Assessing saturated hydraulic conductivity from the dielectrically-predicted dry bulk density

Krissandi Wijaya1*, Purwoko Hari Kuncoro1, Taku Nishimura2, Budi Indra Setiawan3

(1. Faculty of Agriculture, Jenderal Soedirman University, Jl. dr. Soeparno No. 61, Karangwangkal, Purwokerto 53123, Central Java, Indonesia;
2. Graduate School of Agricultural and Life Sciences, The University of Tokyo, Yayoi 1-1-1, Bunkyo-ku, Tokyo 113-8657, Japan;
3. Department of Civil and Environmental Engineering, Bogor Agricultural University, Kampus IPB Darmaga, Bogor 16680, West Java, Indonesia)

Abstract: Direct measurement of saturated hydraulic conductivity ($K_s$) and dry bulk density ($\rho_b$) is often laborious, time consuming, and expensive. Predicted dry bulk density ($\rho_b$-ADR) using amplitude domain reflectometry (ADR) probe has been employed to predict saturated hydraulic conductivity ($K_s$-ADR) after a slight modification on the previously available non-similar media concept (NSMC) model. All the predicted values were validated using the corresponding measured data of 108 undisturbed samples for each of 0-15 and 15-30 cm soil depths observed. The proposed “Dielectric NSMC” model could describe $\rho_b$-ADR dependencies of $K_s$-ADR better than the other proceeding models for both ranges of soil depths observed. The proposed model tended to slightly underestimate the measured saturated hydraulic conductivity ($K_s$-meas) as implied from the negative value of mean difference (MD). The scattering pattern of data which was well supported by the small value of root mean square error (RMSE) and relative root mean square error (rRMSE), however, suggested a small deviation of the $K_s$-ADR from the $K_s$-meas. Thus, the proposed model was considered sufficient for characterizing $\rho_b$-ADR dependencies of $K_s$-ADR. Effectiveness and practical merit of the $\rho_b$-ADR prediction rather than the conventional direct measurement of $\rho_b$ in particular, might promote a certain advantage for smoother fitting parameters.

Keywords: amplitude domain reflectometry (ADR) probe, dry bulk density, non-similar media concept (NSMC), pedo-transfer function, saturated hydraulic conductivity.


1 Introduction

Saturated hydraulic conductivity ($K_s$) is of the main importance of the dynamic status of water in soil (Hillel, 1998). The value of $K_s$ can be used to characterize soil surface runoff, drainage, solute transports, percolation, and runoff (Nishimura et al., 1993; Arya et al., 1998; Malone et al., 2003; Wijaya et al., 2010). Those are determinant for the effectiveness of water and nutrient uptake by plant roots on which the plant growth and yield depend (Kirkham, 2005).

Unfortunately, the measurement of $K_s$ is often found laborious, time consuming, and expensive (Libardi et al., 1980). To deal with this problem, pedotransfer functions for predicting $K_s$ have been extensively expanded by employing the more easily measured soil physical properties: water content (Libardi et al., 1980), effective porosity (Ahuja et al., 1989), moisture retention (Messing,
1989), air permeability (Loll et al., 1999), and texture (Julia et al., 2004).

In particular, Kozeny-Carman equation (Kozeny, 1927; Carman, 1937) has been able to describe the dry bulk density ($\rho_b$) dependencies of $K_s$, and using the similar media concept (SMC) of Miller and Miller (1956), this dependency has been further clarified by Campbell (1985). Miyazaki (1996), on the other hand, has proposed a non-similar media concept (NSMC) for exploring the $\rho_b$ dependencies of $K_s$, and the result was better than these two preceding models. This NSMC model is applicable to the wide range of soil structure: dispersed to aggregated soils (Miyazaki, 1996), helpful for scaling usage (Zhuang et al., 2000), and yet reliable for compact subsoil (Nakano and Miyazaki, 2005).

About the soil $\rho_b$, it is one of the most frequently used parameters for describing the physical properties of soil. The direct measurement, however, is yet laborious and time consuming likewise, and thus, is unfavorable for vast area. Wijaya et al. (2003) have proposed the use of dielectrically-predicted dry bulk density ($\rho_b$-ADR) instead. The $\rho_b$-ADR was assessed from the value of dielectrically-predicted volumetric water content ($\theta_{ADR}$) using an amplitude domain reflectometry (ADR) probe. The model has been found to be applicable for the either of upland and wetland soils (Wijaya et al., 2004; Wijaya and Kuncoro, 2008).

Despite the effectiveness and practical merit of $\rho_b$-ADR prediction, the $\rho_b$-ADR dependencies of $K_s$ has yet been paid less attention. Thus, a model for predicting $K_s$ using $\rho_b$-ADR has been proposed in this study after a slight modification on the previously available NSMC model.

2 Materials and methods

2.1 Land preparation

A plot 7.5 m × 7.5 m of Andisol (Table 1) located at the Experimental Field of Jenderal Soedirman University – Indonesia ($7^\circ14'53''$S, $109^\circ56'24''$E) was selected as a research site. The plot was imaginary divided into 25 rectangles 1.5 m × 1.5 m of which each corner was taken as the point of field measurement and soil sampling (Figure 1). Thus, there were 36 points in total available. Such plot design was performed within a purpose to endorse the study about spatial variability of the data of interest, which will be presented in another paper. The field, in particular, has been followed for years, but annually plowed and harrowed twice for maintenance.

![Figure 1 Field schematic diagram of the sampling and measurement points](image)

Table 1 Physical properties of the soil

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Texture (%)</td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>37.4</td>
</tr>
<tr>
<td>Silt</td>
<td>48.2</td>
</tr>
<tr>
<td>Clay</td>
<td>14.4</td>
</tr>
<tr>
<td>Organic carbon content, C-org (%)</td>
<td>9.4</td>
</tr>
<tr>
<td>Particle density, $\rho_t$ (g cm$^{-3}$)</td>
<td>2.47</td>
</tr>
<tr>
<td>Dry bulk density, $\rho_b$ (g cm$^{-3}$)</td>
<td>0.650</td>
</tr>
<tr>
<td>Total density, $\rho_t$ (g cm$^{-3}$)</td>
<td>0.964</td>
</tr>
<tr>
<td>Water content at field capacity, FC (cm$^3$ cm$^{-3}$)</td>
<td>0.81</td>
</tr>
<tr>
<td>Water content at permanent wilting point, PWP (cm$^3$ cm$^{-3}$)</td>
<td>0.21</td>
</tr>
</tbody>
</table>

2.2 Calibration of ADR probe used

A set of ADR probes that latterly used for field measurement was preliminary calibrated using disturbed samples taken from 0-15 and 15-30 cm soil depths of the targeted field. The samples were mixed and repacked into three acrylic cylinders (11 cm in diameter and 8 cm height) for three levels of soil mass wetness, $w$ (0.30 – 0.80 g g$^{-1}$) and five levels of total density, $\rho_t$ (0.70 – 1.20 g cm$^{-3}$). Determination of soil dielectric constant ($\nu$) was performed...
by perpendicularly embedding sensor rods of the ADR probe into the targeted sample repacked (Figure 2). This measurement took about 15 seconds for a stable data reading, and has been conducted for three replications of each acrylic cylinder.

![Figure 2 Schematic diagram of the ADR probe calibration](image)

A slight amount of samples, namely about 20 g of each, were taken out of the one third-upper, middle, and lower parts of the acrylic cylinder. These samples were weighed to determine \( \rho_t \), and then oven-dried at 105°C for 24 hours to determine \( w \), by which the value of \( \rho_b \) could be calculated from. The data of \( w \) and \( \rho_b \) were then used to determine the value of volumetric water content (\( \theta \)) related. Finally, the data of \( \theta \) and \( v \) were plotted and fitted using least-square method (Figure 3) to obtain the model for predicting \( \theta \) namely \( \theta_{ADR} \) as in Wijaya et al. (2003, 2004):

\[
\theta_{ADR} = 2.5582 v^3 - 4.7527 v^2 + 3.2659 v - 0.3970 \tag{1}
\]

Where: \( \theta_{ADR} \) is dielectrically predicted \( \theta \) (cm\(^3\) cm\(^{-3}\)), and \( v \) is dielectric constant data (volt).

Figure 3 Relationship between volumetric water content, \( \theta \) (cm\(^3\) cm\(^{-3}\)) and dielectric constant, \( v \) (volt) after the calibration of ADR probe conducted

2.3 Soil sampling and measurement

The calibrated ADR probe was applied to the targeted measurement point (Figure 1) for data collection of the soil dielectric constant. This measurement was conducted at 0-15 and 15-30 cm soil depths for three replications. Thus, total number of the data collected was 216 of which 108 data were obtained from the each depth. By recalling Equation 1, the data were then employed to determine the related \( \theta_{ADR} \) as in Wijaya et al. (2003, 2004).

After the measurement, undisturbed soil sample was taken using 100 cm\(^3\) core sampler (5 cm in diameter and 5 cm height) for the same number of replications and a range of soil depth nearby. The sample was weighed to determine \( \rho_t \) and then water saturated for \( K_s \) measurement using the falling head method. Finally, the sample was oven-dried at 105°C for 24 hours to determine the value of \( w \) and \( \rho_b \), by which the value of \( \theta \) related could be calculated from.

2.4 Model development

Previously available NSMC model for predicting \( K_s \) (Miyazaki, 1996) may be written as:

\[
K_s = K_{s0} \left[ \frac{\tau \rho_s}{\rho_b} \right]^{\frac{1}{3}} \left[ \frac{\rho_b}{\rho_b0} \right]^{\frac{1}{3}} - 1 \tag{2}
\]

Where: \( K_s \) is predicted saturated hydraulic conductivity (cm s\(^{-1}\)), \( K_{s0} \) is saturated hydraulic conductivity of the sample reference (cm s\(^{-1}\)), \( \tau \) is shape factor of the solid phase (\( \rho_b/\rho_s < \tau \leq 1 \)), \( \rho_s \) is particle density of the sample reference (g cm\(^{-3}\)), \( \rho_b \) is dry bulk density of the soil sample examined (g cm\(^{-3}\)), and \( \rho_{b0} \) is dry bulk density of the sample reference (g cm\(^{-3}\)).

Meanwhile, the value of \( \rho_{b,ADR} \) could be determined from \( \rho_t \) and \( \theta_{ADR} \) (Wijaya et al., 2003, 2004):

\[
\rho_{b,ADR} = \rho_t - (\theta_{ADR} \rho_w) \tag{3}
\]

Where: \( \rho_{b,ADR} \) is dielectrically-predicted dry bulk density (g cm\(^{-3}\)), \( \rho_t \) is the total density (g cm\(^{-3}\)), \( \theta_{ADR} \) is
dielectrically-predicted volumetric water content (cm$^3$ cm$^{-3}$), and $\rho_w$ is the water density (1 g cm$^{-3}$).

By substituting $\rho_b$ in the Equation 2 with $\rho_{b-ADR}$ in Equation 3, a new model for predicting $K_s$ may take shape as:

$$K_{s-ADR} = K_{s0} \left(\frac{\tau_s}{\rho_{b-ADR}}\right)^2 - 1$$

(4)

Where: $K_{s-ADR}$ is dielectrically-predicted saturated hydraulic conductivity (cm s$^{-1}$). For the sake of simplicity, we suggested to call this new proposed model as “Dielectric NSMC” in order to distinguish it from the previously available NSMC model.

2.5 Model validation

All those dielectrically-predicted parameters $\theta_{ADR}$, $\rho_{b-ADR}$, and $K_{s-ADR}$ were validated using the measured $\theta$, $\rho_b$, and $K_s$, respectively. The analysis was performed upon the value of mean difference (MD), root mean square error (RMSE), relative root mean square error (rRMSE), and deviation times (DT):

$$MD = \frac{1}{n} \sum_{i=1}^{n}(PV_i - MV_i)$$

(5)

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n}(PV_i - MV_i)^2}$$

(6)

$$rRMSE = \frac{RMSE \times 100}{MV}$$

(7)

$$\log_{10} DT = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left[\log_{10}\left(\frac{K_{s-ADR}}{K_s}\right)\right]^2}$$

(8)

Where: PV is the value of dielectrically predicted parameters (cm$^3$ cm$^{-3}$, g cm$^{-3}$, or cm s$^{-1}$), MV is the value of measured parameters (cm$^3$ cm$^{-3}$, g cm$^{-3}$, or cm s$^{-1}$), MV is the mean of measured parameters (cm$^3$ cm$^{-3}$, g cm$^{-3}$, or cm s$^{-1}$), and n is the sample number.

3 Results and discussion

3.1 Dielectrically-predicted volumetric water content ($\theta_{ADR}$)

As shown in Figure 4, there was a tendency for the $\theta_{ADR}$ predicted to slightly overestimate the $\theta_{meas}$ measured. This result was also implied from the positive value of mean difference (MD) as 0.033 and 0.053 for the both soil depths observed 0-15 and 15-30 cm, respectively (Table 2). The small values of RMSE and rRMSE for these depths, however, suggested a small deviation of the $\theta_{ADR}$ from the $\theta_{meas}$, and this was also revealed from the scattering pattern of the data with regard to the 1:1 line in Figure 4. These results, in general, suggested that there was a fairly good agreement between the $\theta_{ADR}$ and $\theta_{meas}$, in line with the result of Wijaya et al. (2003, 2004, 2010).

If we took a look closer to the soil depths, in particular, 15-30 cm depth resulted a slightly higher value of RMSE and rRMSE, rather than the 0-15 cm depth (Figure 4), which implied a bit more discrepancy between the $\theta_{ADR}$ and $\theta_{meas}$ at this former depth. This result was ascribed to the remaining earthworm furrows or dense layer of this untilled 15-30 cm depth, which may in turn affect the volume of soil macro-pores of the soil pores (McColl et al., 1982; Hillel, 1998; Malone et al., 2003; Domingues et al., 2018; Wijaya et al., 2019), and thus, alter the soil $\theta$ as well (Horn et al., 2014; Iiyama and Hirai, 2014). As the $\theta_{ADR}$ tended to slightly overestimate $\theta_{meas}$, it was due to the remaining dense layer to be more dominant than the remaining earthworm furrows, of which the less volume of macro-pores resulted the less value of $\theta_{meas}$. 

![Figure 4 Dielectrically-predicted ($\theta_{ADR}$) vs. measured ($\theta_{meas}$) volumetric water content](image-url)
Table 2 Statistical values for the degree of coincidence between dielectrically-predicted and measured data

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Soil depth (cm)</th>
<th>Sample number (n)</th>
<th>MD</th>
<th>RMSE</th>
<th>rRMSE (%)</th>
<th>DT (times)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta) (cm(^3) cm(^{-3}))</td>
<td>0 – 15</td>
<td>108</td>
<td>0.033</td>
<td>0.053</td>
<td>13.41</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>15 – 30</td>
<td>108</td>
<td>0.053</td>
<td>0.063</td>
<td>15.27</td>
<td>NA</td>
</tr>
<tr>
<td>(\rho_b) (g cm(^{-3}))</td>
<td>0 – 15</td>
<td>108</td>
<td>-0.033</td>
<td>0.053</td>
<td>8.90</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>15 – 30</td>
<td>108</td>
<td>-0.053</td>
<td>0.064</td>
<td>10.27</td>
<td>NA</td>
</tr>
<tr>
<td>(K_s) (cm s(^{-1}))</td>
<td>0 – 15</td>
<td>108</td>
<td>-0.00157</td>
<td>0.011</td>
<td>38.11</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>15 – 30</td>
<td>108</td>
<td>-0.00035</td>
<td>0.009</td>
<td>52.84</td>
<td>1.73</td>
</tr>
</tbody>
</table>

3.2 Dielectrically-predicted dry bulk density (\(\rho_{bADR}\))

There was a tendency for the predicted \(\rho_{bADR}\) to slightly underestimate the measured \(\rho_{b_meas}\) (Figure 5), and this was in line with the negative value of MD as -0.033 and -0.053 for the depth of 0-15 and 15-30 cm, respectively (Table 2). This result, however, was conversely to the case of \(\theta_{ADR}\) to \(\theta_{meas}\) shown in Figure 4. In fact, the soil dry bulk density (\(\rho_b\)) has a negative correlation with the total soil porosity (e.g. Etana et al., 2013; Kuncoro et al., 2014; Domingues et al., 2018), and thus, with the soil volumetric water content (\(\theta\)) as well.

Except for the value of \(\rho_{b_meas}\) less than 0.47 g cm\(^{-3}\), the scattering pattern of the data with regard to the 1:1 line in Figure 5 implies a relatively small deviation of the \(\rho_{bADR}\) from the \(\rho_{b_meas}\), which was further explained by the small value of RMSE and rRMSE for the both depths observed. This result, in general, implied that there was a fairly good agreement between the \(\rho_{bADR}\) and \(\rho_{b_meas}\), and this was in a good agreement with the finding of Wijaya et al. (2003, 2004, 2010).

3.3 Dielectrically-predicted saturated hydraulic conductivity (\(K_{sADR}\)) from the “Dielectric NSMC” model developed

In particular, a slightly higher value of RMSE and rRMSE for the depth of 15-30 cm than the depth of 0-15 cm (Figure 5) implied a bit more discrepancy between \(\rho_{bADR}\) and \(\rho_{b_meas}\) for the former depth. Likewise, this result was thought to be ascribed to the remaining earthworm furrows or the development of dense layers of this untilled 15-30 cm depth (McColl et al., 1982; Hillel, 1998; Malone et al., 2003; Kuncoro et al., 2014; Domingues et al., 2018; Wijaya et al., 2019). As the \(\rho_{bADR}\) tended to slightly underestimate \(\rho_{b_meas}\), more dominant remaining dense layer than the earthworm furrows was the reason for the higher value of \(\rho_{b_meas}\).
suggests that special care should be given when specifying $K_0$ and $\rho_{b0}$ since these values might be site specific and soil type dependent.

Adding to this, the number of $\rho_b$ data of interest used for determining those fitting parameters might also be a point of concern: 108 data of predicted $\rho_{b ADR}$ in this study, but 18 measured $\rho_b$ of repacked disturbed-samples in Miyazaki (1996). It seems that the larger number of $\rho_b$ data used for, the smoother fitting parameters will be resulted. In this manner, therefore, the effectiveness and practical merit of $\rho_b$ prediction using ADR probe proposed in this study provide a certain advantage compared to the conventional direct measurement of $\rho_b$.

![Figure 6 Dielectrically-predicted saturated hydraulic conductivity ($K_{s ADR}$) vs. dielectrically-predicted dry bulk density ($\rho_{b ADR}$)](image)

Table 3 Fitted parameters for the $K_s$ prediction

<table>
<thead>
<tr>
<th>Soil depth (cm)</th>
<th>Sample number (n)</th>
<th>Dielectric NSMC model developed</th>
<th>NSMC model (Miyazaki, 1996)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K_0$ (cm s⁻¹)</td>
<td>$\rho_{b0}$ (g cm⁻³)</td>
</tr>
<tr>
<td>0 – 15</td>
<td>108</td>
<td>0.0653</td>
<td>0.306</td>
</tr>
<tr>
<td>15 – 30</td>
<td>108</td>
<td>0.0412</td>
<td>0.294</td>
</tr>
</tbody>
</table>

Considering about the soil depths, on the other hand, $K_{s ADR}$ of 15-30 cm depth was found a bit more scattered than that of 0-15 cm depth, especially for the higher $\rho_{b ADR}$ namely more than 0.62 g cm⁻³ (Figure 6). This result was in line with the aforementioned higher discrepancy of the $\theta_{ADR}$ and $\rho_{b ADR}$ for this depth of interest (Figure 4 and Figure 5, respectively). This was related to the variety in soil macro-pores volume due to the remaining earthworm furrows or dense layer of this untilled 15-30 cm depth (McColl et al., 1982; Hillel, 1998; Ahuja et al., 1989; Arya et al., 1998; Horn et al., 2014; Domingues et al., 2018; Wijaya et al., 2019).

3.4 Dielectrically-predicted ($K_{s ADR}$) vs. measured ($K_{s mean}$) saturated hydraulic conductivity

There was a tendency for the $K_{s ADR}$ to slightly underestimate $K_{s mean}$ (Figure 7), similar to the case of the aforementioned $\rho_{b ADR}$ to $\rho_{b mean}$ (Figure 5). This result was further supported by the negative value of MD as -0.00157 and -0.00035 for the both depths observed 0-15 and 15-30 cm, respectively (Table 2). Nevertheless, the small values of RMSE and rRMSE for these depths (Table 2) indicated a small deviation of the $K_{s ADR}$ from the $K_{s mean}$. This was in accordance with the scattering pattern of the data with regard to the 1:1 line in Figure 7 except for the $K_{s mean}$ data less than 0.004 cm s⁻¹ of 15-30 cm depth. This result, in general, suggested that there was a fairly good agreement between the $K_{s ADR}$ and $K_{s mean}$. Thus, the developed “Dielectric NSMC” model was considerably sufficient for predicting $K_s$ upon the Andisol soil observed in this study.

![Figure 7 Dielectrically-predicted ($K_{s ADR}$) vs. measured ($K_{s mean}$) saturated hydraulic conductivity.](image)
the lower RMSE value of the latter (Figure 7). This discrepancy might be explained by the higher values of rRMSE and DT of the latter than that of the former (Table 2). In addition, the latter tended to result lower \( K_s_{ADR} \) than the former for a certain \( \rho_b_{ADR} \) given as shown in Figure 6. This result, however, was different from the finding of Nakano and Miyazaki (2005) that \( K_s \) of untilted subsoil was well related with its \( \rho_b \) by the NSMC model and always higher than the \( K_s \) of tilled topsoil.

The difference was probably related to the type of soil observed: Eutric Fluvisol in Nakano and Miyazaki (2005), but Andisol in this study. Further, site specific characteristics of the soil structure might also be a point of concern. Nakano and Miyazaki (2005) have assumed that untilted subsoil tends to be compacted without serious changes in micromorphology of soil pores, while we thought about changes in the volume of macro-pores due to the remaining earthworm furrows or dense layer of this untilted 15-30 cm depth (McColl et al., 1982; Hillel, 1998; Malone et al., 2003; Wijaya et al., 2010; Etana et al., 2013; Domingues et al., 2018; Wijaya et al., 2019).

4 Conclusion

Predicted dry bulk density (\( \rho_b_{ADR} \)) using ADR probe has been employed to predict saturated hydraulic conductivity (\( K_s_{ADR} \)) after a slight modification on the previously available NSMC model. The proposed “Dielectric NSMC” model could describe \( \rho_b_{ADR} \) dependencies of \( K_s_{ADR} \) better than the other proceeding models for both ranges 0-15 and 15-30 cm of soil depths observed. This proposed model tended to slightly underestimate the measured saturated hydraulic conductivity (\( K_s_{meas} \)) as implied from the negative value of MD. The scattering pattern of data which was well supported by the small value of RMSE and rRMSE, however, suggested a small deviation of the \( K_s_{ADR} \) from the \( K_s_{meas} \). In general, therefore, the proposed model was considerably sufficient for characterizing \( \rho_b_{ADR} \) dependencies of \( K_s_{ADR} \).

Acknowledgement

This study was partly supported by the Directorate General for Higher Education of The Ministry of Research, Technology, and Higher Education of Indonesia. The authors are grateful to Dr. Tetsuya Araki for the very valuable discussion and constructive suggestion. Further, the authors acknowledged Dr. Ardiansyah for the inspiring idea during the data collection.

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