Modeling and optimization of airflow resistance through a dry bed of Bambara nuts

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Abstract: Drying is essential to prolong storage life of crops. This study is carried out to provide design information for developing a Bambara nut dryer. Bambara nuts samples were condition to 6%, 8%, 10% and 12% db. Pressure drops along depth of 0.2, 0.4, 0.6, 0.8 and 1 m were obtained using a constructed aerodynamic apparatus. Airflow resistances at these depths were calculated for airflow rates of 0.025, 0.035, 0.045, and 0.055 m^3 s^{-1}. I – Optimal response surface design was used to model and optimize the airflow resistance. Reduces quadratic model was selected among other models to be the best for modeling and optimizing airflow resistance of Bambara nuts. This study showed that airflow rate, bed depth, packing type and moisture content all influenced airflow resistance of Bambara nuts. Optimize airflow resistance values for drying Bambara nuts within and outside the experimental design space were obtained. Cube plots for dryer designers were established for Bambara nuts.

Keywords: modeling, optimization, Bambara nuts, airflow resistance, moisture contents

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1 Introduction

Bambara nuts (*Vigna subterranes* (L) Verdc) belong to the legumes family of Fabaceae or Papilionoideae. Many researchers are of the opinion that Bambara nuts originated from West Africa along the Niger River. Arab traders were believed to had taken it to Madagascar and then to Asia. It was introduced to South America continent in the seventeenth century. A Bambara nut contains 65% carbohydrate and 18% protein content and is the cheapest protein content seed in Sub-Sahara Africa. World production of Bambara nuts increased from 29,800 tonnes in 1972 to 79,155 tonnes in 2015. The highest producers at 2015 in descending order are Mali, Niger, Burkina Faso, Cameroon, and Democratic Republic of the Congo. Exporting countries are Chad, Burkina Faso, Mali, Niger and Senegal. This nut if well harnessed can bring food security in Sub-Sahara Africa (Purseglove, 1992; Baudoin and Mergai, 2001; Brink et al., 2006; Mkandawire, 2007; Kouassi et al., 2010; FAOSTAT, 2015).

Airflow resistance of agricultural grains or seeds is the opposition to flow of air through bulk grain caused by the forces of friction on these grains. Shahbazi (2011) is of the opinion that the knowledge of resistance to airflow through agricultural products is an important consideration in the design of drying, cooling, or aeration systems and proper fan selection for these systems. Several variables like height and bed porosity, strange material, composition and bulk density, accession, variety, kind of surface and seed moisture content, velocity and airflow direction, packing method, grain morphology, affect the design of an aeration system (Crozza and Pagano, 2006). Shedd (1953) produced an airflow resistance for several agricultural grains and seeds. Other researchers among others who had studied air flow resistance of agricultural produce include: Gunasekaran and Jackson (1988), Lukaszuk et al. (2008), Rajabipour et al. (2001). ASAE D272.3 (2007), Teixeira et al. (2015),

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Górnicki and Kaleta (2015a), Górnicki and Kaleta (2015b), Kenghe et al. (2012), Dilmac et al. (2016), Garg and Maier (2006), Nishizu et al. (2017), Bala (2016), and Olatunde et al. (2016).

Modeling is defined as a process by which ideals and concepts of Scientists and Engineers about the natural environment are presented to each other and then make changes to these ideas and concepts over time in response to new evidence and understandings. A model can also be a mathematical representation of a physical, biological or information system. Mathematical modeling is a principled activity (Hartmann et al., 2017; Ambitious science teaching, 2015; Cha et al., 2000; Dym and Ivey, 1980).

Mathematical optimization can be defined as selection of best factors or elements (with regard to some set goals or constrains before the beginning of the selection) among some group of factors and element considered. More generally, optimization includes finding “best available” values of some objective function given a defined domain (or input), including a variety of different types of objective functions and different types of domains (Bolaji et al., 2017; The Nature of Mathematical Programming, 2014; Battiti et al., 2008).

Response surface methodology (RSM) can be described as a technique that involves complex calculation for optimization process. This approach develops a suitable experimental design that integrates all of the independent variables and uses the data input from the experiment to finally come up with a set of equations that can give theoretical value of an output. The outputs are obtained from a well-designed regression analysis that is based on the controlled values of independent variables. Thereafter, the dependent variable can be predicted based on the new values of independent variables. RSM involves the use of the following experimental designs: Central Composite Design (CCD); Box-Behnken (BB); Optimal Designs (Garlapati and Roy, 2017; Gnanasundaram et al., 2016; Said and Amin, 2015; Giovanni, 1983).

Optimal Designs is a flexible design structured to accommodate structured models, categorical factors and irregular (constrained) regions. Optimal Designs includes: I-optimal, D-optimal, A- optimal, modified distance optimal, distance optimal. (Stat-Ease, 2017).

The objective of this study was to model and optimize airflow resistance through a dry bed of Bambara nuts for design consideration of drying machines with bed depths ranges of 0.2-2.5 m. This is because drying Bambara nuts improve its storage life. Improving the storage life of this nut will improve the source of income of farmers in developing countries where this nut is mostly farmed.

2 Materials and methods

2.1 Sample

Bambara nuts (*Vigna subterranean* (L.) verdc) was obtained from Adikpo market, Benue state, Nigeria. Sample was taken to the Agronomy Laboratory of the University of Agriculture, Makurdi, Nigeria for identification. After cleaning, the nuts were conditioned as described by Audu et al. (2017), to 6%, 8%, 10% and 12% db.

2.2 Determination of airflow resistance

Pressure drop of Bambara groundnut was measured using the constructed aerodynamic properties measuring apparatus. The apparatus was set up as shown in Figure 1. A water manometer containing water was connected to the apparatus with a rubber hose to deliver air from the air column to the manometer. A light dimmer switch was attached and calibrated by turning the handle to change the speed of the fan and the airflow rate produced inside the test column was measured using a digital anemometer. Airflow rate of 0.025, 0.035, 0.045, and 0.055 $m^3 s^{-1}$ were marked on the light dimmer switch. To determine the pressure drops of Bambara nut, the sample seeds were poured into the air column (upper chamber of the apparatus) to the desired depth. The packing of the column was achieved by three different methods. The first method involved the pouring of the sample seeds from a height near zero from the top of the apparatus to produce a “loose filled”. The second packing method was achieved by loosely filing the column and then tapping it 10 times. This packing method is termed “slightly dense filled”. The third was also achieved by tapping the column 20 times, this is called “dense filled”. Air was blown through the apparatus with seeds at different depths of 0.2, 0.4, 0.6, 0.8 and 1 m. Measurements of pressure drop read at different depths were taken from the manometer. Pressure drop was calculated using Equation (1).

$$\Delta P = \rho g h \quad (1)$$

where, ΔP = Pressure drop (Pa); ρ = density of air (kg m^{-3}); $h = h_2 - h_1$ = height of water drop on the manometer (m).

Airflow resistance was calculated using Equation (2).

$$\text{Airflow Resistance} = \frac{\Delta P}{L} \quad (2)$$

where, L is the depth (m).

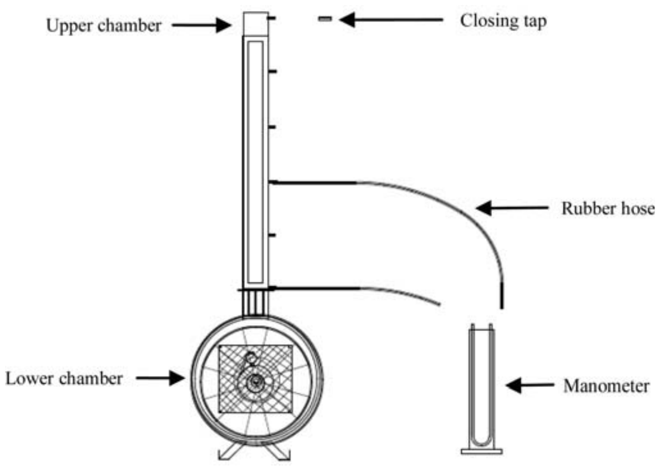

Figure 1 Apparatus constructed for measuring pressure drop

2.3 Experimental design and statistical analysis

The experimental design used for modeling and optimizing airflow resistance was I – Optimal response surface design. All statistical analysis was done using Design Expert 10 software. Optimization was carried out for drying of Bambara nuts in bed depth ranges of 0.2-1 m within experimental design space and 1.5-2.5 m which is outside the experimental design space. Optimization goals were set at moisture (Maximum), airflow (0.025-0.055 and 0.05-0.1 m^3 s^{-1}), bed depth (between 0.2-1 and 1.5-2.5 m), packing types (within experimental range) and airflow resistance (minimum).

3 Result and discussing

Table 1 displayed I – Optimal response surface experimental design used for reporting airflow resistance. Airflow resistance results observed during the experiments range from 14.6-76 Pa m^{-1}. Figure 2 showed the graphical relationships between moisture, airflow rate, packing type and bed depth with airflow resistance. It showed that increase in moisture reduced and increased the resistance of the nuts to airflow at certain moisture levels. This is caused by the variation in shape of the nuts as they absorbed moisture. Airflow resistance reduced as the bed depth increased. This occurred because the air speed required to push the air up the bed as the bed depth increases is not sufficient to cause the nuts to offer any resistance. Figure 2 showed that airflow resistance reduced with packing types. This can be explained by the fact that packing type alters the porosities of the nuts therefore causing the airflow direction to alter. Airflow rate increased the resistance to airflow as shown in one of the graphs because as the speed of the flowing air increased the friction offered by the nuts surface increased due to reduce porosity in the bulk nuts. Similar observations were reported by Teixeira et al. (2015), Górnicki, and Kaleta (2015a), Górnicki, and Kaleta (2015b), Kenghe et al. (2012), Dilmac et al. (2016), and Garg and Maier (2012).

In modeling the airflow resistance of Bambara nuts, five models: special Design Model, linear, two factor interaction (2FI), quadratic and cubic models were considered and statically analyzed by the software (design expert). The modeling design summary was shown in Table 2. Linear and quadratic models were suggested by the software with cubic model being ‘aliased’ (This means that there are not enough unique design points to independently estimate all the coefficients for this model). Quadratic model was chosen instead of linear model because it has low Sequential p-value (the probability that the order terms are modeling noise rather than helping explain the trend in the response) of 7.36×10^{-11}, no lack of Fit p-value (the amount the model predictions miss the observations), and the highest Adjusted R-Squared (A measure of the amount of variation about the mean explained by the model) of 0.977336 (Table 3). Analysis of variance (ANOVA) for the quadratic model was done (Table 4). The quadratic model was found to be significant at $P<0.05$. The insignificant model terms were removed from the ANOVA until only the significant ($P<0.05$) terms were left in the table. The statistical properties of the model were shown in Table 5. The quadratic model had standard deviation of 3.178705 with a mean data value of 36.41425. The coefficient of variation (CV%) (standard deviation expressed as a percentage of the mean) was 8.72929.

Table 1 Experimental design table displaying experimental results

Run	Factor 1 Moisture (%)	Factor 2 Bed depth (m)	Factor 3 Airflow rate (m^3 s^{-1})	Factor 4 Packing types	Response 1 Airflow resistance (Pa m^{-1})	Run	Factor 1 Moisture (%)	Factor 2 Bed depth (m)	Factor 3 Airflow rate (m^3 s^{-1})	Factor 4 Packing types	Response 1 Airflow resistance (Pa m^{-1})
1	6	0.2	0.045	loose	74	21	12	1	0.055	dense	18.8
2	8	0.2	0.055	loose	70	22	6	0.6	0.045	slightly dense	26.67
3	12	1	0.025	loose	17.4	23	8	0.8	0.035	loose	19.75
4	10	0.6	0.025	loose	27.75	24	8	0.6	0.055	dense	27.67
5	6	0.2	0.045	dense	68	25	8	0.8	0.035	slightly dense	19.75
6	6	1	0.055	dense	15	26	8	1	0.035	dense	15.6
7	6	0.2	0.025	slightly dense	64	27	8	0.2	0.055	slightly dense	69
8	12	1	0.025	slightly dense	18.2	28	6	0.6	0.025	dense	30
9	6	1	0.055	slightly dense	19.5	29	10	0.2	0.025	dense	76
10	10	0.2	0.035	loose	68	30	8	0.4	0.035	dense	36.5
11	12	0.6	0.035	dense	29.33	31	6	0.6	0.045	loose	26.33
12	8	1	0.035	dense	15.6	32	10	1	0.045	slightly dense	17.4
13	12	0.6	0.035	dense	29.33	33	12	1	0.055	loose	18
14	10	0.4	0.045	slightly dense	41	34	6	1	0.025	slightly dense	14.6
15	6	0.2	0.025	loose	68	35	6	1	0.025	loose	14.6
16	8	0.4	0.035	dense	36.5	36	8	0.6	0.055	dense	27.67
17	10	0.6	0.025	slightly dense	26	37	12	0.2	0.055	loose	72
18	12	0.2	0.035	slightly dense	76	38	12	0.2	0.055	dense	80
19	6	1	0.055	loose	15.6	39	8	0.8	0.035	loose	19.75
20	12	0.6	0.055	slightly dense	29.67	40	12	1	0.025	dense	17.6

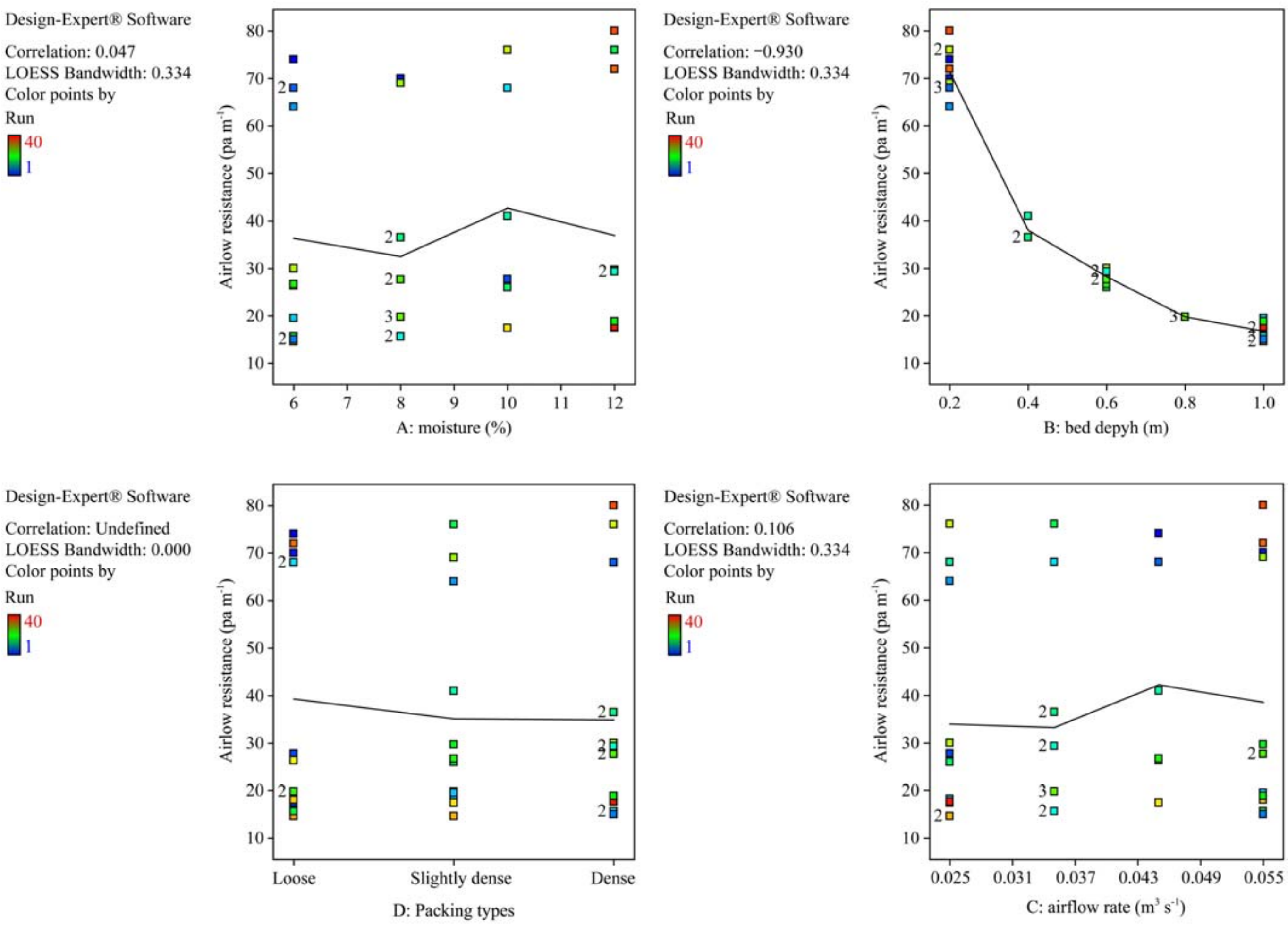

Figure 2 Graphical relationships of airflow resistance with moisture, bed depth, packing type and airflow rate

Table 2 Modeling design summary for airflow resistance of Bambara nuts

Software setting for analysis										
File Version	10.0.1.0									
Study Type	Response surface									
Design Type	I-optimal									
Design Model	Reduced cubic									
Subtype	Randomized									
Runs	40									
Blocks	No blocks									
Factors characteristics before and after analysis										
Name	Units	Type	Subtype	Min	Max	Coded	Values	Mean	Std. dev.	
Moisture	%	Numeric	Discrete	6	12	–1.0=6	1.0=12	8.8	2.388	
Bed depth	m	Numeric	Discrete	0.2	1	–1.0=0.2	1.0=1	0.62	0.323	
Airflow rate	$m^3 s^{-1}$	Numeric	Discrete	0.025	0.055	–1.0=0.025	1.0=0.055	0.039	0.012	
Packing types		Categorical	Nominal	Loose	Dense					
Responses characteristics before and after analysis										
Name	Units	Obs	Analysis	Min	Max	Mean	Std. Dev.	Ratio	Trans	Model
Airflow Resistance	Pa m^{-1}	40	Polynomial	14.6	80	36.414	22.806	5.479	None	R Quadratic

Table 3 Models analysis for airflow resistance

Source	Sequential p-value	Lack of fit p-value	Adjusted R-Squared	Predicted R-Squared	
Design model	6.52×10^{-8}		0.977115	-25.2233	
Linear	2.5×10^{-14}		0.855344	0.828645	
2FI	0.997804		0.81276	0.667927	
Quadratic	7.36×10^{-11}		0.977336	0.956709	Suggested
Cubic			1		Aliased

Table 4 ANOVA for response surface reduced quadratic model

Source	Sum of Squares	Df	Mean Square	F Value	p-value Prob > F	
Model	19921.33	3	6640.44	657.198	1.78×10^{-31}	significant
Moisture	82.62	1	82.62	8.18	0.007	significant
Bed depth	18173.01	1	18173.01	1798.57	2.48×10^{-32}	significant
Bed depth^2	2306.21	1	2306.21	228.24	3.7×10^{-17}	significant
Residual	363.75	36	10.1			
Lack of Fit	363.75	31	11.73			
Pure Error	0	5	0			
Cor Total	20285.08	39				

Table 5 Statistical properties of quadratic models of airflow resistance of Bambara nuts

Statistical parameters	Values
Std. Dev.	3.178705
Mean	36.41425
C.V. %	8.72929
PRESS	442.9788
–2 Log likelihood	201.8186
R-Squared	0.982068
Adj R-Squared	0.980574
Pred R-Squared	0.978162
Adeq Precision	56.93607
BIC	216.5741
AICc	210.9614

The PRESS (Predicted Residual Sum of Squares) of the model which is a measure of how well a particular model fits each point in the design is 442.9788 which is quite high. The –2 Log Likelihood (the coefficient estimates for the chosen model to maximize the likelihood that the fitted model is the correct model) was 201.8186. The R-Squared (measure of the amount of variation around the mean explained by the model), Adj R-Squared (measure of the amount of variation around the mean explained by the model, adjusted for the number of terms in the model) and Pred R-Squared (measure of the amount of variation in new data explained by the model) were 0.982068, 0.980574 and 0.978162 respectively which make the model a good model. Adequate precision which is a signal-to-noise ratio was 56.93607. Ratios greater than 4 indicated adequate model discrimination. The BIC (a large design penalized likelihood statistic used to choose the best model) and AICc (a small to medium penalized likelihood statistic used to choose the best model) were 216.5741 and 210.9614 respectively.

The model equation for airflow resistance of Bambara nut was given below.

Airflow Ristance =

$$99.782+0.6096M-193.535B+105.473B^2 \tag{3}$$

where, M is moisture content in (%) and B is bed depth in (m).

To diagnosis the quadratic model generated, a graph of residuals vs. predicted and predicted vs. actual were plotted (Figure 3). Residuals vs. predicted plot tests the assumption of constant variance. The plot showed a random scatter (constant range of residuals across the graph) which was good. No expanding variance ("megaphone pattern <") was observed in this plot to indicate the need for a transformation. Predicted vs. actual graph helps to detect a value, or group of values, that are not easily predicted by the model. In this plot almost all the points fell within easily predicted region. A typical model 3D Graphs for quadratic model for airflow resistance of Bambara nuts was shown in Figure 4. The three-dimensional model graph showed values of factors and response used in modeling. Higher resistance to airflow was observed across all moisture level at lower depth. Rajabipour et al. (2001) and Dilmac et al. (2016) observed similar trend.

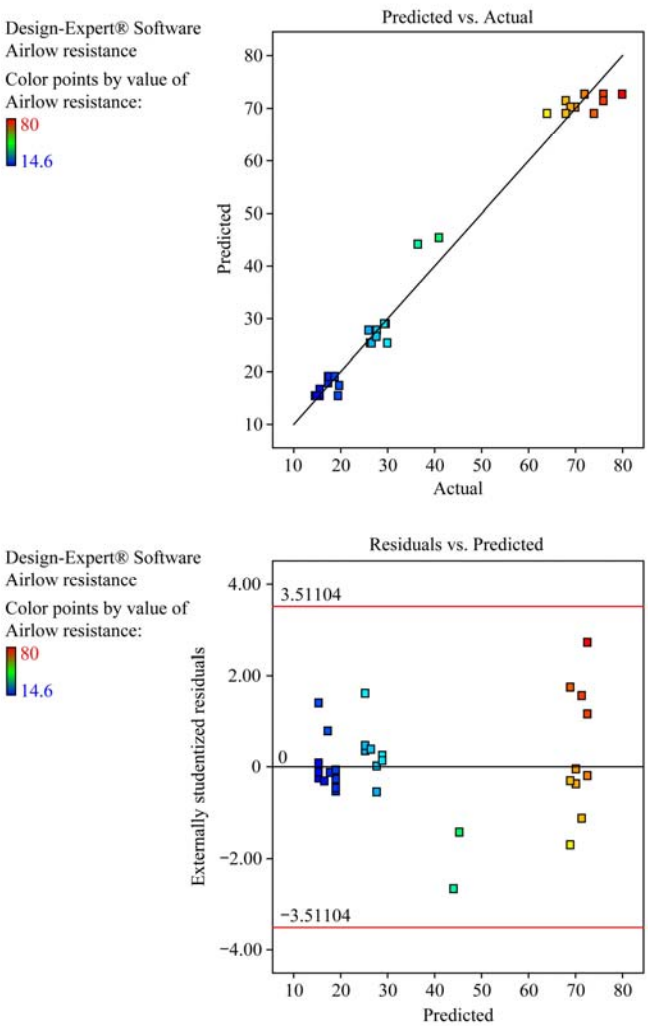

Figure 3 Diagnostic graph for airflow resistance quadratic model of Bambara nuts

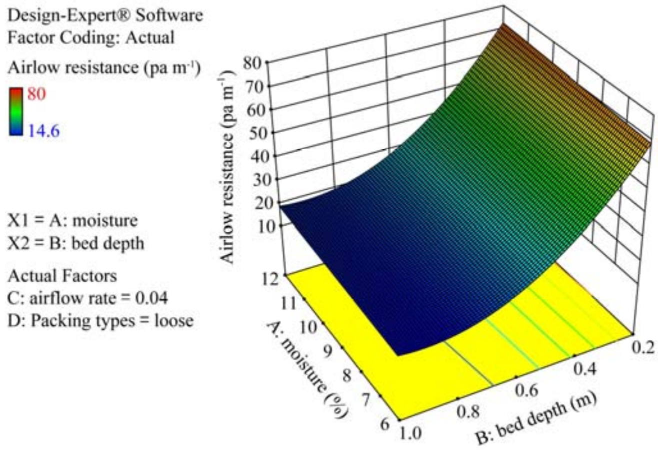

Figure 4 Typical 3D graph for quadratic model for airflow resistance of Bambara

Optimization results obtained were shown in Table 6 and 7. Only results for three combinations for the goals used for the optimization were shown. The result in Table 6 showed that the lowest airflow resistance that could be achieved within a bed depth range of 0.2-1 m in a dry bed is 18.316 Pa m^{-1} at a depth of 1 m, with an airflow rate of 0.039, 0.026 and 0.048 m^3 s^{-1} for Dense, Slightly dense and Loose packing types respectively. This means that when designing a dryer for Bambara nuts the loading system will determine the speed of the fan selected. For a dry bed depth range of 1.5-2.5 m. Table 7 showed that the lowest airflow resistance achieved was 54.108 Pa m^{-1} at a depth of 1.5 m, with an airflow rate of 0.09, 0.07 and 0.09 m^3 s^{-1} for Dense, Loose and Slightly dense packing types respectively. This means that designers of dip beds will encounter a higher air resistance if the flow rate is increased. A typical cube plots for the optimized values were shown in Figure 5 and 6. These cube plots guild dryer designers with selection of other design parameters if one of the parameters is fixed. Figure 5 showed a guild for designing a dryer for depth range of 0.2-1 m if the filling type was dense filling. Figure 6 also showed a guild for designing a dryer for depth range of 1.5-2.5 m if the filling type was dense filling too.

Post analysis was done mathematically from optimized predicted results and the confirmation (Validation) report for the quadratic model was shown in Table 8. Confirmation analysis compared the prediction interval of the model to a follow up optimized predicted sample's average (data mean). If the optimized predicted samples average (data mean) is inside the prediction interval then the model is confirmed or validated. In the confirmation report the optimized predicted data mean of 18.316 Pa m^{-1} lied between the 95% PI (prediction interval) low of 11.58 Pa m^{-1} and 95% PI high of 25.052 Pa m^{-1} for bed depth ranges of 0.2-1 m. Also for bed depth range of 1.5-2.5 m the optimized predicted data mean of 54.108 Pa m^{-1} lied between the 95% PI (prediction interval) low of 41.864 Pa m^{-1} and 95% PI high of 66.352 Pa m^{-1}. Thus, the model has been confirmed or validated.

Table 6 Optimize solutions for drying of Bambara nuts at bed depth range of 0.2-1 m

Constraints						
Name	Goal	Lower limit	Upper limit	Lower weight	Upper weight	Importance
Moisture	Maximize	8	12	1	1	3
Bed depth	In range	0.2	1	1	1	3
Airflow rate	In range	0.025	0.055	1	1	3
Packing types	In range	Loose	Dense	1	1	3
Airflow resistance (Pa m^{-1})	Minimize	14.6	80	1	1	3
Optimize solutions						
Number	Moisture (%)	Bed depth (m)	Airflow rate (m^3 s^{-1})	Packing types	Airflow resistance (Pa m^{-1})	Desirability
1	12	0.918	0.039	Dense	18.316	0.97
2	12	0.918	0.026	Slightly dense	18.316	0.97
3	12	0.917	0.048	Loose	18.316	0.97

Table 7 Optimize solutions for drying of Bambara nuts at bed depth range of 1.5-2.5 m

Constraints						
Name	Goal	Lower limit	Upper limit	Lower weight	Upper weight	Importance
Moisture (%)	Maximize	8	12	1	1	3
Bed depth (m)	In range	1.5	2.5	1	1	3
Airflow rate (m^3 s^{-1})	In range	0.05	0.1	1	1	3
Packing types	In range	Loose	Dense	1	1	3
Airflow Resistance (Pa m^{-1})	Minimize	14.6	80	1	1	3
Optimize Solutions						
Number	Moisture (%)	Bed depth (m)	Airflow rate (m^3 s^{-1})	Packing types	Airflow resistance (Pa m^{-1})	Desirability
1	12	1.5	0.09	Dense	54.108	0.63
2	12	1.5	0.07	Loose	54.108	0.63
3	12	1.5	0.09	Slightly dense	54.108	0.63

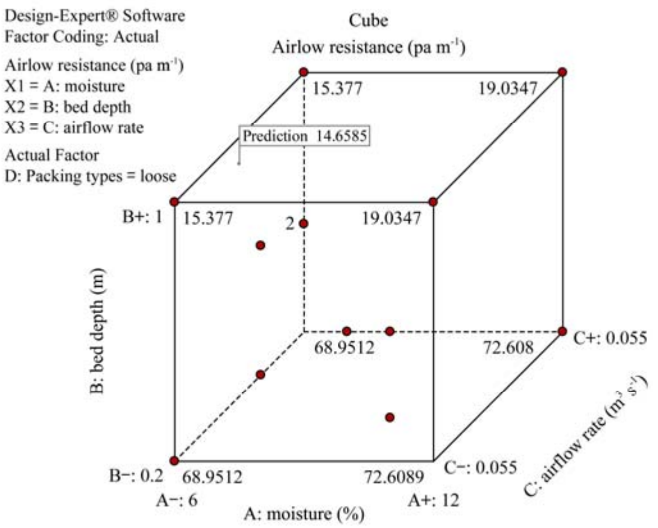

Figure 5 Typical optimize cube graph for airflow resistance of Bambara at depth 0.2-1 m

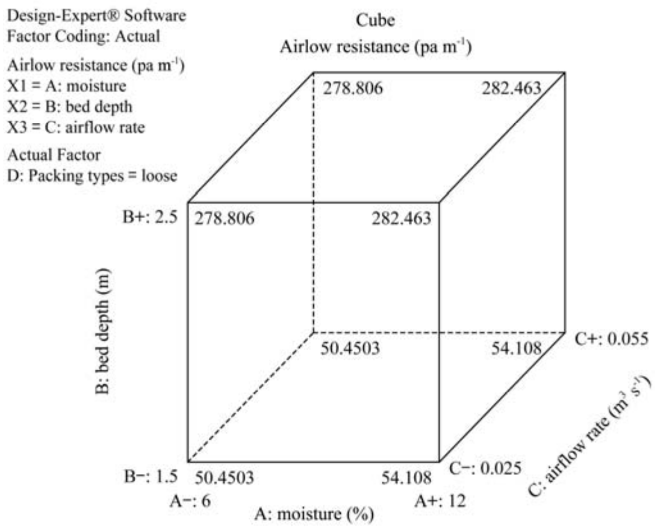

Figure 6 Typical optimize cube graph for airflow resistance of Bambara at depth 1.5-2.5 m

Table 8 Confirmation (Validation) report of model used for optimizing airflow resistance

Two-sided			Confidence = 95%							
Bed depth (m)	Airflow rate (m^3 s^{-1})	Predicted mean	Predicted median	Obs	Standard deviation	n	Standard error of prediction	95% PI low	Data mean	95% PI high
0.2-1	0.025-0.055	18.316	18.316	-	3.179	1	3.321	11.58	18.316	25.052
1.5-2.5	0.05-0.1	54.108	54.108	-	3.179	1	6.037	41.864	54.108	66.352

4 Conclusion

Conclusions drawn from this study are:

- Increase in airflow rate increased airflow resistance, increase in bed depth decreased airflow resistance, while increase in seeds moisture showed a fluctuation in airflow resistance values.
- Quadratic model was the best for predicting air flow resistance of Bambara nuts.
- Optimize airflow resistance values for drying Bambara nuts within the experimental design space was obtained.
- Optimize airflow resistance values for drying Bambara nuts outside the experimental design space was also obtained.
- A cube plots for drying designers was established for Bambara nuts.

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