Mathematical models of geometric sizes of coffee beans as dependent random variables

Roman Kuzminskyj*, Roman Sheremeta

(*Faculty of Mechanical and Energy Engineering, Lviv National Agrarian University, Volodymyr the Great st., 1, Dubliany, Zhovkva district, Lviv region, 80381, Ukraine*)

Abstract: Dimensions of 100 randomly selected coffee beans of the Arabica and Robusta variety were determined by measuring their length (*l*), width (*b*) and thickness (*h*). The results of the measurements were processed by the methods of mathematical statistics. Parameters of distributions of separate sizes as random variables were determined. By the value of the coefficient of variation, the density function of normal distribution (Gaussian distribution) was taken as a model of separate sizes of beans. Models of two-dimensional distributions of beans sizes as independent random variables were presented. The coefficients of correlation between the geometric sizes of beans are calculated. The obtained values of the correlation coefficients indicated that the geometric sizes of beans should be considered as dependent random variables. The mathematical models of geometric sizes of beans as dependent random variables as density functions of their normal distribution were proposed. By values of the sums of squared deviations as a fitting criterion it has been established that the mathematical models of geometric sizes of beans as dependent random variables in the form of density functions of their normal distribution provide better data approximation than the mathematical models of geometric sizes of beans as independent random variables.

Keywords: coffee beans, geometric parameters, mathematical models, distribution function

Citation: Kuzminskyj, R., and R. Sheremeta. 2018. Mathematical models of geometric sizes of coffee beans as dependent random variables. Agricultural Engineering International: CIGR Journal, 20(4): 178–183.

1 Introduction

Beans and seeds as a biological material are very variable. Geometric sizes of beans, berries and seeds of different crops need to be known in order to provide an efficient separation process, as well as for the design of machinery and equipment for transportation, storage and further processing. Sometimes, only two geometric sizes of beans, berries and seeds, such as length and width are distinguished (since the thickness and width of beans, berries and seeds are considered to be the same) (Karimi et al., 2011; Zewdu and Solomon, 2007). Beyond that, only data on the limits of the variation of individual geometric sizes of some beans, berries and seeds are mainly given in the literature (Müller et al., 2015; Zewdu and Solomon, 2007). For all intents and purposes, it is important to specify the probability distribution laws of all three principal dimensions of beans, berries and seeds as random variables, the relationships between individual sizes taking into account additionally.

Researchers from many countries studied the geometric sizes of seeds of different crops (Karaj and Muëller, 2010; Müller et al., 2015; Niveditha et al., 2013). Olukunle and Akinnuli (2012) collected the geometrical statistical data for coffee seeds and beans and obtained that for coffee seeds their average length, width and thickness were 9.78, 7.24 and 5.23 mm with standard deviation of 0.67, 0.59 and 0.49 respectively while that for coffee beans their average length, width and thickness were 8.19, 6.11 and 4.60 mm with standard deviation of 0.8, 0.59 and 0.5 respectively. El-Gendy et al. (2011) measured the main dimensions (length, width, thickness) of three categories of Arabian coffee seeds (Himy, Esmaely and Ma'arepy) and founded that the highest

Received date: 2018-02-15 **Accepted date:** 2018-04-16

***Corresponding author:** Roman Kuzminskyj, Dr. sc., Volodymyr the Great st., 1, Dubliany, Zhovkva district, Lviv region, 80381, Ukraine, +380968600796, Email: rkuzminsky@gmail.com.

frequency for length was 10 mm, for width - 7 mm and for thickness - 7 mm. Severa et al. (2012) analyzed twenty different coffee types originating from 13 different countries and confirmed a dominant importance and the relevance of length-to-width ratio.

In many scientific papers, the effect of relative humidity on the dimensions of seeds of different crops was investigated (Karimi et al., 2009; Milani et al., 2007; Sanchez-Mendoza et al., 2008; Sokolovskyj et al., 2012; Zewdu and Solomon, 2007). In particular many researches examined the geometric sizes of beans, berries and seeds as random variables (Gursoy and Guzel, 2010; Karimi et al., 2011; Mansouri et al., 2015; Mirzabe et al., 2013). For example, as a result of the statistical processing of measurements of geometric sizes of winter wheat seeds, the laws of distribution of length, width and thickness of seeds of separate varieties were established (Sokolovskyj et al., 2012). The two-dimensional Weibull distribution was used to describe the geometric sizes of raisin berries, assuming that individual geometric sizes were independent random variables (Karimi et al., 2011). Obtaining two-dimensional distributions of geometric sizes of seeds is important for ensuring the proper quality of separation processes (Kuzminskyj et al., 2014).

The aim of the study is to establish the relationship between separate geometric sizes of coffee beans and to obtain mathematical models of their distribution based on the results of experimental studies. Such a mathematical model will allow you to accurately thaw the size of coffee beans to model transport processes during storage, processing (separation, grinding) and packaging.

2 Material and methods

The geometric dimensions of coffee beans (see Figure 1) were determined by measuring length (l), width (b) and thickness (h).

Figure 1 Dimensions of a coffee bean

Dimensions of 100 randomly selected coffee beans of the Arabica and Robusta variety were measured using an microscope Sigeta Forward (see Figure 2) with an increase from 10 to 500 times.

Figure 2 Microscope Sigeta Forward

Density functions of the normal distribution of separate dimensions as random variables are as follows:

$$f(x) = \frac{1}{\sigma[x]\sqrt{2\pi}} e^{-\left[\frac{(x-M[x])^2}{2\sigma[x]^2}\right]} \tag{1}$$

and the integral function of their distribution:

$$F(x) = \frac{1}{\sigma[x]\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\left[\frac{(x-M[x])^2}{2\sigma[x]^2}\right]} dx \tag{2}$$

where, M[x] – mathematical expectation, mm; σ[x] – mean square deviation, mm; x – random variable (length, width or thickness of the beans), mm.

For example, considering the length and width of the coffee beans as independent normally distributed random variables, taking into account the type of differential function (1), the model of their two-dimensional distribution is obtained:

$$f(l,b) = f(l)f(b) \tag{3}$$

$$f(l,b) = \frac{1}{2\pi\sigma[l]\sigma[b]} e^{-0,5\left[\frac{(l-M[l])^2}{\sigma[l]^2}+\frac{(b-M[b])^2}{\sigma[b]^2}\right]} \tag{4}$$

Given the correlation coefficient r_{lb} between the length l and width b of the bean, the two-dimensional normal distribution model will have the form:

$$f(l,b) = \frac{1}{2\pi\sigma[l]\sigma[b]\sqrt{1-r_{lb}^2}} e^{\frac{-1}{2\left(1-r_{lb}^2\right)}\left[\frac{(l-M[l])^2}{\sigma[l]^2}+\frac{(b-M[b])^2}{\sigma[b]^2}-2r_{lb}\frac{l-M[l]}{\sigma[l]}\cdot\frac{b-M[b]}{\sigma[b]}\right]} \tag{5}$$

where, $M[l]$, $M[b]$ – mathematical expectation of the length and width of beans, respectively, mm; $\sigma[l]$, $\sigma[b]$ –

mean square deviation of the length and width of beans respectively, mm; r_{lb} – the value of the correlation coefficient between l and b.

The correlation coefficient is calculated by the formula:

$$r_{xy} = \frac{\sum(x - M[x])(y - M[y])}{\sqrt{\sum(x - M[x])^2 \sum(y - M[y])^2}} \quad (6)$$

where, x, y – geometric dimensions (length, width or height), mm; $M[x]$, $M[y]$ – mathematical expectations of the sizes x and y respectively, mm.

The criterion of the sum of squared deviations (*SSD*) is used in order to compare the results of the approximation of experimental data on the measurement of bean grain sizes by two-dimensional distributions of their geometric sizes, both independent and dependent random variables. The criterion of fitting is calculated by the formula:

$$SSD = \sum_{i=1}^{k_i}\left(\sum_{j=1}^{k_j}(m_{i,j} - m_{\mathrm{T}\,i,j})^2\right) \quad (7)$$

where, i, j – sequence count of intervals of geometric sizes of beans, as random variables; k_i, k_j – number of intervals of separate geometric sizes of beans; $m_{i,j}$, $m_{\mathrm{T}\,i,j}$ – the number of data in separate intervals of geometric sizes of beans, obtained respectively as the results of measurements and according to the calculation results using the theoretical models in the form of two-dimensional distributions of random variables.

3 Results and discussion

By the values of coefficient of variation of separate geometric sizes of coffee beans, a hypothesis about their normal distribution (Gaussian distribution) was proposed.

The differential functions of the normal distribution of l, b and h for the coffee beans of the Arabica variety were as follows:

$$f(l) = \frac{1}{0.458\sqrt{2\pi}} e^{-\left[\frac{(l-9.25)^2}{2\cdot 0.458^2}\right]} \quad (8)$$

$$f(b) = \frac{1}{0.135\sqrt{2\pi}} e^{-\left[\frac{(b-7)^2}{2\cdot 0.135^2}\right]} \quad (9)$$

$$f(h) = \frac{1}{0.108\sqrt{2\pi}} e^{-\left[\frac{(h-3.6)^2}{2\cdot 0.108^2}\right]} \quad (10)$$

Figure 3 showed histograms and density functions for the l, b and h of coffee beans.

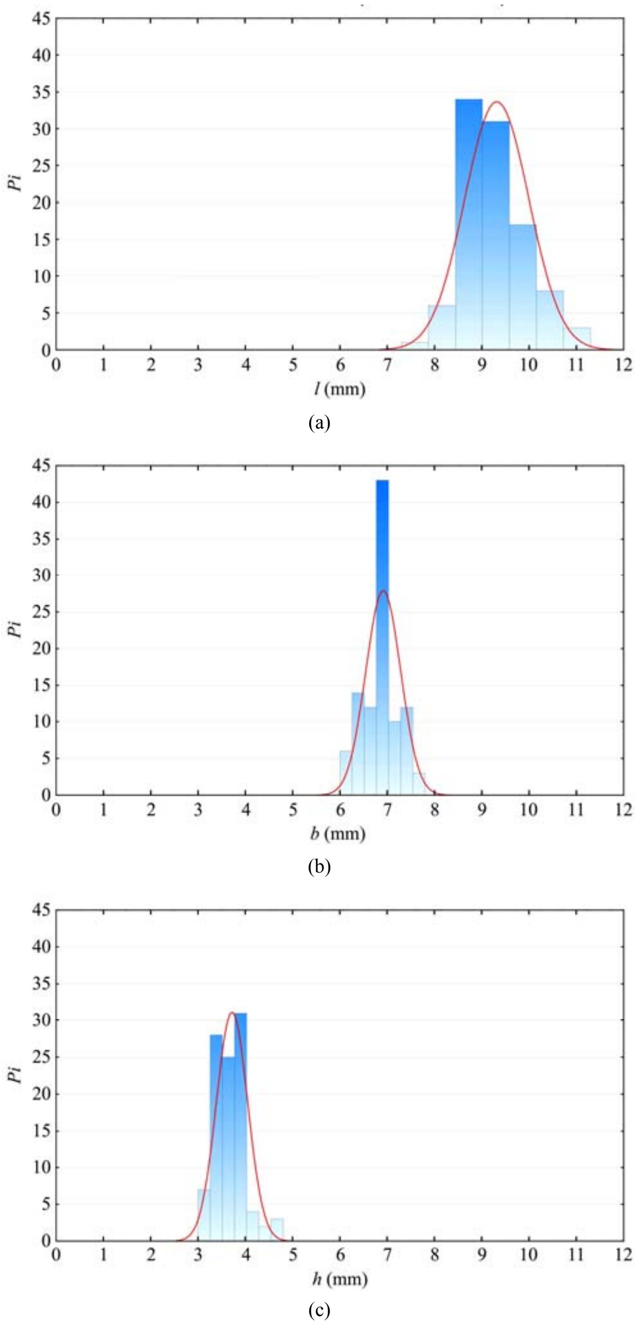


Figure 3 Histograms and density functions of probability distribution of geometric sizes of coffee beans of the Arabica variety

Parameters of the normal distribution of the geometrical sizes (length, width and thickness) of coffee beans of the Arabica and Robusta variety were given in Table 1.

The values of the correlation coefficients between the separate dimensions of beans of the studied varieties of coffee were given in Table 2.

Since the values of the correlation coefficient between the separate dimensions of coffee beans are $r = 0.3$-0.5, that indicates a steady correlation between these sizes according to Cheddok scale. Consequently, such correlation must be taken into account in the model of a two-dimensional normal distribution of beans sizes.

Table 1 Parameters of normal distribution of geometric sizes of coffee beans

Culture, variety	Dimensions of seed	Parameters	
		$M[x]$ (mm)	$\sigma[x]$ (mm)
Coffee beans Arabica variety	Length l (mm)	9.25	0.6767
	Width b (mm)	7.0	0.3671
	Thickness h (mm)	3.6	0.3289
Coffee beans Robusta variety	Length l (mm)	10.55	1.0438
	Width b (mm)	8.0	0.6304
	Thickness h (mm)	4.6	0.542

Table 2 The values of the coefficients of correlation between the separate dimensions of the studied varieties of coffee beans

Culture, variety	Correlation coefficients		
	Between length and width	Between width and thickness	Between length and thickness
Coffee beans Arabica variety	0.379	0.147	0.262
Coffee beans Robusta variety	0.124	0.052	0.168

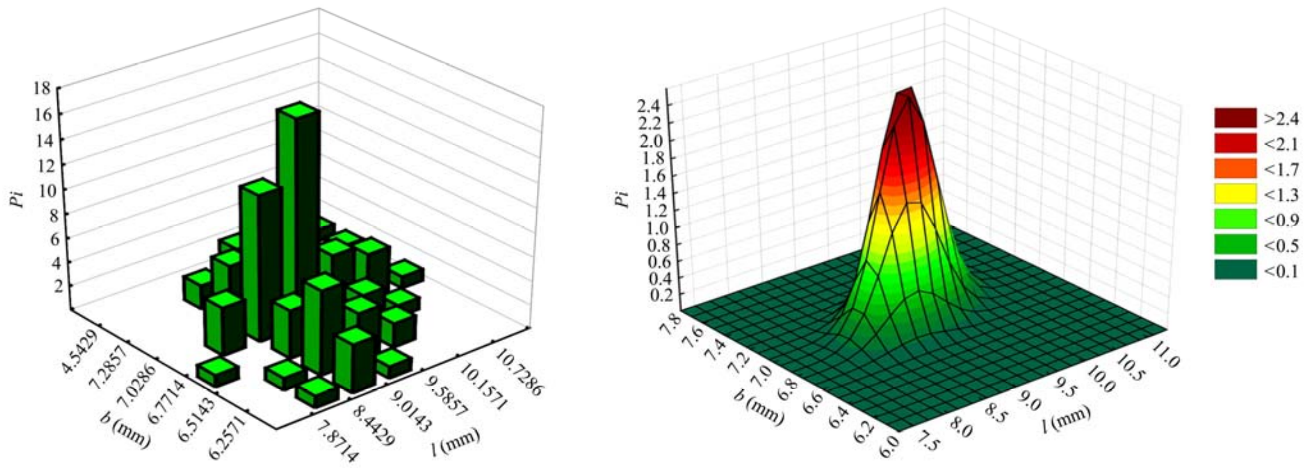
Thus, the two-dimensional normal distribution models of geometric dimensions of coffee beans of the Arabica variety were as follows:

$$f(l,b) = \frac{1}{2\pi 0.458\cdot 0.135\sqrt{1-0.379^2}} e^{\frac{-1}{2(1-0.379^2)}\left[\frac{(l-9.25)^2}{0.458^2}+\frac{(b-7)^2}{0.135^2}-2\cdot 0.379\frac{l-9.25}{0.458}\cdot\frac{b-7}{0.135}\right]} \tag{11}$$

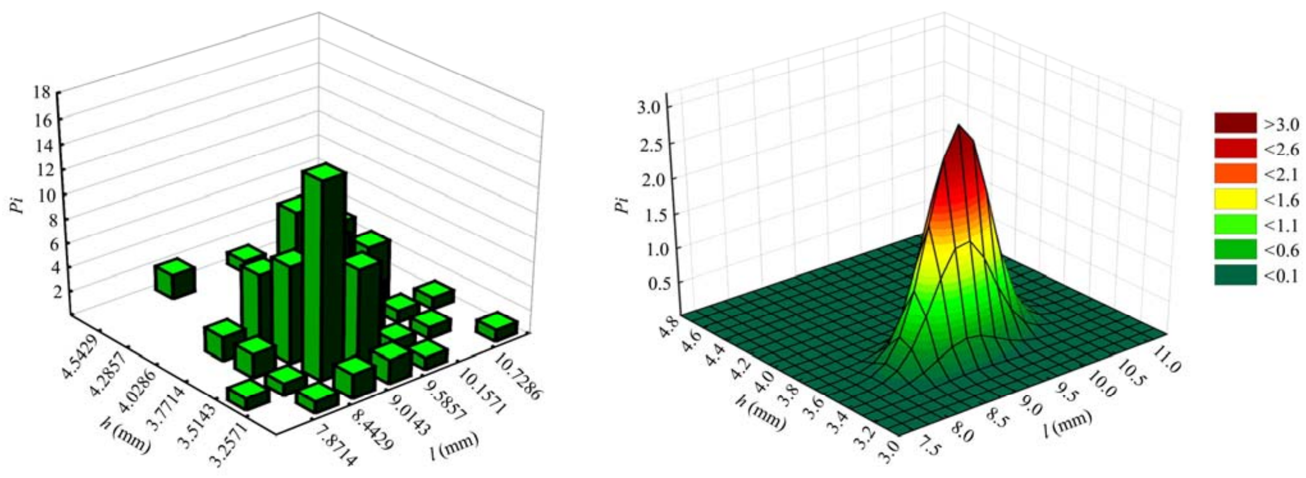
$$f(l,h) = \frac{1}{2\pi 0.458\cdot 0.108\sqrt{1-0.262^2}} e^{\frac{-1}{2(1-0.262^2)}\left[\frac{(l-9.25)^2}{0.458^2}+\frac{(h-3.6)^2}{0.108^2}-2\cdot 0.262\frac{l-9.25}{0.458}\cdot\frac{h-3.6}{0.108}\right]} \tag{12}$$

$$f(b,h) = \frac{1}{2\pi 0.135\cdot 0.108\sqrt{1-0.147^2}} e^{\frac{-1}{2(1-0.147^2)}\left[\frac{(b-7)^2}{0.135^2}+\frac{3.6^2}{0.108^2}-2\cdot 0.147\frac{b-7}{0.135}\cdot\frac{h-3.6}{0.108}\right]} \tag{13}$$

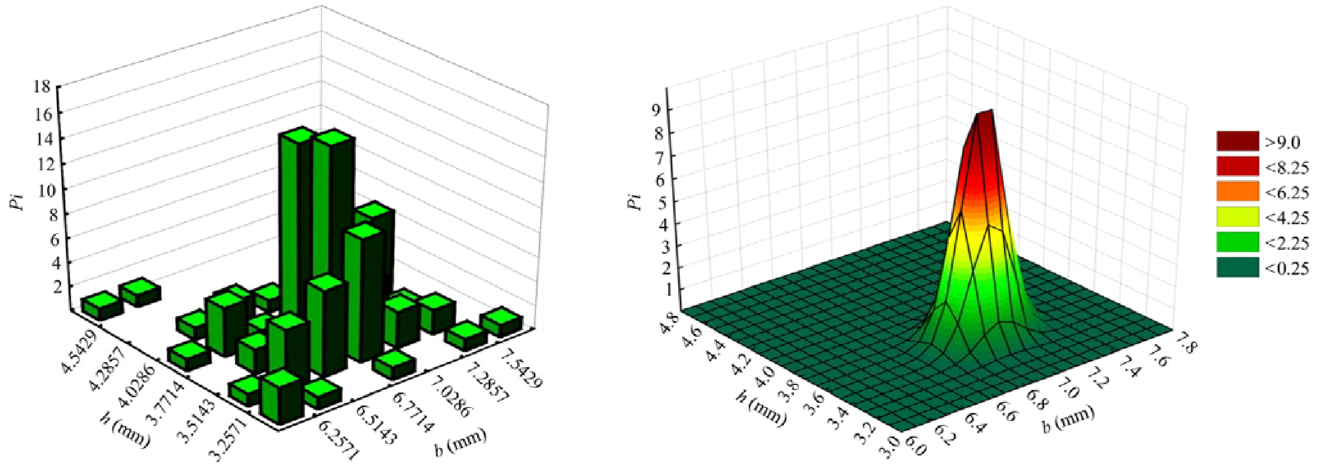
The models of two-dimensional normal distribution of the geometric sizes of coffee beans of the Arabica variety as dependent random variables was presented on Figure 4.



(a) $f(l, b)$



(b) $f(l, h)$



(c) $f(b, h)$

Figure 4 Histograms and density functions of probability distribution of geometric sizes of coffee beans of the Arabica variety as dependent random variables

The lines of levels of two-dimensional distribution of l and b of coffee beans of the Arabica variety as dependent random variables is shown on Figure 5. It can be seen that largest density of two-dimensional distribution to be in evidence for $b \approx 7.0$ mm and $l \approx 9.2$ mm.

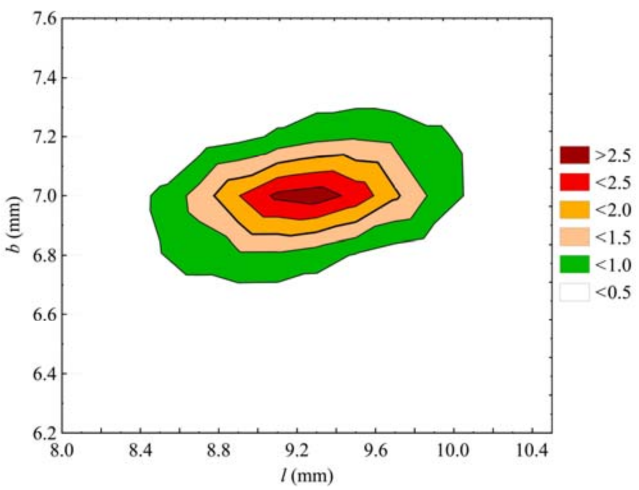


Figure 5 The lines of levels of density functions of two-dimensional probability distribution of l and b of coffee beans of the Arabica variety as dependent random variables

The values of sums of squared deviations between experimental and theoretical frequency of occurrences, which has been calculated according to two-dimensional distributions of geometric sizes of the studied varieties of coffee beans, respectively, both independent and dependent random variables were given in Table 3.

The obtained SSD values indicated that mathematical models of geometric sizes of beans as density functions of the normal distribution of two-dimensional normal distribution of the geometric sizes of studied cereal crops beans as dependent random variables provide a higher accuracy of approaching empirical data.

Table 3 The values of sums of squared deviations between experimental and theoretical frequency of occurrences in approximation of geometric sizes distributions

Culture, variety	*SSD* of approximation of geometric sizes distributions					
	Length and width		Width and thickness		Length and thickness	
	1	2	1	2	1	2
Coffee beans of the Arabica variety	762.66	1391.96	2373.06	3857.85	2032.04	2652.32
Coffee beans of the Robusta variety	206.21	766.4	867.87	1795.94	496.99	802.73

Note: 1 – as independent random variables; 2 – as dependent random variables.

4 Conclusion

In order to take into account the considerable variability of the sizes of coffee beans, these sizes should be regarded as random variables. To improve the separation quality, it is necessary to calculate separation processes using two-dimensional distributions of geometric parameters of coffee beans as random variables. The values of the correlation coefficient between the separate geometric sizes of coffee beans indicate a stable correlation between them. The mathematical models of geometric sizes of coffee beans as density functions of their normal distribution as dependent random variables provide a higher accuracy of approaching and therefore that models can be recommended for use in the calculation and design of technological equipment for the processing and storage of coffee beans.

References

El-Gendy, H. A., A. I. Elrayes, and S. F. Abdel-Hammed. 2011. Engineering properties of some Arabian coffee varieties. *Egyptian Journal of Agricultural Research*, 89(2): 615–625.

Gürsoy, S., and E. Güzel. 2010. Determination of physical properties of some agricultural grains. *Research Journal of Applied Sciences, Engineering Technology*, 2(5): 492–498.

Karaj, S., and J. Müller. 2010. Determination of physical, mechanical and chemical properties of seeds and kernels of *Jatropha curcas* L. *Industrial Crops and Products*, 32(2): 129–138.

Karimi, M., K. Kheiralipour, A. Tabatabaeefar, G. M., Khoubakht, M. Naderi, and K. Heidarbeigi. 2009. The effect of moisture content on physical properties of wheat. *Pakistan Journal of Nutrition*, 8(1): 90–95.

Karimi, N., A. Arabhosseini, M. H. Kianmehr, and J. Khazaei. 2011. Modelling of raisin berries by some physical and statistical characteristics. *International Agrophysics*, 25: 141–147.

Kuzminskyj, R., O. Sokolovskyj, and R. Sheremeta. 2014. Modeling geometrical parameters of seed crops using multidimensional distributions. In *The role of science in improving the technological level and efficiency of the agro industrial complex of Ukraine: Theses International Science-Practice Conference,* 91–94. Ternopil, 15-16 May.

Mansouri, A., A. H. Mirzabe, and A. Ráufi. 2015. Physical properties and mathematical modeling of melon (*Cucumis Melo* L.) seeds and kernels. *Journal of the Saudi Society of Agricultural Sciences*, 16(3): 218–226.

Milani, E., M. Seyed, A. Razavi, A. Koocheki, V. Nikzadeh, N. Vahedi, M. Moein Fard, and A. Gholamhossein Pour. 2007. Moisture dependent physical properties of cucurbit seeds. *International Agrophysics*, 21(2): 157–168.

Mirzabe, A. H., J. Khazaei, G. R. Chegini, and O. Gholami. 2013. Some physical properties of almond nut and kernel and modeling dimensional properties. *Agricultural Engineering International: CIGR Journal*, 15(2): 256–265.

Müller, M., Š. Horníčková, P. Hrabě, and J. Mařík. 2015. Analysis of physical, mechanical and chemical properties of seeds and kernels of *Jatropha curcas*. *Research in Agricultural Engineering*, 61(3): 99–105.

Niveditha, V. R., K. R. Sridhar, and D. Balasubramanian. 2013. Physical and mechanical properties of seeds and kernels of *Canavalia* of coastal sand dunes. *International Food Research Journal*, 20(4): 1547–1554.

Olukunle, O. J., and B. O. Akinnuli. 2012. Investigating some engineering properties of coffee seeds and beans. *Journal of Emerging Trends in Engineering and Applied Sciences* (*JETEAS*), 3(5): 743–747.

Sánchez-Mendoza, J., A. Dominguez-Lopez, S. Navarro-Galindo, and J. A. Lopez-Sandoval. 2008. Some physical properties of roselle (*Hibiscus sabdariffa* L.) seeds as a function of moisture content. *Journal of Food Engineering*, 87(3): 391–397.

Severa, L., J. Buchar, and Š. Nedomová. 2012. Shape and size variability of roasted Arabica coffee beans. *International Journal of Food Properties*, 15(2): 426–437.

Sokolovskyj, O., R. Kuzminskyj, and R. Sheremeta. 2012. Effect of relative humidity on the geometrical parameters of the seeds of wheat and rye. *Motrol - Commission of Motorization and Energetics in Agriculture*, 14(4): 64–70.

Zewdu, A. D., and W. K. Solomon. 2007. Moisture-dependent physical properties of tef seed. *Biosystems Engineering*, 96(1): 57–63.