A two degree of freedom damped fruit tree model

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Abstract: A simple two degree of freedom fruit tree model was built and some of its behaviour was compared with real cherry trees. The model represented from one hand the rooting system with a certain amount of soil and of the trunk, from the other hand the main branches and limb. The calculated results for the model have shown good accordance with the test results of the measured real tree: in both cases two peaks in the amplitude and acceleration vs. frequency diagrams were clearly recognizable. Using the equation of the model, the effect of shaker parameters and shaking frequency can be studied, which enables more accurate design of the shaker machine.

Keywords: fruit tree, modelling, mechanical shaking, shaker machine

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1 Introduction

Since shaker harvest of some fruit varieties is practiced the attempt to describe mathematically the trees is also present. One of the first approaches was published by Fridley and Adrian (1966) who suggested the replacement of tree at shaking cross section by a one degree of freedom three-element model.

Important contribution to the modelling was made by Horváth and Sitkei (2001). They recognised that the trunk cannot be regarded as a vertical cantilever. It translates and rotates during shaking and moves a certain amount of soil around the tree. They measured the translations of the tree while shaking the trunk at different heights and then calculated its virtual turning point.

Láng (2008) has composed a simple tree structure model of a trunk and main roots. It included mass, spring and damping elements, all reduced to the external end of the main roots. The model was virtually shaken and acceleration and displacement amplitudes versus shaking frequency were calculated. The real cherry tree was also shaken and the same data were recorded. The acceleration and displacement amplitude vs. frequency functions were similar for both the virtual and real trees which proved the accuracy of the model. This model however didn't include the limbs, so no data can be achieved of the amplitude and acceleration of the primary and secondary branches.

Castro-Garcia et al.(2008) performed dynamic analysis on 17 olive trees using modal testing techniques. Modal parameter identification was focused in the range of shaking frequencies used by the most trunk shakers. The first two modes of vibration of the main tree frame were identified with damping ratios of 26.9% and 17.1% and natural frequencies of 20.2 and 37.7 Hz, respectively. During the testing, the olive trees behaved like a damped harmonic oscillator with predominantly mass damping in these modes. Similar tests were carried out by Fenyvesi and Fenyvesi (2008) on grapes.

In order to study the influence of different shakers on the dynamic response of an olive variety in Tunisia, Bentaher et al. (2013) has undertaken a finite element numerical modeling. The tree was modeled by three-dimension beams, each of them having two nodes

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and six freedom degrees for each node. For each part of the tree, the wood's mechanical characteristics were determined. Orbital and multidirectional shakers were the mechanical harvesting tools tested. They found that the orbital shaker gave the better mean response of the fruits. However, the responses were more homogeneous for the multidirectional shaker. The use of high frequencies of excitation improves the response of tree.

Galili at al. (2001) used a two degree of freedom model to describe the interaction between tree trunk and shaker which allowed relative motion between them. This model however regarded the tree as a whole unit.

The objective of this paper is to introduce a simple two degree of freedom model which enables the calculation of acceleration and amplitude of trunk and primary branches for a vase form fruit tree. To check the model its calculated data have to be compared with data measured on real fruit trees. The two degree of freedom model enables also the calculation of power demand at different shaking frequencies both for trunk and limb.

2 Material and methods

2.1 Theoretical background

The two degree of freedom model of the fruit tree was composed from one hand of the rooting system with a certain amount of soil and of the trunk (index 1), from the other hand of the main branches and limb moving together when shaking the tree (index 2) , as is shown in Figure 1.

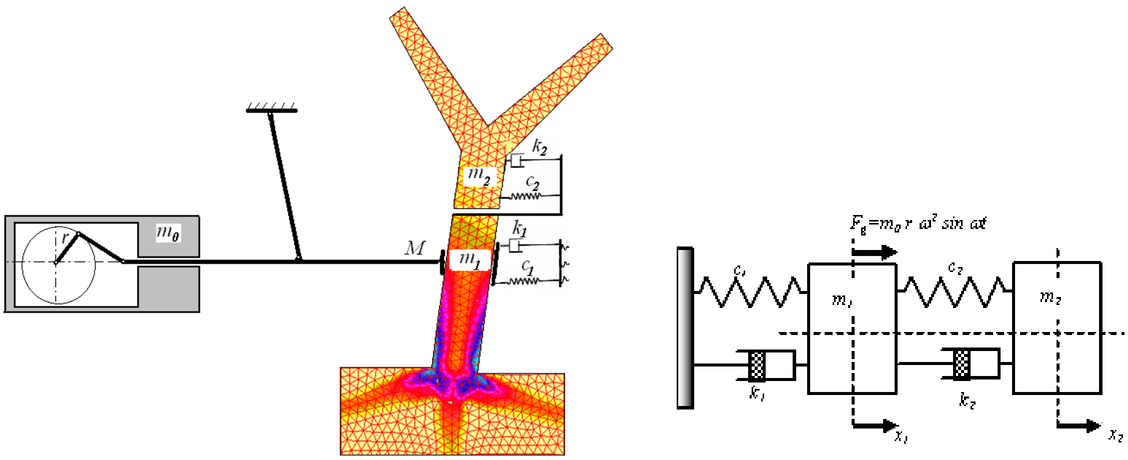

Figure 1 The two degree of freedom fruit tree model composed of trunk and limb

Wherer

c_1 is the spring constant of the trunk and routing system, m/N;

c_2 is the spring constant of the main branches, m/N;

m_1 is the reduced mass of the rooting system, soil around it and trunk, as well as the mass of shaker boom, kg;

m_2 is the reduced mass of the main branches and limb, kg;

k_1 is the viscous damping coefficient of the trunk and routing system, Ns/m;

k_2 is the viscous damping coefficient of the main branches, Ns/m;

m_0 is the unbalanced mass of the inertia shaker, kg;

M is the mass of the shaker boom, kg;

R is the eccentricity of the unbalanced mass, m;

ω is the angular velocity of the rotating unbalanced mass, 1/s;

x_1 and x_2 are the displacements of the trunk and limb respectively, m;

$F_g=m_0r\omega^2\sin\omega t$ is the periodical vibrating force, N.

The kinetic system of equations for the model is as follows:

$$m_1\ddot{x}_1 + (k_1 + k_2)\dot{x}_1 + (\frac{1}{c_1} + \frac{1}{c_2})x_1 - k_2\dot{x}_2 - \frac{1}{c_2}x_2 = m_0 r\omega^2 \sin\omega t \quad (1)$$

$$m_2\ddot{x}_2 + k_2\dot{x}_2 + \frac{1}{c_2}x_2 - k_2\dot{x}_1 - \frac{1}{c_2}x_1 = 0 \quad . \quad (2)$$

Disregarding the description of the steps, after the necessary transformations and substitutions, the particular solution related to the mass m_1 is:

$$x_1 = x_{1p} = A_1 \sin(\omega t + \Psi) \quad (3)$$

Where ,

$$A_1 = \frac{m_0}{m_1} r\omega^2 \sqrt{\frac{(\frac{1}{m_2 c_2} - \omega^2)^2 + (\frac{k_2}{m_2}\omega)^2}{(\omega^4 - E_2\omega^2 + E_0)^2 + (E_3\omega^3 - E_1\omega)^2}} \quad (4)$$

$$\Psi = arctg\,\frac{b}{a} \quad (5)$$

$$a = \frac{H_1 Z_1 - H_2 Z_2}{H_1^2 + H_2^2}, \quad b = \frac{H_2 Z_1 + H_1 Z_2}{H_1^2 + H_2^2} \quad (6)$$

$$H_1 = (\omega^4 - E_2\omega^2 + E_0), \quad H_2 = (E_3\omega^3 - E_1\omega) \quad (7)$$

$$Z_1 = \frac{m_0}{m_1} r\omega^2 [\frac{1}{m_2 c_2} - \omega^2], \quad Z_2 = \frac{m_0}{m_1 m_2} k_2 r\omega^3 \quad (8)$$

$$E_0 = \frac{1}{m_1 m_2} \cdot \frac{1}{c_1 c_2}, \quad E_1 = \frac{1}{m_1 m_2}[\frac{k_2}{c_1} + \frac{k_1}{c_2}] \quad (9)$$

$$E_2 = \frac{1}{m_1 m_2}(\frac{m_2}{c_1} + \frac{m_1 + m_2}{c_2} + k_1 k_2) \quad (10)$$

$$E_3 = \frac{1}{m_1 m_2}[m_2 k_1 + (m_1 + m_2)k_2] \quad (11)$$

The kinetic equation for m_2 can be written as

$$m_2\ddot{x}_2 = m_0 r\omega^2 \sin\omega t - (m_1\ddot{x}_1 + k_1\dot{x}_1 + \frac{1}{c_1}x_1) \quad (12)$$

The particular solution for $x_2 = x_{2p}$ will result in:

$$x_2 = -A_2 \sin(\omega t + \Phi) \quad (13)$$

Where,

$$A_2 = \frac{B_2}{\omega^2}, \quad B_2 = \sqrt{J^2 + K^2} \quad (14)$$

$$J = \frac{1}{m_2}[m_0 r\omega^2 + (m_1\omega^2 - \frac{1}{c_1})a + k_1 b\omega] \quad (15)$$

$$K = \frac{1}{m_2}[-k_1 a\omega + (m_1\omega^2 - \frac{1}{c_1})b] \quad (16)$$

$$\Phi = arctg\,\frac{K}{J} \quad (17)$$

Finally for the accelerations applies:

$$a_1 = \omega^2 A \sin(\omega t + \Psi) \quad (18)$$

$$a_2 = -\omega^2 A_2 \sin(\omega t + \Phi) \quad (19)$$

2.2 Field tests

Experiments with inertia type shaker in a 10-year-old cherry orchard were carried out to measure accelerations and to calculate amplitudes of trunk and main branches at different shaking frequencies. For this, accelerometers were fixed on trees with average trunk diameters of 13.5 cm at 80, 110, 160, 190 and 240 cm height (Figure 2).

Acceleration versus time functions were recorded during mechanical shaking of trees at 80 cm trunk height in the frequency range from 4,8 to 15 Hz.

The parameters of the slider crank type shaker machine were as follows: the total unbalanced mass m_0=115 kg, the eccentricity of the unbalanced mass r=25 mm. The mass of the shaker boom (attached to the trunk) M=75 kg.

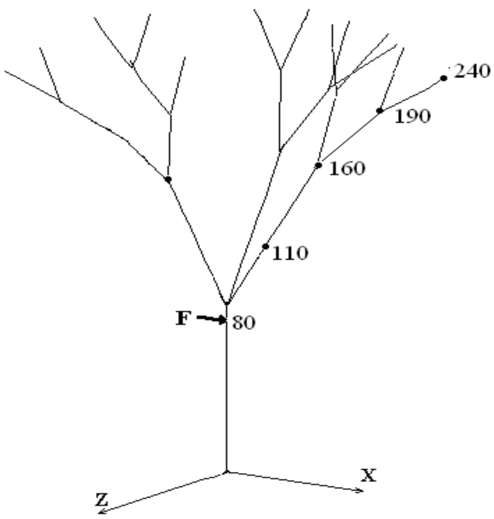

Figure 2 Height of acceleration measurements (cm)

The spring constants c_1 of tested trunks were measured statically, applying different horizontal forces to them at 80 cm height. As the average value of three tests resulted c_1 =1.8 E-06 m/N as spring constant of the trunk.

The three main branches (Figure 2) were regarded as truncated cones contacted directly to the trunk by their bottom end. The larger diameters were taken for 11 cm, their smaller ones for 2.5 cm, their length 140 cm. The center of gravity of a main brunch resulted for 38.5 cm above their bottom end. The average spring constant c_2 was measured by applying force to the branches at their centre of gravity and recording their displacement. As a result of tests the average spring constant of main branches resulted in c_2=7.0 E-06 m/N.

The reduced mass m_r of the rooting system, soil around it and trunk was measured as follows: the limb of a tree was removed at 80 cm height; the remaining trunk was supplied with an accelerometer at its top. Then it was displaced for about 25 mm horizontally and released. Meanwhile acceleration versus time curve was recorded. The action was repeated with an extra mass m_e, fixed to the top of the trunk. Acceleration versus time curve was recorded in this arrangement as well. Using FTT the natural frequencies f_1 f_2 of the two test arrangements was identified. Solving the Equation 20, called Rayligh's method, the mass m_r could be calculated:

$$m_r = \frac{m_e}{\frac{f_1^2}{f_2^2} - 1} = 385 \quad kg \tag{20}$$

Where,

m_r is the reduced mass of the rooting system, soil around it and trunk, kg;

f_1 and f_2 are the natural frequency of the trunk without and with extra mass, Hz;

m_e = 13.5 kg, the extra mass.

With those data the trunk mass resulted in m_1= m_r +M=277+75=352 kg.

The reduced mass m_2 of the main branches were calculated using the data achieved by determining the centre of gravity of them. The total volume of the three elements was 3,425 cm^3, their total mass 28.5 kg.

The average dumping coefficient k_1 was calculated off the running out acceleration versus time curves of the shaken tree trunks, measured at 80 cm trunk height. The equation applied (21):

$$k_1 = \frac{2m_1\Delta}{t_c} \tag{21}$$

Where,

m_1 is the reduced mass of the rooting system, soil around it and trunk, as well as the mass of shaking rod, kg;

Δ is the logarithmic decrement of the system, measured on the diagrams;

t_c is the cycle time of the vibration, s.

With the average logarithmic decrement of 1.26, cycle time t_c =0.08 s; the dumping coefficient k_1 = 4012 Ns/m.

The dumping coefficient of the main branches k_2 was measured similarly to k_1: the running out acceleration versus time curves of branches were evaluated. Replacing Δ=0.82 into Equation (21), k_2 = 705 Ns/m.

3 Results

By replacing shaker machine and fruit tree parameters into Equations (1)-(19), displacement amplitude versus frequency, as well as acceleration versus frequency diagrams could be drawn for both trunk and main branches of the model tree (Figures 3 and 4). As expected, in both cases two natural frequencies are recognizable, one at about 6 Hz, another at about 12 Hz. Beyond the second natural frequency both the trunk and limb displacement are decreasing.

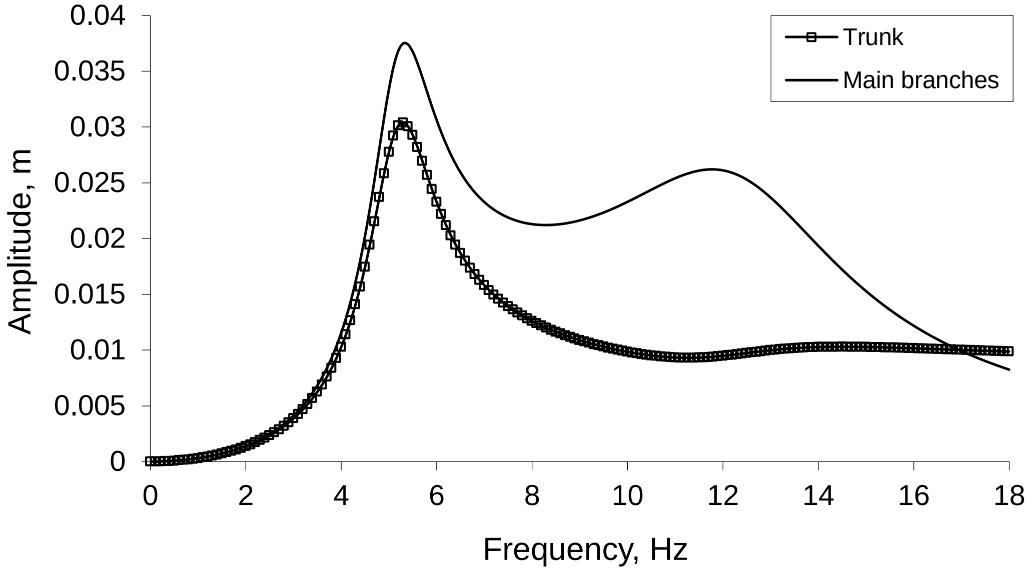

Figure 3 Displacement amplitudes versus frequency diagrams for the trunk and limb of the model

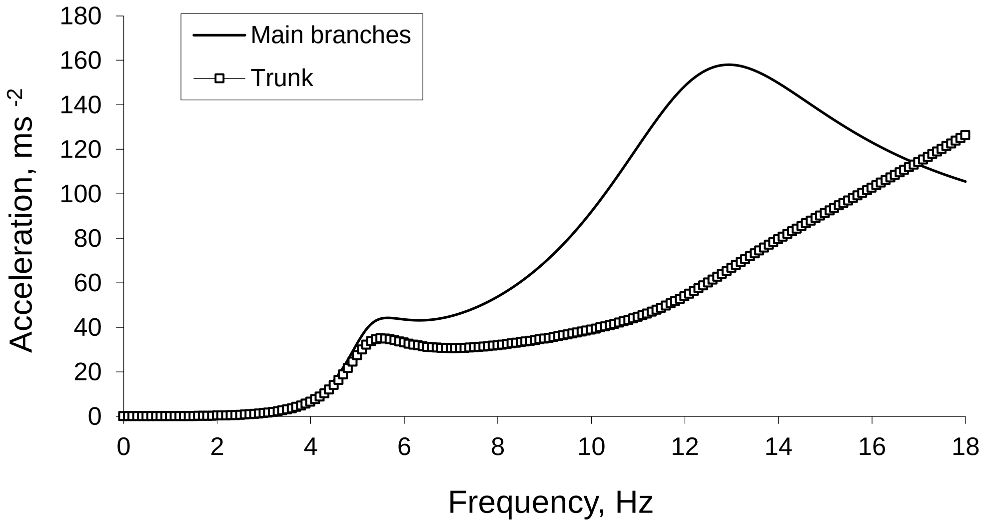

Figure 4 Acceleration versus frequency diagrams for the two parts of themodel tree

The theoretical average power demand of shaking the model tree can be calculated, applying the Equation (22) (Horváth and Sitkei, 2001):

$$P_{s,av} = \frac{1}{2} m_{1,2} \ A_{1,2}^{2} \omega^{3} \tag{22}$$

Where,

$m_{1,2}$ are the reduced masses of trunk and main branches, respectively, kg;

$A_{1,2}$ are the displacement amplitudes of trunk and main branches, respectively, m.

As the diagrams in Figure 5 indicate, the theoretical average power demand of shaking for the trunk at the first and second natural frequency is about the same. In case of the limb, more power is used at 12 Hz than at 6 Hz.

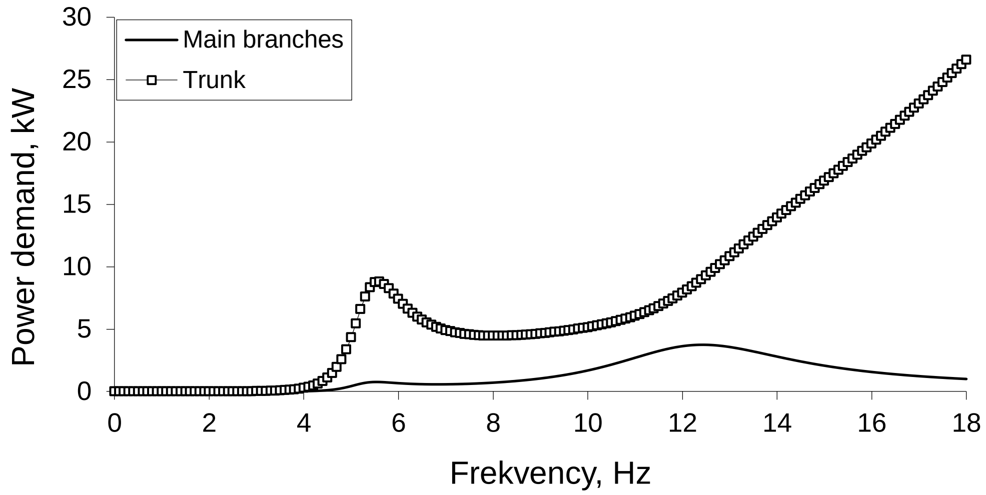

Figure 5 Theoretical power demands for trunk and limb in function of shaking frequency

Figure 6 shows the acceleration values at different heights of a real cherry tree between 4, 8 and 15 Hz shaking frequencies. The first acceleration peak is recognizable on three of five curves at 6 and 7 Hz shaking frequencies, the second peak can be clearly seen at 12 Hz at all curves.

Difference between measured and real acceleration values can be recognised in the tendency of trunk acceleration in the higher frequency range (Figures 4 and 6). The reason for this may be the dumping effect of the foliage of the limb on real trees.

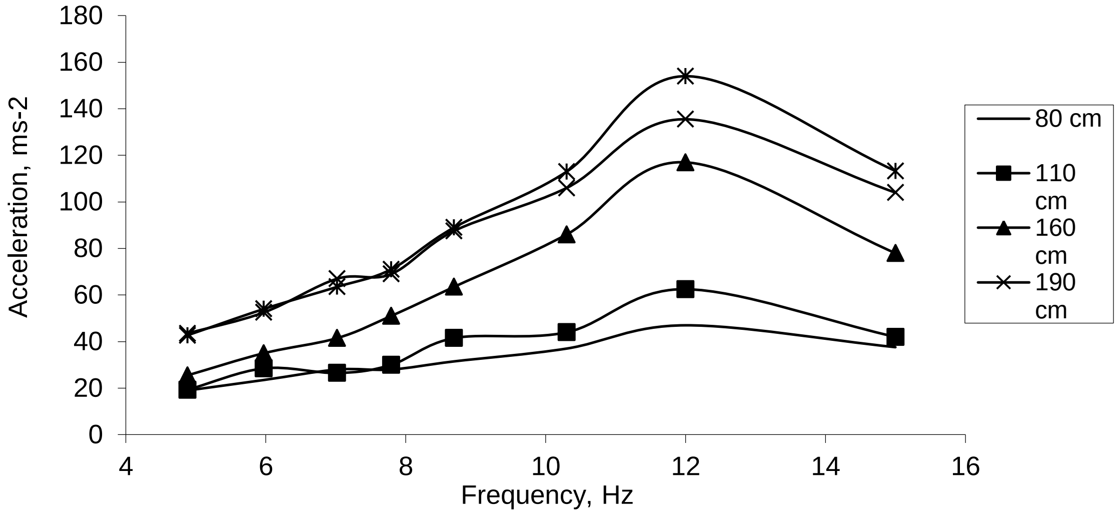

Figure 6 Acceleration versus frequency diagrams for the five different parts of a real cherry tree

The difference between measured and calculated first and second resonance frequencies may be explained by the asymmetry of the real tree structure and by the inaccurate measuring methods of the parameters for trunk and limb of the model tree.

4 Conclusions

The two degree of freedom model proved to be applicable to describe a real fruit tree more accurate than the model with one degree of freedom. It enables from one hand to test the effect of shaker machine parameters, such as unbalanced masses and the eccentricity of them as well as the mass of shaker boom on limb amplitude and acceleration.

From the other the natural frequencies of the shaker-fruit tree system can be defined. For the practice it means, that to achieve appropriate amplitude and acceleration of the branches, the tree should be shaken at its second natural frequency.

According to the diagram in Figure 3; shaking trees at low frequencies large amplitudes can be achieved. However, they don’t lead to high fruit detachment because of the low acceleration at those frequencies.

The diagrams in Figure 5 indicate that the total power use for shaking is much the same at about 6 and 12 Hz, meanwhile the acceleration is much higher at 12 Hz (Figures 4 and 6). This gives further argument to shake the trees on their second natural frequency. Increasing frequency further, more and more power would be used for shaking the trunk, and less and less for the branches.

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