infty t f(t)dt = \lambda \int_0^\infty t e^{-\lambda t} dt\,, \tag{7}$$

at integration by parts we shall obtain:

$$m_t = \frac{1}{\lambda}\,.$$

Finding the physical essence of **λ,** we can see that the looseness is reciprocal of the bulk density and it depends on the intensity of soil tillage

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Dispersion of consecutive events of the timespan in case of exponential distribution is expressed as:

$$D_t = \int_0^\infty t^2 f(t)dt - \frac{1}{\lambda^2} = \int_0^\infty \lambda t^2 e^{-\lambda t} dt - \frac{1}{\lambda^2}\,, \tag{9}$$

integrating it, the final formula will be obtained:

$$D_t = \frac{1}{\lambda^2}. \tag{10}$$

As the root-mean-square deviation (standard deviation) is equal to the root square from the dispersion, we obtain:

$$\sigma_t = \frac{1}{\lambda}, \tag{11}$$

It means that the mathematical expectancy according to the equation (1) is equal to standard deviation, and variation coefficient according to exponential distribution as follows:

$$V_t = \frac{\sigma_t}{m_t} = 1, \tag{12}$$

in other words, coefficient of variation is equal to 100%.

For describing the longitudinal distribution of soil bulk density, the time ***t*** in equations (5) and (6) should be replaced by the length of the way ***l*** and as a result we shall obtain:

$$F(l) = 1 - e^{-\lambda l},$$

$$f(l) = dF(l)/dl = \lambda e^{-\lambda l}, \; l \rangle 0 \tag{13}$$

where ***F(l)*** is the function of distribution of the soil bulk density and ***f(l)*** is distribution density of soil bulk density. This can be expressed as follows:

$$m_l = \frac{1}{\lambda}, \qquad D_l = \frac{1}{\lambda^2},$$

$$\sigma_l = \frac{1}{\lambda} \quad \text{and} \quad V_l = \frac{\sigma_l}{m_l} = 1. \tag{14}$$

It should be noted that this description corresponds to an ideal case. In the general case (according to time), the actual stream of random events is not stationary (Ventsel, 2002; Lurie, 1970). The intensity of stream and mathematical expectancy are depending on the observation point in the field which is indirectly proven by a relatively slow stabilization of the coefficient of variation (Fig. 1). Thus, for better understanding, we have:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{m(t+\Delta t) - m(t)}{\Delta t} = m'(t)\,. \tag{15}$$

The probability of events does not depend only on the length of the given segment but depends also on its location and can be expressed as follows:

$$a = \int_{t_0}^{t_0+\tau} \lambda(t)dt\,. \tag{16}$$

In this case the function of distribution of stochastic events appears as:

$$F_{t_0}(t) = 1 - e^{-\int_{t_0}^{t_0+t} \lambda(t)dt}\,, \tag{17}$$

and density of distribution of stochastic events is:

$$f_{t_0}(t) = \lambda(t_0 + t)e^{-\int_{t_0}^{t_0+t} \lambda(t)dt} \tag{18}$$

The obtained two-parametric distribution is no more the exponential distribution and it depends on the value of parameter t_0 and function **λ(t)** which first can be approximated as linear:

$$\lambda(t) = a + bt \tag{19}$$

The obtained structure of non-stationary Poisson´s stream is somewhat complicated in comparison with that of a more simple stationary Poisson stream. As to the function **λ(t),** the content and form can be given that characterize the actual process rather precisely, so with such a distribution a very accurate approximation of actual longitudinal distributions is possible, if time ***t*** in equations (15)…(19) will be replaced by length of the way ***l***.

The principal characteristic of non-stationary Poisson´s stream is the absence of aftereffects between events. It means that any stochastic event does not engender any influence for the period of time up till the next event.

Random agrotechnological processes can be simulated and registered as function **λ = f(l)** of intensity of a stream of stochastic events from the length of the passed way. The causal factors of the variation of soil bulk density are: dynamic load of the tractor wheels, changes of speed of vehicles, the scheme of movement and other factors.

3.4. Approximation of the Variation of Field Distributions by Erlang´s Distribution

Generally a stream of random events with limited aftereffects known as Palme stream, can be analyzed and approximated by the distributions belonging to the gamma-distributions family. In the case of an integer of exponent $\boldsymbol{k}$ we have a special case of Palme stream representing Erlang stream and corresponding to Erlang distribution (Ventsel, Ovcharov, 2000). If the length of the way is as argument, then in this case the density of distribution of biparametric Erlang distributions appears as follows:

$$f_k(l) = \frac{\lambda(\lambda l)^{k-1} e^{-\lambda l}}{(k-1)!} \tag{20}$$

In the case of Erlang distributions, the mathematical expectancy is expressed in the form of:

$$m_l = \frac{k}{\lambda}, \tag{21}$$

dispersion:

$$D_l = \frac{k}{\lambda^2} \tag{22}$$

and standard deviation:

$$\sigma_l = \sqrt{\frac{k}{\lambda^2}}. \tag{23}$$

In our case Erlang distribution has rather interesting properties. If the form parameter or exponent k=1, then Erlang distribution turns into exponential distribution (13) and the first and the second central moments (21 and 22) acquire the same form as the corresponding central moments (14) of exponential distribution. It means that the measure of variation of the field distribution of seeds due to aftereffects, is the difference of the form parameter k from 1.

3.5. Approximation of Field Distributions by Normalized Erlang´s Distribution

Proceeding from the aforesaid, we started with an assumption, that for approximation of longitudinal distributions in random agrotechnological processes, rated Erlang distributions with a varying form parameter (exponent ***n)***, are most appropriate. Their density of distribution is expressed as per time as below:

$$\widetilde{g}_{(n)}(t) = \frac{n\lambda(n\lambda t)^{n-1}}{(n-1)!} e^{-n\lambda t} \quad . \tag{24}$$

Here, according to (24) the time can be replaced by the length of the way and in this case for finding corresponding digital characteristics of the sample of longitudinal distributions approximated by rated Erlang distributions we obtain the following expressions:

mathematical expectancy: $$\widetilde{m}_x = \frac{1}{\lambda}, \tag{25}$$

dispersion: $$\widetilde{D}_x = \frac{1}{n\lambda^2}, \tag{26}$$

standard deviation: $$\widetilde{\sigma}_x = \frac{1}{\lambda\sqrt{n}} \tag{27}$$

and coefficient of variation: $$\widetilde{V}_x = \frac{\widetilde{\sigma}_x}{\widetilde{m}_x} = \frac{1}{\lambda\sqrt{n}} \times \frac{\lambda}{1} = \frac{1}{\sqrt{n}} \tag{28}$$

Use of rated Erlang distributions for formalization of distributions obtained in random agrotechnological processes is made especially convenient by the independence of the distributions scale parameter from the form parameter or, in other words, the increase of exponent ***n*** of distribution does not influence its mathematical expectation.

In the case of form parameter ***n* = 1**, we are to deal with exponential distribution (13, 14) which is a special case of both Erlang distributions and gamma-distributions.With the approach of exponent ***n*** to infinity, the dispersion of distribution comes nearer to zero and rated Erlang distribution turns into a discrete distribution. Thus, rated Erlang distribution covers, with certain assumptions, in whatever random agrotechnological processes variation of corresponding intervals of homogeneous distributions within the limits of V = 0...100 %. For ourselves it was a surprise, that at rated Erlang distribution the factor of variation was equal to unit, divided on a root square from a degree of distribution (31). In the literature available to us such relationship is absent. Neither have we found references by other authors to the use of rated Erlang distributions in random agrotechnologilical processes at approximation of corresponding intervals of distributions.

3.6. Approximation of Field Distributions by means of Gamma- distributions

The density of distribution for gamma-distributions is expressed as below:

$$f_k x = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)} \quad , \tag{29}$$

where $$\Gamma(k)=\int_0^{\infty} e^{-t}t^{k-1}dt \tag{30}$$

and the first and second central moments are expressed as follows:

$$m_x=\frac{k}{\lambda} \tag{31}$$

$$D_x=\frac{k}{\lambda^2} \tag{32}$$

from here it is possible to deduce the formula for finding the interrelation of the factor of variation with form parameter ***k*** of distribution

$$V_x=\sqrt{\frac{1}{k}}\,, \tag{33}$$

Normalization of gamma-distribution makes it possible to approximate discretionary longitudinal distributions whose factor of variation remains within the limits of **V = 0.... ∞.**

Gamma-distributions allow to approximate als continuous-time discrete distributions.

3.7. Approximation of Field Distributions by Means of Normalized Gamma-distributions

The main numerical characteristics of normalized gamma-distribution can be found according to formulas (31) and (32) analogously to (25...(27):

mathematical expectancy: $$\tilde{m}_x=\frac{1}{\lambda}\,, \tag{34}$$

dispersion: $$\tilde{D}_x=\frac{1}{k\lambda^2}\,, \tag{35}$$

standard deviation: $$\tilde{\sigma}_x=\frac{1}{\lambda\sqrt{k}} \tag{36}$$

and coefficient of variation: $$\tilde{V}_x=\frac{\tilde{\sigma}_x}{\tilde{m}_x}=\frac{1}{\lambda\sqrt{k}}\times\frac{\lambda}{1}=\frac{1}{\sqrt{k}}\,, \tag{37}$$

where unlike rated Erlang distribution degree ***n*** which should be any whole positive number more than zero, the degree of distribution k may be any whole positive rational number different than zero. Due to the latter property of the rated gamma-distribution, its staging in density of distribution disappears and in the case of exponent of degree ***k*** less than one, it is possible to approximate distributions with the coefficient of variation more than one (100 %).

Proceeding from the aforesaid, it is possible to conclude that random processes can be completely described (modelled) and it is possible to estimate the quality of distributions by means of two quantitative characteristics from which the first is intensity $\boldsymbol{\lambda}$ of a stream and the second - exponent $\boldsymbol{k}$ of specified gamma-distribution or coefficient of variation.

3.8. Fast Method for Definition of the Theoretical Substitute Distribution of Parameters of Processes

Proceeding from the precondition that a concrete distributidon of the sample of random events can be approximated by means of rated gamma-distribution, then for finding substitute distribution, it is expedient to use the algorithm below:

- Calculation of the arithmetic mean of the members of the representative sample;
- Calculation of the standard deviation of the sample members;
- Finding of the coefficient of variation of the sample having divided standard deviation by the arithmetic mean;

From the interrelation (37) exponent $\boldsymbol{k}$ of density of distribution of the rated gamma-distribution can be expressed, which simultaneously is the form parameter of the said distribution and is defined by the formula:

$$k = \frac{1}{V^2} \tag{38}$$

Proceeding from the formula above, it follows that:

- The found form parameter defines the function of density of distribution of the theoretical distribution we are interested in;
- At digital integration of the function of distribution density within the set limits, it is possible to find, if necessary, distribution probabilities interesting for us;

For transferring the obtained results into the respective scale, it is necessary to multiply the integrated numbers by the arithmetic mean of the members of the investigated sample.

4. RESULTS AND DISCUSSION

Concerning the above mentioned method we have two different situations of compactobility of the soil with relative variability of soil bulk density:

a) low;
b) high.

It is important that overall soil compaction smoothes out to some extent the variability of bulk density. Examples of these two opportunities are shown in Figures 1 and 2.

The next situation is shown in Figure 2 of the case with organic soil in the middle part of the field. Depending on this, the bulk density in comparison with low variability situation (Figure 1) is different from the difference between maximum and minimum bulk density 0.92 Mg m^{-3} (average = 1.10 Mg m^{-3}, and standard deviation = 0.25 Mg m^{-3}), at the same time analogous difference of bulk densities in the case of low variability 0.60 Mg m^{-3} (average = 1.21 Mg m^{-3}, and standard deviation = 0.12 Mg m^{-3}) whereby there is a significant difference. So the question stands: which of the above mentioned situations is more preferable for soil conditions? In our opinion it depends, firstly, on differences of soil associations, secondly, an organic soil is more sustainable to the influence of various mobile technical means used during vegetation period.

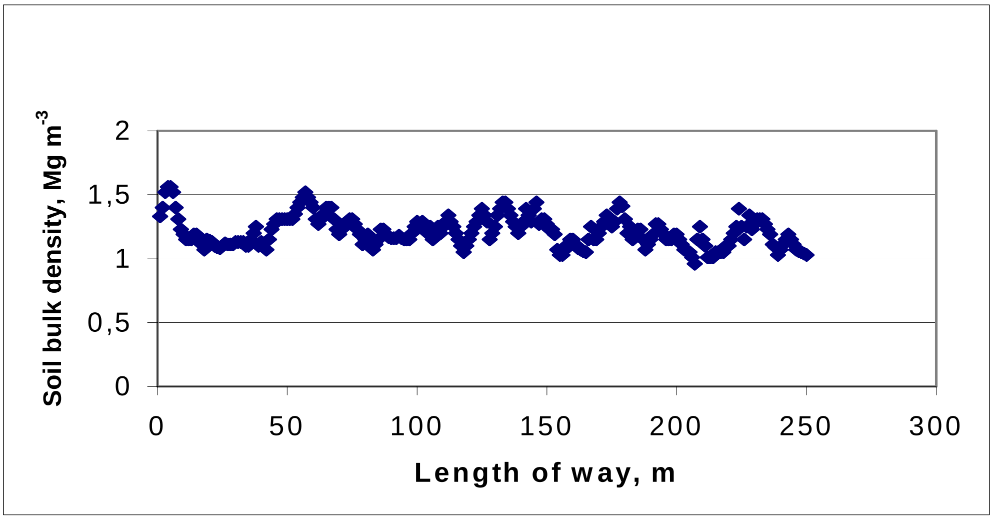

Figure 1. Variation of soil bulk density and length of way 255 m (low variability).

Notes (belong to Figure 1):

1) the bulk density has been observed in permanent observation point at Reiu (58°19′N, 24°38′E) in Pärnu County (Estonia) in 1988 (field experiments by R. Lehtveer and E. Nugis). The soil Eutri-Histic Gleysol has been compacted at random by heavy tractors;
2) at this length of way for calculating of the intensity of soil compaction λ we have chosen at random sample sizes in growing order with step of 5 m;
3) the soil compaction has decreased a high-frequency fluctuation of bulk density;
4) intensity of soil compaction λ depends on the location of the chosen sample;
5) the bulk density distribution is not stationary;
6) as for the character of variation, we are to deal with the Palme nonstationary stream with limited aftereffects of random events.

It is important to note (Figures 3 and 4) that distributions for these cases are also quite different. In a compacted field with low variability of soil bulk density the histogram of distribution (Figure 3) is less than normal distribution compared with high variability situation of soil bulk density (Figure 4).

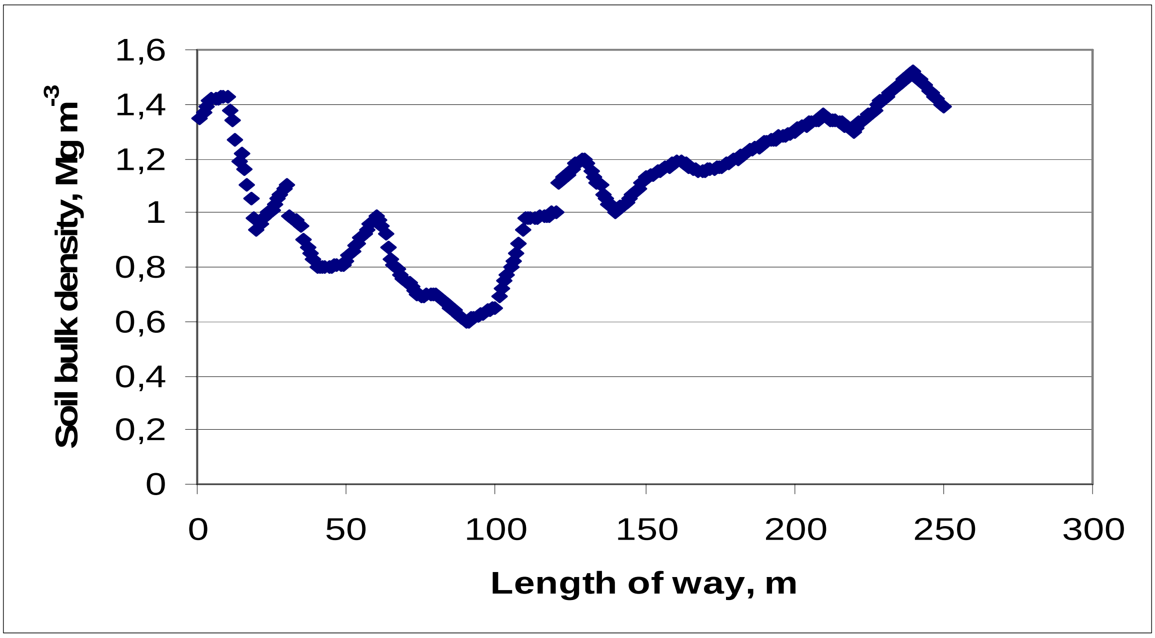

Figure 2. Variation of soil bulk density and length of way 255 m (high variability).

Note (belongs to Figure 2):

> The bulk density has been observed in permanent observation point at Adavere (58°43´N, 25°54´E) in Jõgeva County (Estonia) in 1988 (field experiments by R. Lehtveer and E. Nugis). The soil Albeluvisol has been compacted at random by heavy tractors.

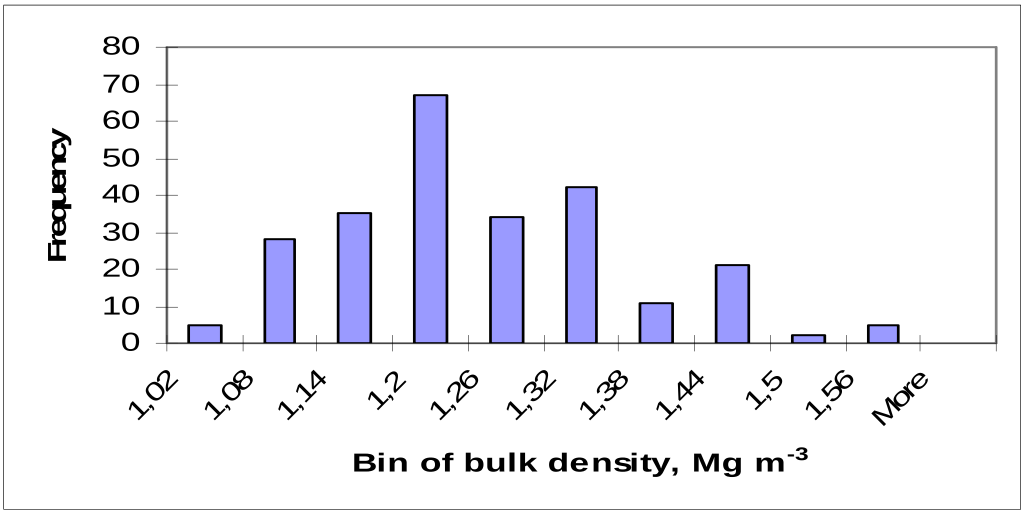

Figure 3. Histogram of distribution, case of low variability of soil bulk density.

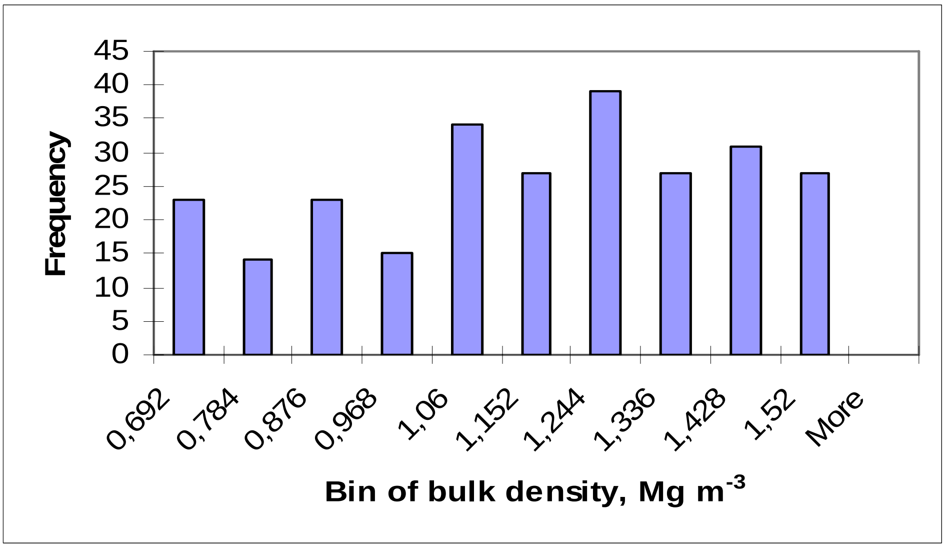

Figure 4. Histogram of distribution, case of high variability of soil bulk density.

Note (belongs to Figure 4)**:**

In this case we have also Palme nonstationary stream with limited aftereffect situation.

As it appears from Figures 3 and 4, the last histogram of distribution in Figure 4 differs greatly from normal distribution here that cannot be said about the situation shown in Figure 3. Here we have an extra proof of non-existence of normal distribution which actually is a mix of different distributions (Feller, 1971).

It appears that in dependence of increase of the size ***n*** of sampling, there is a stabilization of fluctuations of the soil bulk density within the limits of certain borders of its size (Figures 5 and 6). So it is possible to notice that the stabilization of fluctuations comes earlier if the variability of soil bulk density is lower.

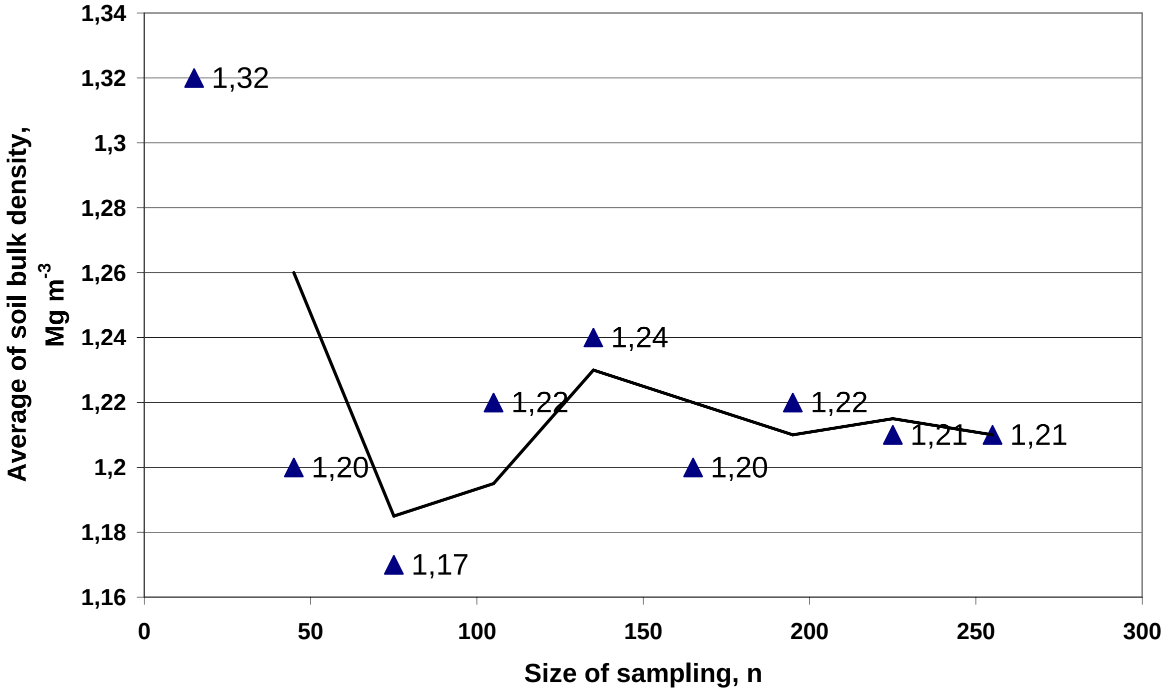

Figure 5. Relation between the arithmetic mean of soil bulk density (low variability) and size of sampling (the sampling in this realization n = 255 has been chosen at random in which from n =15 up to n =225 it was increased by step of 30).

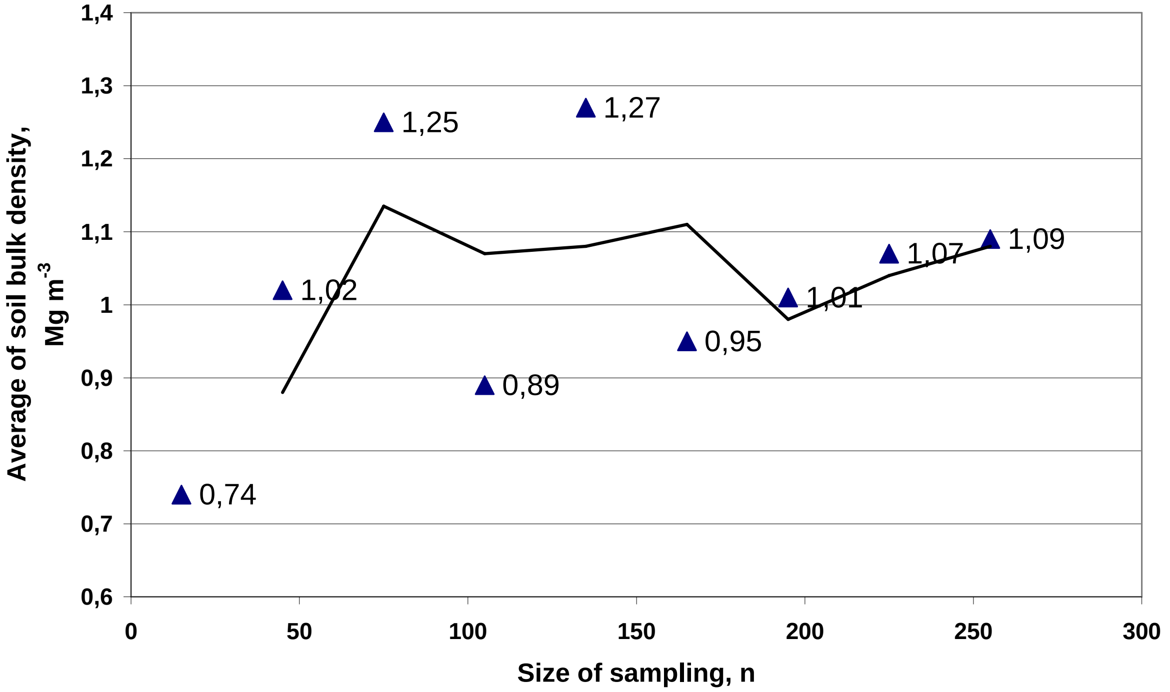

Figure 6. Relation between the arithmetic mean of soil bulk density (high variability) and size of sampling (the sampling in this realization n = 255 has been chosen at random in which from n =15 up to n =225 it was increased by step of 30).

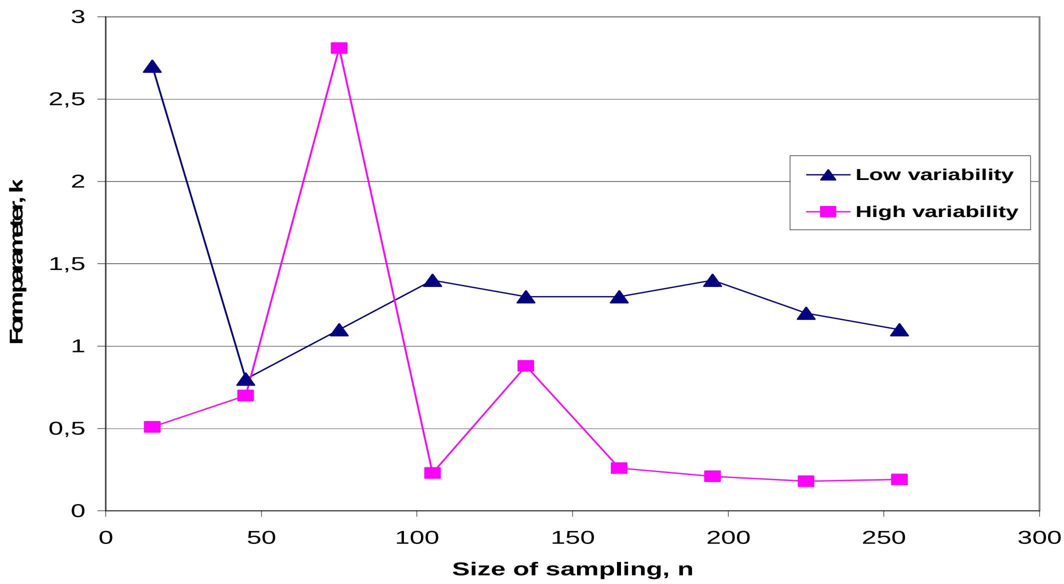

Figure 7. Changes of form parameter k depending on the size of the sample in the case of normalized gamma-distribution for two different cases of variability of the soil bulk density.

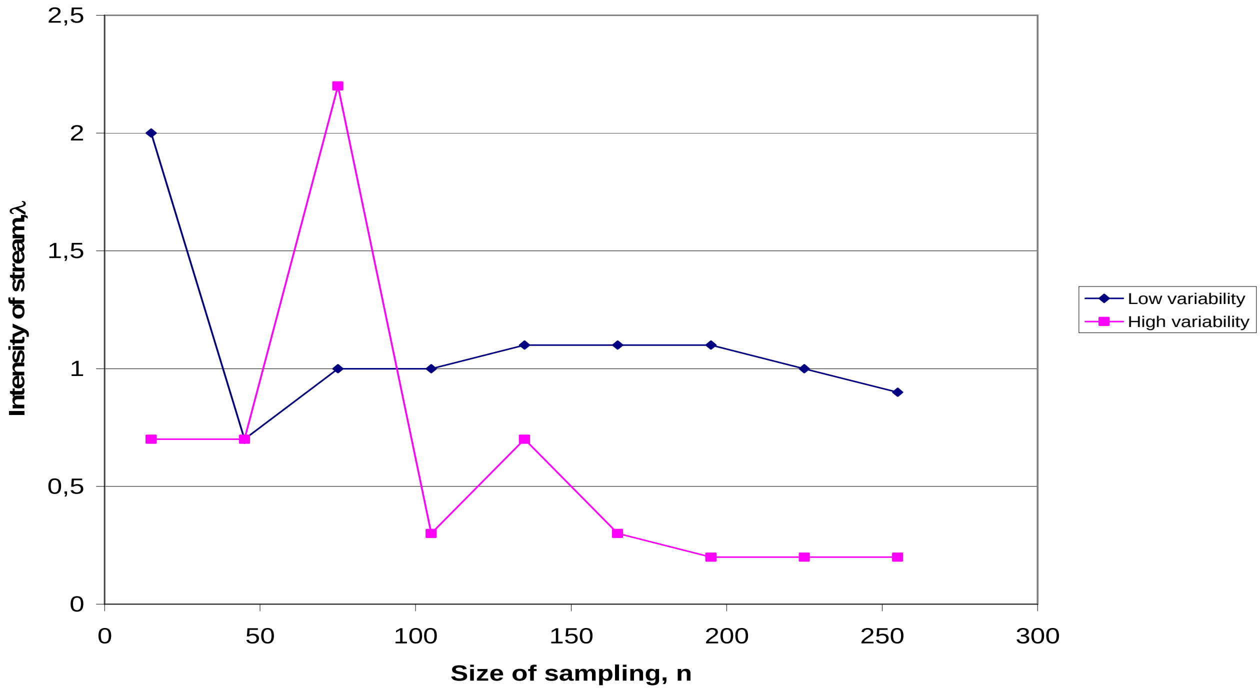

Figure 8. Changes of the inensity of stream λ depending on the size of the sample in the case of gamma-distribution for two different cases of variability (low and high) of the soil bulk density.

As can be seen from Figures 7 and 8, in both cases low and high variability curves are remarkably similar, whereas in the case of low variability the difference is greater but not essentially. This refers to the fact that when using the fast method for definition of parameters of the theoretical substitute distribution of rpocesses offered by us (see Paragraph 3.8) only form parameter $\boldsymbol{k}$ can be assessed. The Figure 7 also shows that form parameter $\boldsymbol{k}$ is not equal to 1 ($\boldsymbol{k} \neq \boldsymbol{1}$). Thus, we do not have the specific case of Erlang and gamma-distribution. Actually we have a normalized Erlang distribution which enables us to apply the above mentioned algorithm (see Paragraph 3.8).

4.1. The Intensity of Stream

The changes of the basic statistical characteristics depending on the sample size can be seen in Table 1. According to this table, it can be concluded that the changes of the coefficient of variation are more essential in case of high variability (max – min = 17.4). If compared with low variability, the same difference is equal to 5.1 which is about 3.4 times smaller.

Thus, the most likely stable situation for low variability of bulk density anticipated between 1.20 ...1.22 Mg m^{-3} and sampling size was achieved 165 – 255, whereas the soil was compacted at random, and soil type did not vary much within the sample.

Table 1. Main statistical data for two cases of variability of the soil bulk density Mg m^{-3} depending on the size of the sample, ***n*.**

Parameter	Case of varia-bility	Size of the samle, ***n***								
		15	45	75	105	135	165	195	225	255
Average, Mg m^{-3}	low	1.32	1.20	1.17	1.22	1.24	1.20	1.22	1.21	1.21
	high	0.74	1.02	1.25	0.90	1.27	0.95	1.01	1.07	1.10
Standard Mg m^{-3}	low	0.08	0.13	0.11	0.10	0.11	0.11	0.10	0.12	0.12
	high	0.10	0.12	0.07	0.19	0.14	0.19	0.22	0.25	0.25
Variation, %	low	6.1	11.2	9.5	8.6	8.7	8.7	8.5	9.1	9.7
	high	14.0	11.9	6.0	21.0	10.7	19.7	21.9	23.4	23.1

As expected, in case of high variability of soil bulk density, here soil conditions were much more varying. Therefore, with the same at random compacted soil, changes of association exceed the influence of soil compaction. Also it should be noted that for specified gamma-distribution the intensity λ of a stream (Figure 7) for all cases is equal to **1** ($\lambda = 1/m_x$ =1) and it does not depend on the size of sampling ***n*** . It means that we do not have the fact of a stationary process, thereby, we have a Palme stream with constrained aftereffects of stochastic events.

Anyway, it appears that agrotechnological processes with high variability and stochastic events having asymmetric distribution can be described with sufficient accuracy and it is not the basis for prediction of events in principle not taking place.

5. CONCLUSIONS

Compaction by wheels of modern machine technologies has a negative effect on soil physical properties compared to non-compacted areas. As for determining the rational parameters related to agrotechnological processes and also including precision farming requirements (concerning our further investigations), our work is based on the methods of mathematical statistics and new viewpoints developed by St.-Petersburg school. An approximation by a normalised gamma-distribution is adjusted to describe the distributions of records of the suitable results concerning agrotechnological processes. We have found that a normalized gamma-distribution is much wider and suitable in comparison with a normal distribution. It is possible to approximate, including normal, as well as others, down to discrete distributions with continuous time. Thus, approximation of form parameter ***k*** to infinity and gamma-distribution are described with a regular stream, which actually occurs rather seldom. So for example for soil physical properties, the level of the bulk density can be submitted as function of time or the passed way.

Our conclusion is that the offered technique of the description of random distributions allows us to carry out also transition from these random distributions to random processes. All this concerns as well the results related to bulk density of the soil at two permanent observation points.

As for any particular soil, there are certain borders of maximum and minimum of bulk density and it is possible to conclude, that distribution of data on bulk density is not subjected to the law of normal distribution. This theory serves as basis for creating methodology for control of agrotechnological processes in real time.

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