Periodic Analysis of Solarium-cum-Greenhouse

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ABSTRACT

In this paper, an attempt has been made to evaluate the performance of solarium-cum-greenhouse by using the periodic analysis. Based on basic energy balance of each component of the solarium cum-greenhouse, a program in Matlab has been developed to evaluate room air, plant/isothermal mass temperature and thermal load levelling by matrix inversion. Temperatures at different layers of basement including air gap, phase change material (PCM) and soil have also been estimated. It is inferred that the temperature at bottom of PCM is highest due to minimum heat loss through the insulation. It is also observed that the best thermal load levelling is achieved for larger thickness of PCM.

Keywords: Energy storage, heat loss, PCM, solar energy, solarium-cum-greenhouse, thermal load

1. INTRODUCTION

Greenhouse technology has evolved to create a favorable environment to cultivate a desirable crop year around. The temperature inside the greenhouse can be increased and decreased as per heating and cooling of the greenhouse air. The use of transparent cover for thermal heating is one of the passive concepts (direct gain) used in design of a building Morse (1881), Balcemb et al. (1977). The over heating of an environment inside the building had been controlled by indirect thermal heating (Fuchs, 1974). The combination of direct and indirect heating is generally known as solarium. The solarium cum greenhouse can be used for indirect heating / cooling of the living space. This is good for comfort living, low energy budget and also very aesthetic. Off-season vegetables can be grown in the greenhouse and substantial savings in the kitchen budget can be made Gupta and Tiwari (2004). In solarium-cum-greenhouse latent heat storage is one of the most efficient ways of storing thermal energy. In a latent heat thermal storage (LHTS) system, during phase change process, the surface heat flux decreases due to increase thermal resistance of growing layer of the molten / solidified medium. A large number of passive solar greenhouses use latent heat storage materials undergoing reversible phase change solid-liquid. In recent years, several researchers, Ye and Ge (2000), Xiao et al. (2002), Qin et al. (2003), Kumari et al. (2005) have studied the thermal properties of several PCMs. The most frequently used phase change material for these purposes is $CaCl_2.6H_2O$, because it is a low cost material with meeting point at 29.8 ^oC and has fairly high heat storage capacity of 150KJ/kg by Marinkovic et al. (1998). Phase change materials (PCM) use chemical bonds to store and release heat. The solid-liquid PCMs perform like conventional storage materials; thus temperature rises as they absorb solar heat. Unlike conventional (sensible) storage materials, when PCMs reach the temperature at which they change phase, they absorb large amounts of heat without getting hotter.

In the present study, thermal performance of greenhouse cum solarium with PCM floor has been investigated theoretically in terms of room air temperature, plant / isothermal mass temperature and thermal load leveling (TLL). It is seen that the solarium room air temperature is maximum in the case of PCM used below the floor.

2. WORKING PRINCIPLE OF SOLARIUM

Figure 1(a) shows a cross-sectional view of a passive thermal heating of a greenhouse- cum-solarium. There is a partition wall between the greenhouse and the living space. The partition can be transparent (fig.1a), opaque (fig. 1b), semi transparent, movable or fixed depending upon the temperature desired inside the greenhouse and the living space. Solar radiation, after reflection from greenhouse cover, is transmitted inside the greenhouse. The solar radiation falling on north wall partition may be reflected, conducted or transmitted depending upon the material of north partition wall. The thermal energy either transmitted or conducted into the living space can be utilized to heat the room air in the living space. The radiation falling on the floor of the greenhouse and reflected radiation from north partition wall may be utilized to heat the greenhouse air temperature. The distribution of solar radiation on north partition wall and floor also depends upon the width and the height of the Solarium. In figure 1(b) the transmitted solar radiation through north canopy cover is generally significant during the winter month due to low altitude angle. This can be retained inside the greenhouse by providing brick north wall and more energy can be stored using phase change material with this wall (Tiwari, 2002). In figure1(c) energy storage in the solarium cum greenhouse may be enhanced by impregnating some suitable PCM in the floor. PCM can absorb solar energy at daytime while PCM changes from solid to liquid, and releases the energy and freezes back to solid when the room temperature falls down at evening (Belen, 2003).

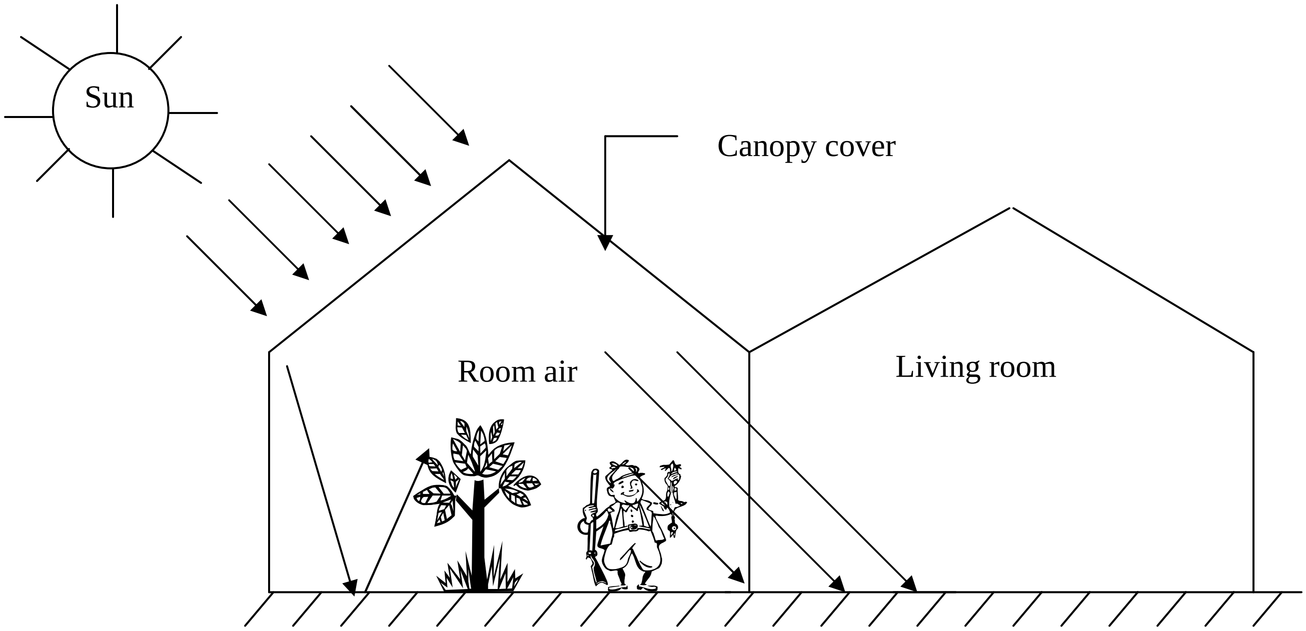

Figure 1(a). Schematic view of Greenhouse-cum-Solarium.

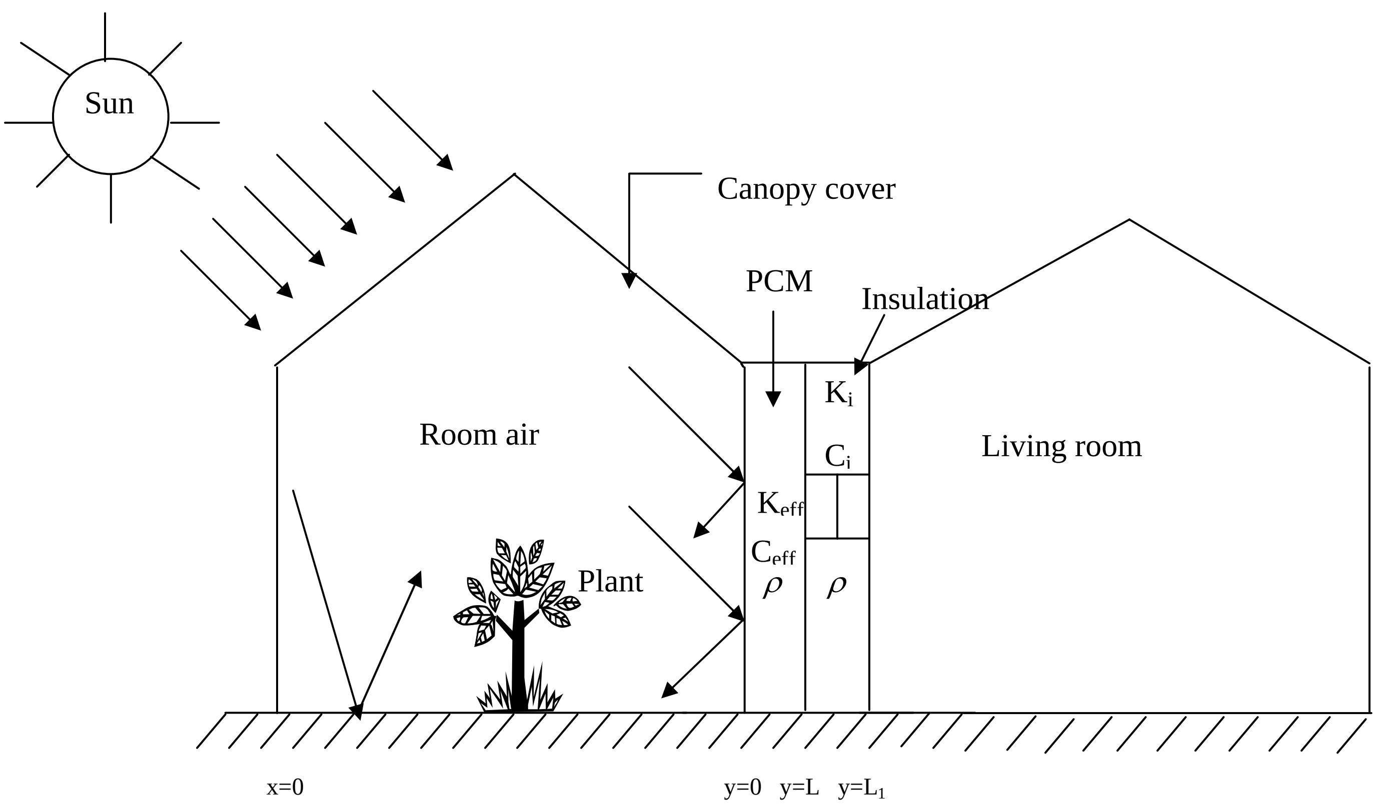

Figure 1(b). Schematic view of Greenhouse-cum-Solarium with insulated PCM north wall.

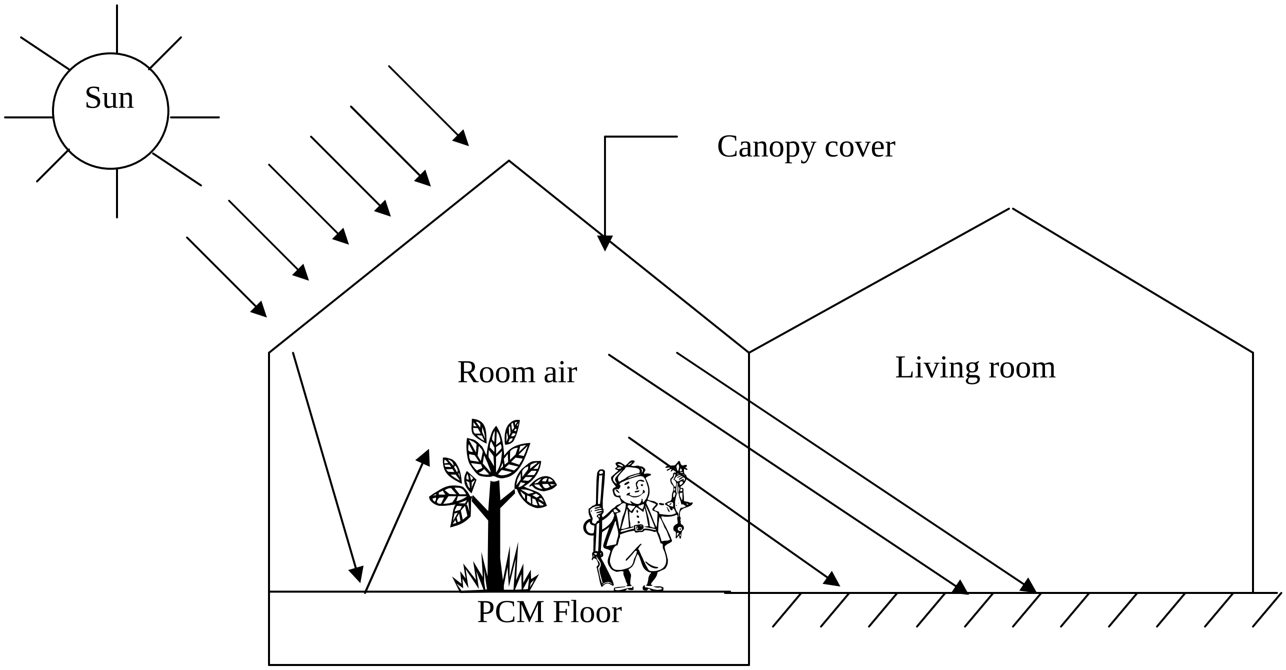

Figure 1(c). Greenhouse-cum-Solarium with basement having air-PCM.

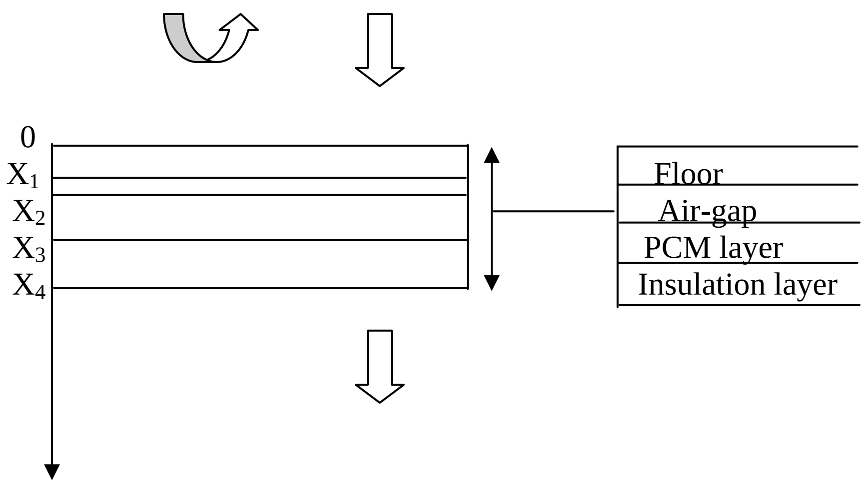

Figure 1(d). Schematic view of PCM floor.

3. THERMAL ANALYSIS

In order to write the energy balance equations for different component of the system, the following assumptions have been made.

(1) The orientation of the greenhouse is east-west.

(2) Heat transfer through walls and floor is one dimensional.

(3) Absorptivity and heat capacity of the enclosed air is neglected.
(4) Thermal properties of material/air are temperature independent.

Energy balance equations for different components of the greenhouse system, as shown in the figure 1(c), has been written as below:
At x=0

$$\alpha_g(1-\gamma_g)(1-F_p)(1-F_N)(1-\gamma)\tau S(t) = A_1 h_1 (T_1|_{x=0} - T_r) - K_1 A_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=0} \quad (1)$$

where $S(t)=\sum I_i A_i$, I_i is the sum of beam and diffuse radiation given in fig. 2 on i[th] walls and roofs of a solarium cum greenhouse shown in figure 1(c) (Liu and Jordan, 1962).

At x=x_1
Continuity of the flux

$$-K_1 A_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=x_1} = -K_2 A_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=x_2} \quad (2)$$

$$-K_2 A_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=x_2} = C_a A_1 [T_1(x=x_1) - T_2(x=x_2)] \quad (3)$$

At x=x_3
Continuity of the flux

$$-K_2 A_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=x_3} = -K_3 A_3 \left.\frac{\partial T_3}{\partial x}\right|_{x=x_3} \quad (4)$$

Continuity of Temperature

$$T_2|_{x=x_3} = T_3|_{x=x_3} \quad (5)$$

At x=x_4
Continuity of the flux

$$-K_3 A_3 \left.\frac{\partial T_3}{\partial x}\right|_{x=x_4} = -K_4 A_4 \left.\frac{\partial T_4}{\partial x}\right|_{x=x_4} \quad (6)$$

Continuity of Temperature

$$T_3|_{x=x_4} = T_4|_{x=x_4} \quad (7a)$$

As $x_4 \to \infty$, T_4 is finite. (7b)

Room Air

$$A_p h_p (T_p - T_r) + A_1 h_1 (T_1|_{x=0} - T_r) = h(t)\sum A_i (T_r - T_a) + \dot{m}_a C_a (T_r - T_a) + h_d A_d (T_r - T_a) \quad (8)$$

In the above equation,

$$\dot{m}_a C_a = \frac{V\rho}{t} C_a = \frac{V*1.2}{\left(\frac{3600}{N}\right)} * 1000 = \frac{1}{3} NV = 0.33NV$$

Plant Mass

$$\alpha_p(1-\gamma_p)F_p(1-F_N)(1-\gamma)\tau S(t) = [\dot{q}_{ep} + \dot{q}_{cp} + \dot{q}_{rp}]A_P + M_p C_p \frac{dT_p}{dt}$$

or,

$$\alpha_p(1-\gamma_p)F_p(1-F_N)(1-\gamma)\tau S(t) = A_p h_p(T_p - T_r) + M_p C_p \frac{dT_p}{dt} \qquad (9)$$

Since solar radiation and ambient air temperature are periodic in nature and hence, parameters depending on these can be represented mathematically in the form of Fourier series Threlkeld (1970), Baldasano (1998) as:

$$S(t) = S_{to} + \text{Re}\sum_{n=1}^{6} S_{TN} e^{in\omega t} \qquad (10)$$

$$T_a = T_{ao} + \text{Re}\sum_{n=1}^{6} T_{AN} e^{in\omega t} \qquad (11)$$

where,

$$T_{AN} = T_{an} e^{-i\psi_n},\ S_{TN} = S_{tn} e^{-i\phi_n},\ \psi_n = \tan^{-1}\left(\frac{B_n}{A_n}\right) \text{ and } \phi_n = \tan^{-1}\left(\frac{B_n}{A_n}\right)$$

Fourier coefficients namely $S_{to}, S_{tn}, \psi_n, T_{ao}, T_{an}$ and ϕ_n have been evaluated for a typical day of winter month as shown in figure 2.and presented in table 1.

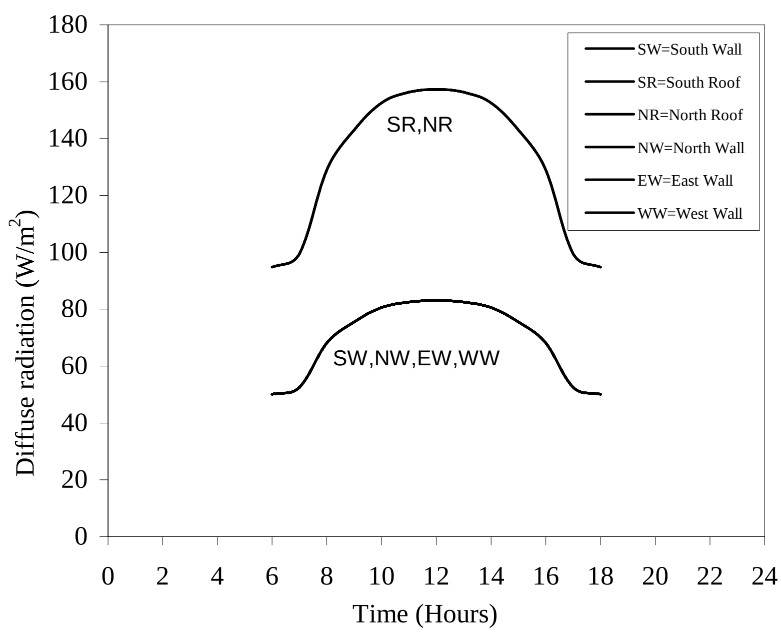

Figure 2. Hourly variation of diffuse radiation on different walls and roofs of greenhouse for a typical winter day in New Delhi.

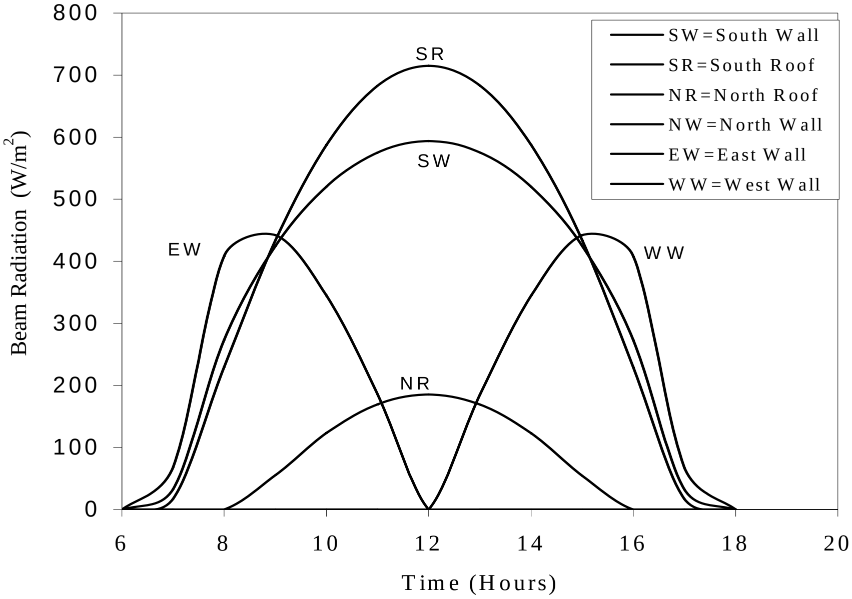

Figure 2 (a). Hourly variation of beam radiation on different walls and roofs of greenhouse for a typical winter day in New Delhi.

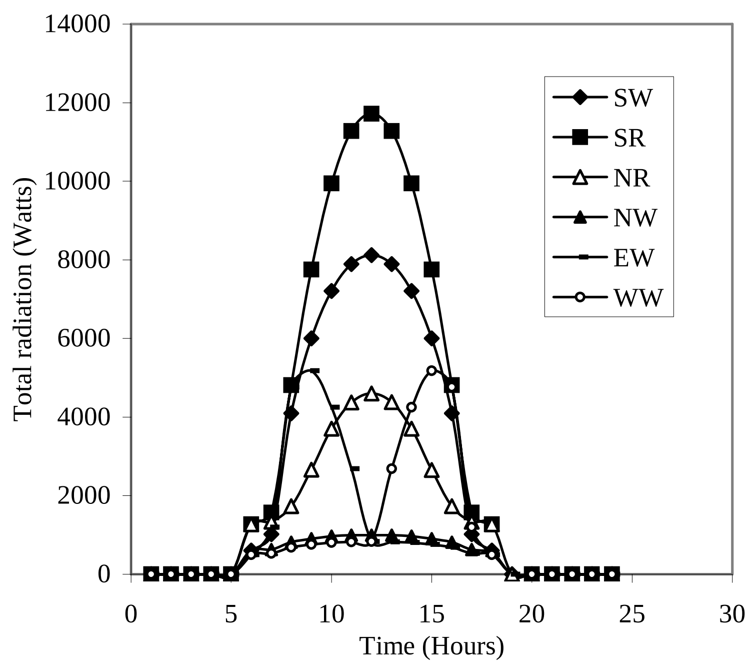

Figure 2 (b). Hourly variation of total solar radiation incident on greenhouse for a typical winter day in New Delhi.

Due to periodic nature of S(t) and T_a, T_1, T_2, T_3 , T_4 ,T_p and T_r can also be expressed as follows:

$$T_1 = A_1' x + B_1' + \sum \left(C_{1n} e^{\beta_{1n} x} + D_{1n} e^{-\beta_{1n} x} \right) e^{in\omega t} \quad (12)$$
$$T_2 = A_2' x + B_2' + \sum \left(C_{2n} e^{\beta_{2n} x} + D_{2n} e^{-\beta_{2n} x} \right) e^{in\omega t} \quad (13)$$
$$T_3 = A_3' x + B_3' + \sum \left(C_{3n} e^{\beta_{3n} x} + D_{3n} e^{-\beta_{3n} x} \right) e^{in\omega t} \quad (14)$$
$$T_4 = A_4' x + B_4' + \sum \left(C_{4n} e^{\beta_{4n} x} + D_{4n} e^{-\beta_{4n} x} \right) e^{in\omega t} \quad (15)$$
$$T_p = T_{po} + \mathrm{Re} \sum_{n=1}^{6} T_{pn} e^{in\omega t} \quad (16)$$
$$T_r = T_{ro} + \mathrm{Re} \sum_{n=1}^{6} T_{rn} e^{in\omega t} \quad (17)$$

where,

$$\beta_{mn} = \sqrt{\frac{n\omega \ \rho_m C_m}{2K_m}}(1+i)$$

where m=1,2,3 and 4 for floor, effective PCM, insulation and ground respectively.
Using the condition given in equation. (7b) in equation (15), it gives

$A_4 = 0, C_{4n} = 0$ and hence

$$T_4 = B_4 + \sum D_{4n} e^{-\beta_{4n} x} e^{in\omega t} \quad (18)$$

Using equations (10-18), time independent and dependent parts of equations (1-9) form 9×9 matrices. The matrix for time independent and time dependent part are given in Appendix A and B.

Table 1(a). Fourier coefficient of total solar intensity falling on greenhouse.

n	0	1	2	3	4	5	6
S_{tn}	9308.55	-14024	5179	176.9971	-937	521.99	-289.99
ϕ_n		3.1417	6.2835	0.0057	3.1417	6.2838	3.1424

Table 1(b). Fourier coefficient of ambient air temperature.

n	0	1	2	3	4	5	6
T_{an}	13.42	-4.08	0.48	0.01	0.01	-0.02	-0.01
ψ_n		4.06	1.17	0.00	5.94	3.31	3.31

3.1 Thermal load leveling

Due to time dependent nature of the room temperature (T_r), the fluctuation in room temperature plays a vital role. The thermal load levelling TLL provide an idea about the fluctuation of room air temperature inside the greenhouse. The thermal load levelling can be defined as follows:

$$\text{TLL}=\frac{T_{r,\max}-T_{r,\min}}{T_{r,\max}+T_{r,\min}} \tag{19}$$

For a given temperature difference between maximum and minimum, the denominator should be a maximum for low value of TLL in winter condition. The TLL will be maximum for summer conditions by Singh and Tiwari (2000). Therefore, TLL is an important factor for optimizing the heating parameter.

4. RESULTS AND DISCUSSION

Methodology of Computations

Hourly variation of beam and diffuse radiation available on different walls/roof of solarium cum greenhouse has been given in figure 2(a). These have been computed by using Liu and Jordan formula (1962). The total radiation on different walls/roof has been shown in figure 2(b). The sum of the radiation of figure 2(b) gives the total radiation falling on the greenhouse. The Fourier coefficient of total radiation falling on the solarium-cum-greenhouse has been given in table 1(a). Further, the Fourier coefficient of ambient air is given in table 1(b). The methods for evaluating these constants are given by Tiwari (2003).

Table 2. Input parameters used for computation.

Parameters	Values	Parameters	Values
τ	0.65	A_p	72 m^2
A_1	24 m^2	K_1	0.52 W/m^{o}C
K_2	0.53 W/m^{o}C	K_3	.057 W/m^{o}C
K_4	0.52 W/m^{o}C	F_n	0.40
F_p	0.80	γ_g	0.10
C_1	1880 J/kg ^{o}C	C_2	6670 J/kg ^{o}C
C_3	1000 J/kg ^{o}C	C_4	1880 J/kg ^{o}C
α_n	0.6	α_p	0.4
α_g	0.4	h_p	15 W/m^{2o}C
h_d	1 W/m^{2o}C	h_1	5.7 W/m^{2o}C
C_a	1758 J/kg^{o}C	V	60
N	1	C_p	4190 J/kg^{o}C
ω	7.2×10^{-5}		

Now equations (I-10) and (I-11) have been computed by using Matlab for matrix inversion for a given design parameters of table 2 and fourier coefficients of table 1. After knowing the constants from equations (I-10) and (I-11), the various temperatures given by equations (12-17) can be evaluated for a given sets of parameters.

Figure 3 shows the hourly variation of the isothermal mass (plant) and room air for the following conditions:

(i) Solarium without north wall (fig. 1a)
(ii) Solarium with PCM north wall (fig. 1b)
(iii)Solarium with basement having air-PCM (fig. 1c)

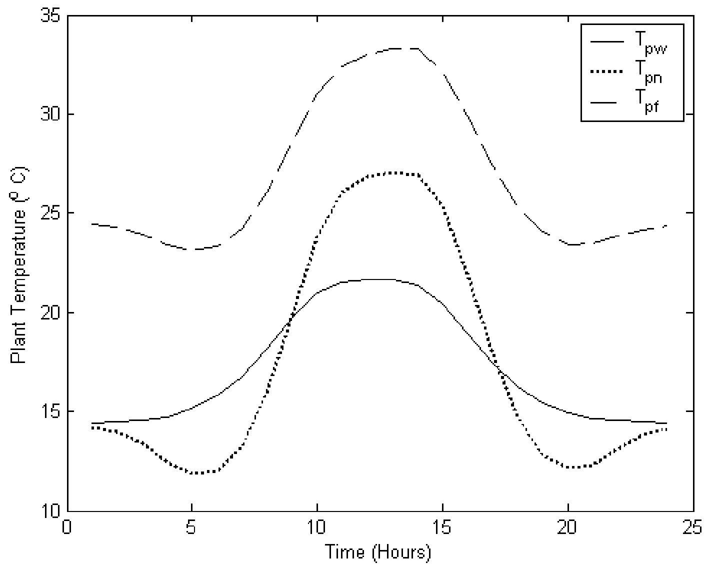

Figure 3(a). Hourly variation of plant temperature without north wall, with PCM north wall and with PCM floor.

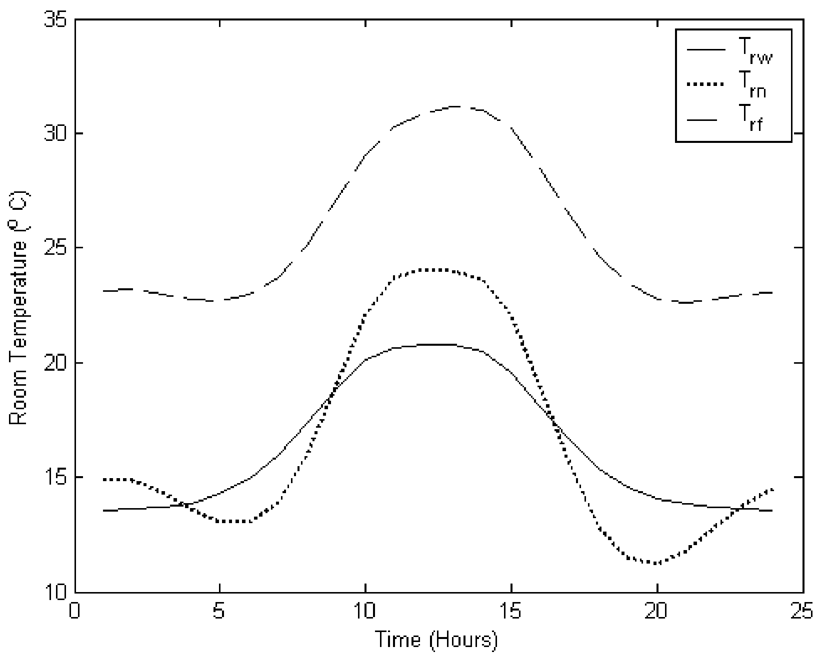

Figure 3(b). Hourly variation of room temperature without north wall, with PCM north wall and with PCM floor.

It is observed that the temperature of isothermal mass is higher than room air temperature during the day time due to direct available solar energy to the plant. Further, the isothermal mass and room air temperature is significantly higher in the case (iii) due to minimum heat loss through the ground. It is further noted that the plant and room air temperature with PCM north wall (case (ii)) is higher by about 5^{o}C during the sunny period and lowered by 3^o C during off-sunny period. This can be due to minimum heat transferred from inside to outside during sunny period and from outside to inside during off-sunny period. Further, it is to be noted that the PCM used below floor with air gap and insulation gives much better performance in comparison with other two cases. Hourly variation of temperature at different layers of basement has been shown in figure 4 for (i) L_3=0.20m and L_4=0.10m and (ii) L_3=L_4=0.5m. It is inferred that the temperature at bottom of PCM is highest due to minimum heat loss through the insulation. The top of PCM layer is also about 80 ^{o}C. It can be seen that the maxima of temperature shifted with increase of thickness below the floor due to increase of heat capacity. It is important to note that the temperature at top of PCM is lower than the floor temperature as expected. However, the temperature at bottom of PCM is higher than the temperature at top of PCM due to accumulation of thermal energy at bottom of PCM. This energy can also be utilized directly by flowing air through air gap inside solarium. It is further to be noted that the reduction in PCM thickness reduces the maximum temperature due to its low heat capacity (fig. 4b).

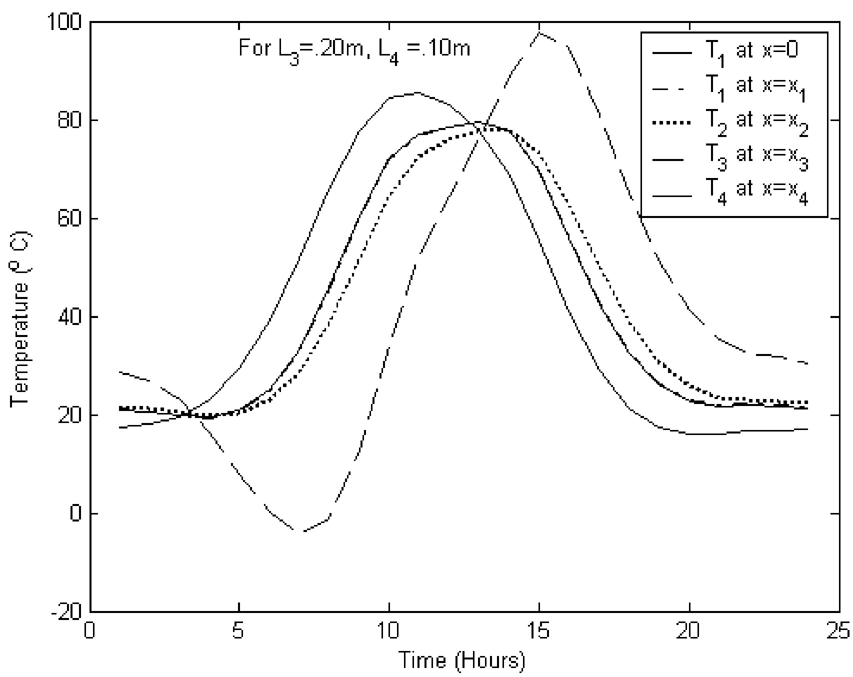

Figure 4(a). Hourly variation of temperature distribution at different surfaces of floor varying thickness of PCM and insulation, keeping total thickness of the floor constant L_3= 0.20 m and L_4 = 0.10 m.

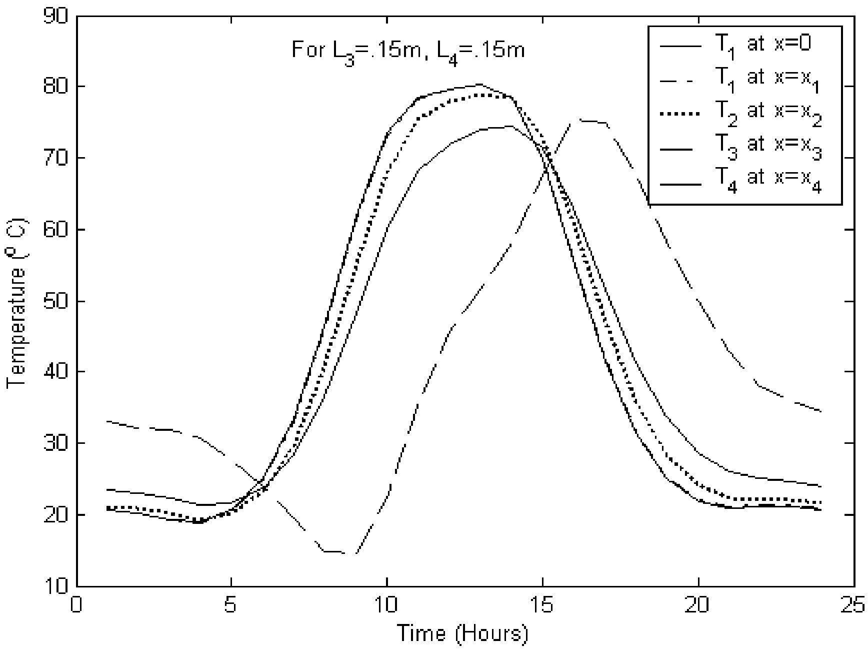

Figure 4(b). Hourly variation of temperature distribution at different surfaces of floor varying thickness of PCM and insulation, keeping total thickness of the floor constant $L_3 = 0.15$ m and $L_4 = 0.15$ m.

Figure 5 shows hourly variation of the plant and room temperature of the greenhouse for different $\frac{L_3}{L_3 + L_4}$, varying the thickness of PCM and insulation layer with total thickness of 0.40m. It is clear that the plant temperature is maximum for least thickness of PCM i.e. L_3= 0.10 m and L_4= 0.20 m and correspondingly the room temperature is maximum for L_3= 0.10 m and L_4= 0.20 m as shown in figure 5(b).

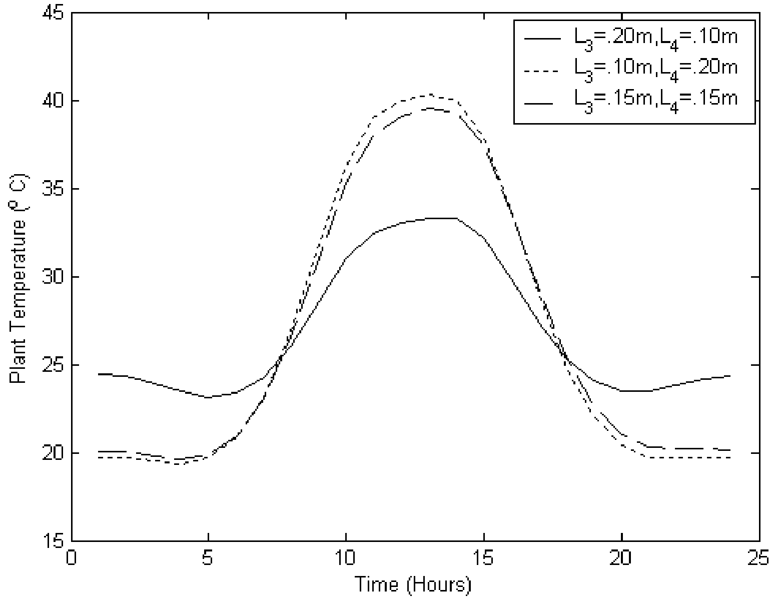

Figure 5(a). Hourly variation of plant temperature at different surfaces of floor varying thickness of PCM and insulation, keeping total thickness of the floor constant.

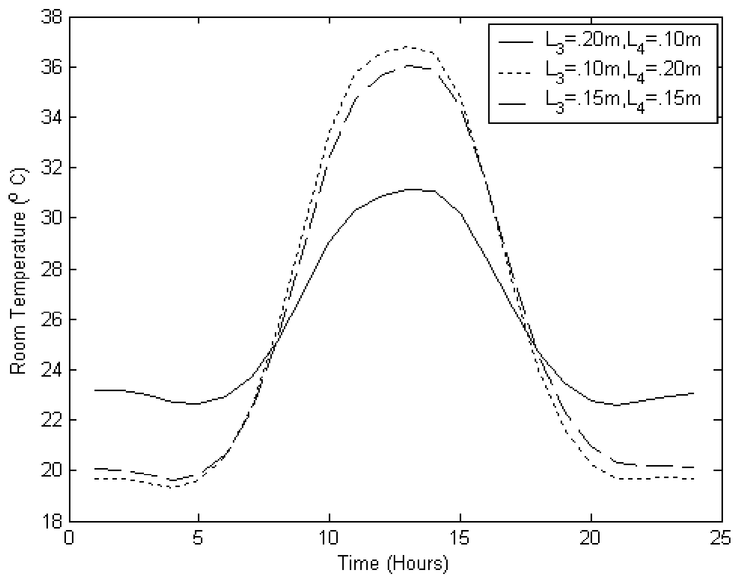

Figure 5(b). Hourly variation of room temperature at different surfaces of floor varying thickness of PCM and insulation, keeping total thickness of the floor constant.

Variation of the thermal load levelling (TLL) with $\frac{L_3}{L_3 + L_4}$ is shown in figure 6. It is clear that the best thermal load levelling which has minimum value is achieved for larger thickness of PCM i.e., $\frac{L_3}{L_3 + L_4}$ for thermal heating of room air temperature.

Further, it is important to note that under this condition the fluctuation or swing in greenhouse room air temperature is minimum and room air temperature is maximum. This condition may be referred as optimum or critical condition.

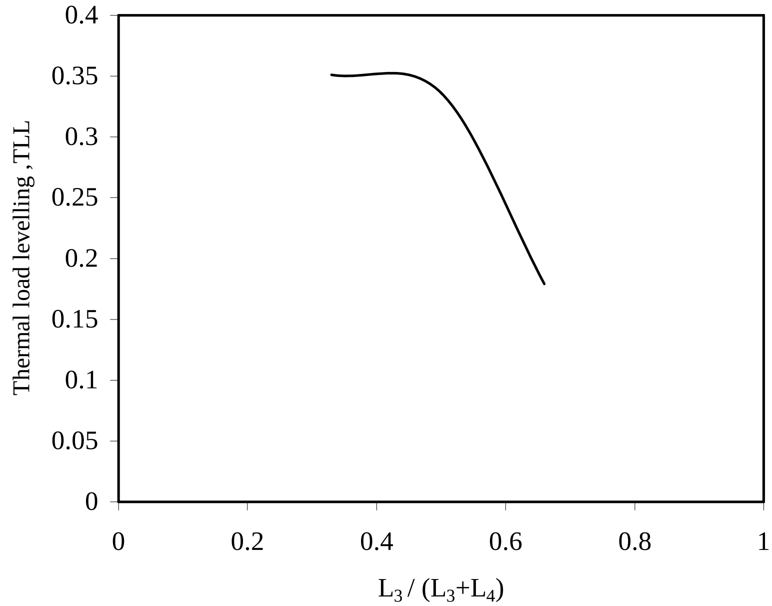

Figure 6. The variation of thermal load levelling (TLL) with different thickness of PCM and insulation, keeping the total floor thickness constant.

5. CONCLUSION

On the basis of present study, it has been observed that the PCM with insulation floor gives the better performance in the terms of plant temperature, room air temperature (fig.3) and thermal load levelling (fig. 6).

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Nomenclature

A_i area of walls and roofs of greenhouse (m^2)
A_d area of door (m^2)
A_p total surface area of plants (m^2)

A_i area of walls and roofs of greenhouse (m^2)
C specific heat (KJ/kg °C)
C_a thermal conductance of air gap
F_n fraction of solar energy falling at north wall
F_p fraction of solar energy falling on the plants
h_d overall heat transfer coefficient from door to ambient air (W/m^2 °C)
h(t) overall heat transfer coefficient from room air to ambient air through canopy (W/m^2 °C)
I_i total radiation on different walls and roofs of greenhouse.
K_1 thermal conductivity of floor (W/mK)
K_2 effective thermal conductivity of PCM (W/mK)
K_3 thermal conductivity of insulation (W/mK)
L_1 thickness of floor (m)
L_2 thickness of air gap (m)
L_3 thickness of effective PCM floor (m)
L_4 thickness of insulation (m)
M_p mass of the plant (kg)
N number of air changes
n n_{th} harmonic
S(t) total solar radiation available on greenhouse canopy cover (W)
S_{to} Time independent Fourier coefficient of total solar radiation on greenhouse
S_{tn} Time dependent Fourier coefficient of total solar radiation on greenhouse
t time (s)
T temperature (^oC)
T_a ambient air temperature (°C)
T_{ao} Time independent Fourier coefficient of ambient temperature (^oC)
T_{an} Time dependent Fourier coefficient of ambient temperature (^oC)
T_1 temperature of floor (°C)
T_2 effective temperature of PCM floor (°C)
$\overline{T}$ hourly average temperature (^oC)
T_3 insulation temperature (^oC)
T_4 ground temperature (^oC)
T_p plant temperature (°C)
T_r room air temperature (°C)
$T|_{x=0}$ floor surface temperature of greenhouse (°C)
V volume of greenhouse (m^3)

Greek Letters

α absorptivity
τ transmissivity of canopy cover
γ reflectivity
ρ density

Appendix

(A) Time Independent Part

The time independent part of equations (1)-(9) can be written in the following form:

$$\alpha_g(1-\gamma_g)(1-F_p)(1-F_N)(1-\gamma)\tau S_{to} = -A_1K_1A_1' + A_1h_1B_1' - A_1h_1T_{ro} \qquad \text{(I-1)}$$

$$-K_1A_1' = -K_2A_2' \qquad \text{(I-2)}$$

$$-K_2A_2' = C_aA_1'x_1 + C_aB_1' - C_aA_2'x_2 - C_aB_2' \qquad \text{(I-3)}$$

$$-K_2A_2' = -K_3A_3' \qquad \text{(I-4)}$$

$$A_2'x_3 + B_2' = A_3'x_3 + B_3' \qquad \text{(I-5)}$$

$$-K_iA_3' = 0 \qquad \text{(I-6)}$$

$$A_3'x_4 + B_3' = B_4' \qquad \text{(I-7)}$$

$$(A_1h_1)B_1' - (A_ph_p + A_1h_1 + h_iA_i + 0.33NV + h_dA_d)T_{ro} + A_ph_pT_{po} = (-h_iA_i - 0.33NV - h_dA_d)T_{ao}$$

(I-8)

$$\alpha_p(1-\gamma_p)F_p(1-F_N)(1-\gamma)\tau S_{to} = A_ph_pT_{po} - A_ph_pT_{ro} \qquad \text{(I-9)}$$

The above equations have been arranged into 9×9 matrix as:

$$\begin{bmatrix} -A_1K_1 & A_1h_1 & 0 & 0 & 0 & 0 & 0 & 0 & -A_1h_1 \\ K_1 & 0 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_aK_1 & C_a & (K_2 - x_2C_a) & -C_a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_2 & 0 & -K_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -x_3 & -1 & x_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -x_4 & -1 & 1 & 0 & 0 \\ 0 & -A_1h_1 & 0 & 0 & 0 & 0 & 0 & -A_ph_p & X \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_ph_p & -A_ph_p \end{bmatrix} \begin{bmatrix} A_1' \\ B_1' \\ A_2' \\ B_2' \\ A_3' \\ B_3' \\ T_{po} \\ T_{ro} \end{bmatrix} =$$

$$\begin{bmatrix} \alpha_g(1-\gamma_g)(1-F_p)(1-F_N)(1-\gamma)\tau S_{to} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ (h_i A_i + 0.33NV + h_d A_d)T_{ao} \\ \alpha_p(1-\gamma_p)F_p(1-F_N)(1-\gamma)\tau S_{to} \end{bmatrix} \qquad \text{(I-10)}$$

where $X = -(A_p h_p + A_f h_f + h_i A_i + 0.33NV + h_d A_d)$

(b) Time Dependent Part

Similarly the coefficients of the time dependent part can be obtained by solving the following matrix:

$$\begin{bmatrix}
(-A_1K_1\beta_1 + A_1h_1) & X_1 & 0 & 0 & 0 & 0 & 0 & 0 & -A_1h_1 \\
K_1\beta_1\exp(\beta_1x_1) & X_2 & -K_2\beta_2\exp(\beta_2x_2) & X_3 & 0 & 0 & 0 & 0 & 0 \\
C_a\exp(\beta_1x_1) & X_4 & X_5 & X_6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_2\beta_2\exp(\beta_2x_3) & X_7 & -K_3\beta_3\exp(\beta_3x_3) & X_8 & 0 & 0 & 0 \\
0 & 0 & -\exp(\beta_2x_3) & X_9 & \exp(\beta_3x_3) & X_{10} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & K_3\beta_3\exp(\beta_3x_4) & X_{11} & X_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & -\exp(\beta_3x_4) & X_{13} & \exp(-\beta_4x_4) & 0 & 0 \\
-A_1h_1 & A_1h_1 & 0 & 0 & 0 & 0 & 0 & -A_ph_p & X_{14} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & X_{15} & -A_ph_p
\end{bmatrix}$$

$$\begin{bmatrix} C_{1n} \\ D_{1n} \\ C_{2n} \\ D_{2n} \\ C_{3n} \\ D_{3n} \\ D_{4n} \\ T_{pn} \\ T_{rn} \end{bmatrix} = \begin{bmatrix} \alpha_g (1-\gamma_g)(1-F_p)(1-F_N)(1-\gamma)\tau S_{tn} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ (h_i A_i + 0.33NV + h_d A_d)T_{an} \\ \alpha_p (1-\gamma_p) F_p (1-F_N)(1-\gamma)\tau S_{tn} \end{bmatrix} \qquad \text{(I-11)}$$

where $X_1 = (A_1 K_1 \beta_1 + A_1 h_1)$

$X_2 = -K_1 \beta_1 \exp(-\beta_1 x_1)$

$X_3 = K_2 \beta_2 \exp(\beta_2 x_2)$

$X_4 = C_a \exp(-\beta_1 x_1)$

$X_5 = C_a(-\exp(\beta_2 x_2) + K_2 \beta_2 \exp(\beta_2 x_2))$

$X_6 = C_a(-\exp(-\beta_2 x_2) - K_2 \beta_2 \exp(-\beta_2 x_2))$

$X_7 = -K_2 \beta_2 \exp(-\beta_2 x_3)$

$X_8 = K_3 \beta_3 \exp(-\beta_3 x_3)$

$X_9 = -\exp(-\beta_2 x_3)$

$X_{10} = -\exp(-\beta_3 x_3)$

$X_{11} = -K_3 \beta_3 \exp(-\beta_3 x_4)$

$X_{12} = -K_4 \beta_4 \exp(-\beta_4 x_4)$
$X_{13} = -\exp(-\beta_3 x_4)$
$X_{14} = A_1 h_1 + A_p h_p + h_i A_i + h_d A_d + 0.33NV$
$X_{15} = M_p C_p \Sigma in\omega + A_p h_p$

The matrices given in appendix have been solved by using Matlab 6.1 for constants $A_1', B_1', A_2', B_2', A_3', B_3', B_4', T_{po}, T_{ro},$ and $C_{1n}, D_{1n}, C_{2n}, D_{2n}, C_{3n}, D_{3n}, D_{4n}, T_{pn}, T_{rn}$. for a given design parameters in table 1 and climatical parameters in figure 2. After knowing these constants, the hourly variation of T_1, T_2, T_3, T_4, T_p and T_r can be evaluated from equations (12-17).