Modeling optimal irrigation scheduling under conjunctive use of canal water and poor quality groundwater in semi-arid region of northwestern India

Junaid N. Khan* and A. K. Jain

(Department of Soil and Water Engineering Punjab Agricultural University, Ludhiana-141004. Punjab, India)

Abstract: In the state of Punjab, India available water resources are inadequate to meet the irrigation needs of the crops. Optimal irrigation scheduling includes allocation of limited water supply to several crops so, as to maximize the net benefits and reduce the stress of the crops during its growing season. Dynamic programming technique of optimization has been adopted for seasonal allocation of water for multiple crops (Wheat, Barley, Mustard and Gram). The stochastic nature of canal water releases of Golewala distributary for 20 years (1982-2001) was estimated by gamma distribution. Based on this expected values of canal water releases were computed as 3766.41, 4138.76, 4422.2, 4674.5 and 4918.95 hectare – meter (ha-m) corresponding to 10%, 20%, 30%, 40% and 50% risk levels of canal water releases in the distributary. The conjunctive use of canal water along with bad quality ground water offers sustainable water allocation option based on water production function. The seasonal allocation is done corresponding to different combinations of canal water and ground water at different risk levels of canal water. The seasonal water has been further redistributed on weekly basis by making use of dated water production functions and soil water balance equation. The potential evapotranspiration was estimated by Penman Montieth method and actual evapotranspiration was estimated on the basis of soil moisture balance in the study area. Economic co-efficient, crop areas, and crops growth stage stress effects are included in the mathematical formulation at both levels. The weekly allocation takes into account the initial moisture content along with limitations in terms of channel capacity, available water supply and soil storage capacity. The allocation of water was 97% and 3% for wheat and mustard crop respectively. Model did not allocate water to barley and gram crops in the catchment area. The seasonal water was redistributed on weekly basis with different risk levels of potential evapotranspiration. The weekly allocation of water varied from 0 – 22.5 mm for 10% risk level of evapotranspiration. The risk level of evapotranspiration did not much affect the allocation and varied from 278.08– 79.01 for full season. The net returns for 10% and 50% risk levels of canal water and 30% ground water were 8.51% and 32.42% higher than existing net returns observed in the command area. The increase in the ground water amount beyond 30% tends to have an adverse effect on the yield of the crops.

Keywords: cropping pattern, conjunctive use, dynamic programming, optimization, salinity

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1 Introduction

Optimal irrigation scheduling is one of the techniques by which proper utilization of resources can take place. Various optimization techniques have been used to arrive at an optimal cropping pattern for optimal use of land and water resources for maximization of net benefits from irrigated agriculture (Singh, 2012a). Application of linear programming (LP) technique for irrigation management has been very popular (Singh, 2012b). An LP based economic-engineering optimization model was used by Khare et al. (2007) to investigate the scope of conjunctive use of surface water and ground water for a link command in Andra Pradesh, India. Md. Azamathulla et al. (2008); Karamouz et al. (2009); and Yang et al. (2009) used similar approaches for the management of water resources for sustainable irrigated agriculture. An integrated soil water balance algorithm was developed and coupled to a non-linear optimization model by Montazar et al. (2010) in order to carry out

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***Corresponding author: Junaid N. Khan,** Department of Soil and Water Engineering Punjab Agricultural University, Ludhiana-141004. Punjab, India. Email: junaidk1974@gmail.com

water allocation planning in complex deficit agricultural water resources systems based on economic efficiency criterion. Li et al (2011) developed and used a robust multistage interval- stochastic programming method to the planning of regional water management systems. Existing optimal on-farm irrigation schedules generally use dynamic programming for optimization. The primary aspect of irrigation scheduling is to produce potential yields from all the crops under limited water supply resources. When the available water is not adequate to meet the crop water demands for the season, water deficits during some periods in the season cannot be avoided. A deficit during the critical growth stage of the crop will have a more profound effect on the yield than during non-critical growth stage.

The conjunctive use problem can be formulated as a combined simulation-optimization model. The decision variables of the optimization model are the optimum cropping pattern and water allocation. Dynamic programming is one of the best optimization tools for the optimal allocation of land and water resources in irrigated agriculture for maximizing the objectives of the water resources system while satisfying the hydraulic, hydrologic and operational constraints (Chavez-Morales et al., 1987; Vontaya et al., 1997; El- Awar et al., 2001; Khare et al., 2007; Regulwar and Gurav, 2011). Poor quality groundwater can be used conjunctively with good quality canal water to fulfill crop water demand and maximize net annual returns particularly in the arid and semi-arid regions where good quality soil and water resources are limited (Singh and Panda, 2012). The conjunctive use of surface water and ground water has been considered as an important factor for optimal utilization of water resources in a canal command area (Yang et al., 2009; Mantazar et al., 2010; Lu et al., 2011). Conjunctive use of canal water and poor quality ground water can lead to about 51.3% -12.5% increase in net annual return from the area at 10% -90% probability of exceedance of rainfall and canal water availability (Raul and Panda, 2013).

A DP-DP iterative approach based on seasonal water production function (Panda et al,. 1996) has been adopted in the study which takes in to account the depth of applied water and its salinity level. It is assumed that the crop yields are limited only by water applied and salinity (Matanga et al., 1979). The intraseasonal weekly irrigation intervals have been taken as per calendar year. Irrigations are assumed to be given at the beginning of these intervals. The area occupied by each crop is specified at the beginning of the season based on the existing cropping pattern. i.e. being followed in the command area. The specific objective of the study was to formulate a dynamic programming model for optimal seasonal and weekly irrigation allocation subject to seasonal and intraseasonal constraints on water supply and land allocation. The irrigation programs are derived at the beginning of the season at specified risk levels of canal water and potential evapotranspiration.

2 Study area

The present study area lies between 30 º- 53’ to 30º- 51’ N latitude and 74 º- 34’ and 79º- 50’ E longitude. It is a canal command area bounded by the Golewala distributory which lies in south western plain region of Punjab (Figure 1). The region is semi-arid in nature. The mean monthly temperature varies over a wide range, minimum air temperature during winter (January) reaches as low as 4.7 ℃ whereas monthly maximum air temperature in summer reaches as high as 45 ℃. The average rainfall is 440 mm with two thirds occurring during June through September. The soils of the study area are formed primarily from the alluvium carried by the river Sutlej. Soil texture varies from clay loam to sand. The culturable and gross command area of the Golewala distributory are 28,700 and 29,800 ha respectively. The year wise weekly canal water releases (ha-m) for 20 years from 1982-2001 are taken into consideration for considering the stochastic nature of canal water releases.

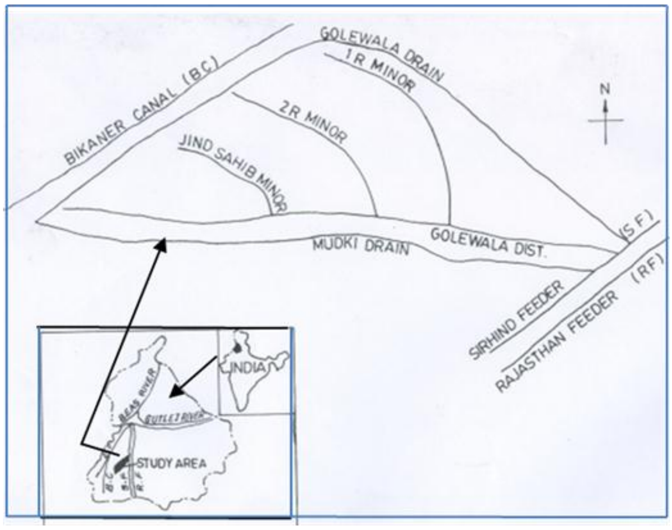


Figure 1 Location map of the study area

The canal water which is the major source of irrigation in the command area is able to meet only 56% of the irrigation demand at 90% probability of exceedence level. Since, canal water supplied in the study area was inadequate hence; some percentage of saline groundwater was used in conjunction with canal water to reduce the shortages of irrigation water. The salinity of canal water in the command area was (0.41 dS/m) and that of the ground water as (3.91dS/m). The groundwater draft was calculated by multiplying the ground water available in the Rabi season by the number of tube wells that are present in the command area. On the basis of the value 10%, 20% and 30% of the groundwater draft was calculated for irrigation purposes along with the available canal water.

3 Model development

The primary objective is to allocate limited seasonal water to the crops grown in the Golewala command area located in the southwest part of the Punjab State. The main objective is to allocate water and area in such a way so that the net returns are maximized, with respect to applied constraints.

The allocation problem is decomposed into two levels seasonal and intraseasonal termed as module I and module II. In the first module seasonal allocation of water and land for multiple crops has been done by using water production function of various crops grown in the study area and dynamic programming technique. The objective function is to maximize the net returns of the crops grown in the command area.

In the second module weekly irrigation programs were obtained for the allocated seasonal supplies are derived from module I. Module I provides the input to module II. In the following sections the conceptual basis of these optimization models is briefly described before presenting their mathematical formulation.

3.1 Allocation of seasonal water and area to multiple crops (module I)

The seasonal water and area allocation has been done corresponding to four major crops, i.e. Wheat, Barely, Mustard and Gram grown in the study area in Rabi season. The limits on the area to be allocated were prescribed based on the cropping pattern followed for about two decades by incorporating the local requirements. The water production functions of the crops are as shown as Equations (1) to (4):

1. **WHEAT**

$$Y = 11.4383 + 1.6899W + 1.9973S - 0.0197W^2 - 0.2882S^2 - 0.0084WS \tag{1}$$

2. **BARLEY**

$$Y = 18.119 + 1.1956W - 0.948S - 0.0258W^2 + 0.0322S^2 - 0.0069WS \tag{2}$$

3. **MUSTARD**

$$Y = 9.2671 + 1.0636W - 0.6057S - 0.0179W^2 + 0.0322S^2 - 0.0069WS \tag{3}$$

4. **GRAM**

$$Y = 10.813 + 0.834W - 2.0045S - 0.0141W^2 + 0.2709S^2 - 0.1912WS \tag{4}$$

Y = crop yield, Kg/ha; W = depth of applied water, cm and S = salinity of applied water, dS/m.

The model is solved for various quantities of seasonal water supply (V_k) and area available area (A_k). The decision variables in the dynamic programming are area and the water to be allocated to various crops. The allocation has been done corresponding to canal water and ground water available in the command area. The year wise weekly canal water releases in golewala distributory (ha-m) for 20 years from 1982-2001 are taken into consideration for considering the stochastic nature of canal water releases. The data was fitted to gamma distribution and the scale and shape parameters were found out as 0.00509 and 25.4097 respectively. Based on these parameters the expected values of canal water releases were computed as 3766.41 ha-m, 4138.76 ha-m, 4422.2 ha-m, 4674.5 ha-m and 4918.95 ha-m corresponding to 90%, 80%, 70%, 60% and 50% probabilities of exceedence respectively.

The long-term average value of canal water release in rabi season was found out to be 4984.17 ha-m. As the ground water in the Golewala command area is of poor quality, a limited portion of groundwater is used along with canal water. The various combinations of available water to be allocated in the command area have been taken as:

1. Canal water
2. Canal water along with 10% groundwater.
3. Canal water along with 20% ground water.
4. Canal water along with 30% ground water.

The optimal allocation of water 'V_0' to different crops were obtained by two dimensional dynamic programming using the following recursive Equation 5, Equation 6 and Equation (7):

$$B^*(X_k, A_k) = A_k \, PRO_k \, Y_{kmax} \, Y_k \, (X_k) \quad (5)$$

$$f^*(X_k, A_k) = \max feasible_{(X_k.A_k)} \quad [B^*(X_k, A_k) + f_{k-1}(Q - X_k, A - A_k)] \quad (6)$$

$$F_n{}^*(X_N, A_N) = \max feasible_{(X_N.A_N)} \quad [B^*(X_N, A_N)] \quad (7)$$

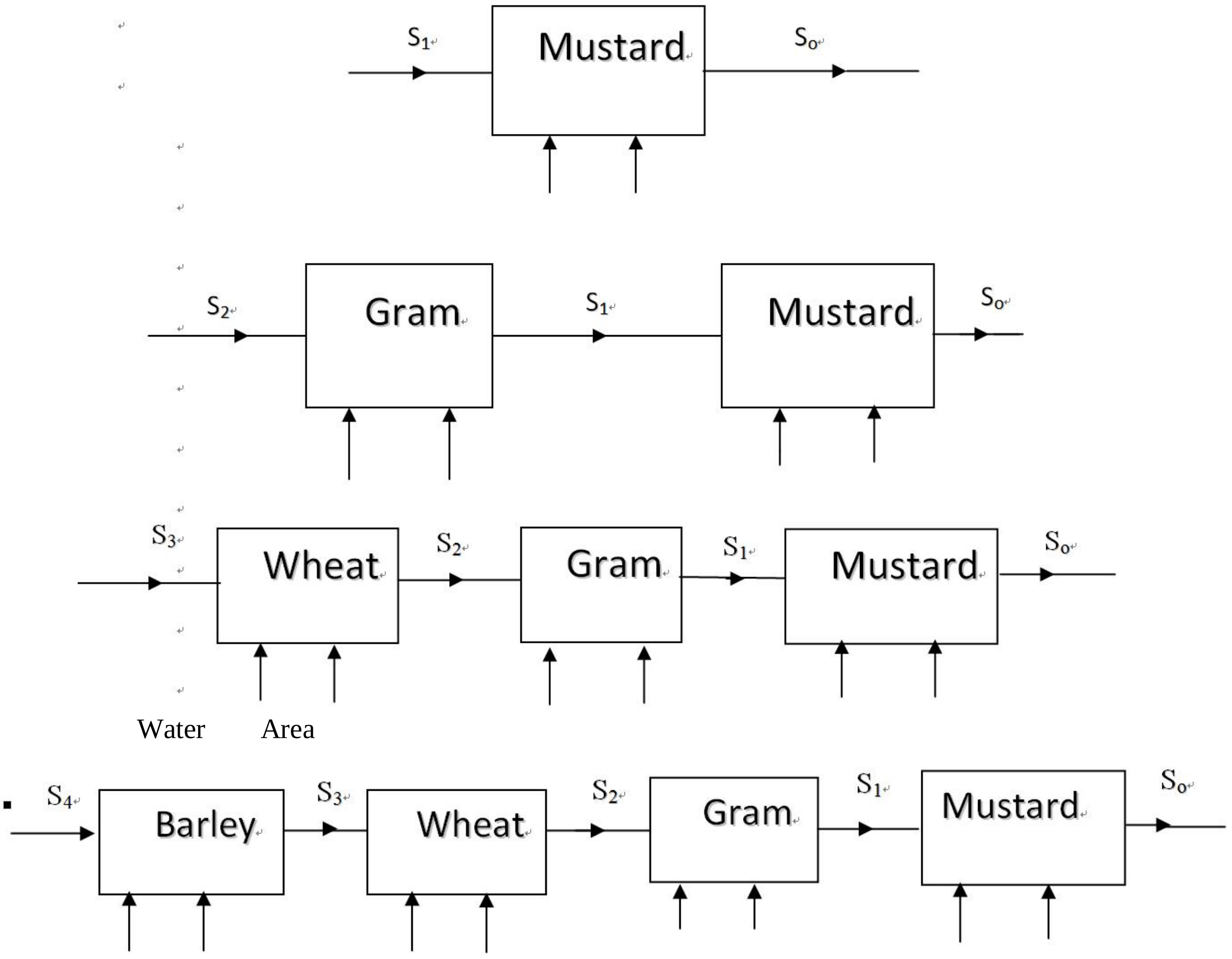
A_k= area allocated to crop k, ha; PRO_k = profit for crop k, Rs.; Y_k = maximum obtainable yield corresponding to crop k, kg/ha.; Y_k (X_k) relative yield. X_k = depth of irrigation water applied, cm. B^* (X_k , A_k) = net profit for allocated amounts of water and area.

The area constraints were fixed as Equation (8) and Equation (9):

$$\sum A_k \leq A \quad (8)$$

$$\boldsymbol{A_k} \leq \boldsymbol{A_{kmax}} \quad (9)$$

$A_{k\ max}$ = maximum area that can be applied to particular crop, ha; A =total area available in the command area, ha. Figure 2 shows the flow chart for seasonal allocation of land and area to multiple crops in a canal command area.



S_1, S_2, S_3, S_4 = State variables

Water, Area = Decision variables

Figure 2 Water allocation for rabi season using dynamic programming with two dimensions (available water, area)

3.2 Single crop intraseasonal allocation (module II)

The problem of irrigation scheduling for an individual crop is usually solved by incorporating relationship between water stress and yield called dated water production function in a dynamic programming models (Jones, 1983). A multiplicative dated water production function derived from sensitivity factors for water stress in physiological growth stages of crops is used (Rao et al., 1988). Table 1 shows the basic data of the crops grown in the Golewala command area.

Table 1 Basic data of crops.

Crop characteristics	*Wheat*	Mustard	Barley	Gram
Avg. date of sowing	3rd Nov	3rd Nov	3rd Nov	23rd Oct
Length of the crop development stage, days	150	150	125	160
Length of the crop development stage, days	25:35:60:30	15:4565:25	15:25:55:30	25:50:55:30
Crop coefficient at different stage, Kc	0.34:0.695: 1.05:0.65	0.34:0.61: 0.88:0.82	0.34:0.695: 1.05:0.65	0.26:0.63: 1.0:0.623
Yield response factor, Kt	0.26:0.6:0.6:0.5	0.3:0.55:0.6:0.3	0.2:0.55:0.45:0.2	0.2:0.9:0.7:0.2
Maximum root depth, cm	120	125	165	135
Product price, Rs/kg	13.5	30.0	9.8	30.0
Maximum area, ha.	20050.0	460.0	515.0	688.0

To obtain weekly irrigation programs, the single crop model is solved in two stages. In the first stage dated water production function is maximized by dynamic programming to obtain water allocation to growth stage periods. Available water supply and soil moisture at the beginning of each growth stage are the two state variables used. The water allocation are subjected to constraints imposed by the soil water balance model and irrigation system in the second stage, the water allocated to each growth stage is reallocated to the weeks comprising the growth stage in a sequential order.

The dated water production function used is as Equation (10) (Rao et al. 1988):

$$\frac{Y}{Y_{max}} = \prod_{t=1}^{NP}\left[1 - K_t\left(1 - \frac{AET}{PET}\right)\right] \qquad (10)$$

PET= potential evapotranspiration, mm; AET = actual evapotranspiration, mm K_t = yield response factor; Y_{max}= maximum yield obtainable, kg/ha; and Y = Actual yield obtainable, kg/ha.

The values of potential evapotransporation ($PET_{i,j}$) are estimated by the procedure using FAO – 56 Penman Montieth method (Allen et al., 1998). The actual evapotransporation (AET_{ij}) are estimated from soil water balance model. Details of the soil water balance model are given by Rao (1987). The actual evapotranspiration in each week is found as following Equations (11) to (14):

$$ET_0 = \frac{0.408\,\Delta\,(R_n - G) + \gamma\,\frac{900}{T+273}u_2(e_s - e_a)}{\Delta + \gamma(1+0.34)U_2} \qquad (11)$$

$$PET = ET_0 * K_c \qquad (12)$$

$$AET_{ij} = PET_{ij} \quad for \quad Z_{ij}W_{ij} \geq (1-P)W_c Z_{ij} \qquad (13)$$

$$AET_{ij} = \frac{W_{ij}\,Z_{ij}\,PET_{ij}}{(1-p)W_c Z_{ij}} \quad for\ Z_{ij}W_{ij} < (1-P)W_c Z_{ij} \qquad (14)$$

Then , by soil water balance equation as Equation 15:

$$W_{i,j+1},\, Z_{i,j+1} = W_{i,j}\,Z_{i,j} + R_{i,j} + X_{i,j} + W_o\left(Z_{i,j+1} - Z_{i,j}\right) - AET_{i,j} - DP_{i,j} - Rn_{i,j} \qquad (15)$$

Where, ET_0 = reference evapotranspiration, mm/day; R_n = net radiation at the crop surface, MJ / m^2 /d; G = soil heat flux density, MJ / m^2 /d; T = mean daily air temperature at 2 m height, ℃; U_2 = wind speed at 2m height, m/ s; e_s = saturation vapour pressure, kPa; e_a = saturation vapour pressure, kPa; e_s-e_a = saturation vapour pressure deficit, kPa; Δ = slope vapour pressure curve, kPa/ ºC; γ = psychrometric constant, kPa /ºC; P = soil water depletion factor. Z i,j = root length, cm; R_{ij}= expected rainfall, mm; X_{ij}= applied irrigation, mm; W_c = Field capacity, mm/cm; Rn_{ij}= Run-off in particular interval, mm; Dp_{ij}= deep percolation losses, mm; W_0= available soil water at beginning of season, mm/cm.

The values of Z_{ij}, $Z_{i,\ j+1}$ are calculated from root growth model as developed by Borg and Grimes (1986). The Equation (16) for root growth model is given as follows:

$$Z = RD_m\left[0.5 + 0.5\,Sin\left(3.03\,\frac{DAP}{DTM} - 1.47\right)\right] \quad (16)$$

Where DAP = current day after planting; DTM = days to maturity; RD_m= maximum rooting depth, cm and Z= current rooting depth, cm.

The expected rainfall (R_{ij}) and irrigation (X_{ij}) are lumped and input to the reservoir at the beginning of the interval. (Rn_{ij}) run-off losses are neglected. The irrigation depth (X_{ij}) applied at the beginning of the interval is subjected to the following constraints. It is zero if the available soil water in the absence of irrigation (but after including the expected rainfall) is adequate to maintain the evapotranspiration at its potential rate up to the end of the interval otherwise, it is limited by the soil storage capacity, or the remaining supply from the water allocated to the growth stage (Xi) or the delivery capacity of the irrigation channel (AWC_{ij}) during the interval I see Equation (17) and Equation (18):

$$X_{i,j} = 0 \quad if \quad Z_{i,j}W_{i,j} \geq (1-p)W_c Z_{i,j} + PET_{i,j} \qquad (17)$$

$$X_{i,j} = min\begin{cases} W_c Z_{i,j} - W_{i,j-1}Z_{i,j-1} - R_{i,j-1} - R_{i,j} - W_o \Delta Z_{i,j} \\ \quad (Available\ soil\ water\ storage) \\ X_i - \sum_{t=1}^{j-1} U_{it}(available\ water\ supply) \\ AWC_{i,j}(channel\ capacity) \end{cases} \qquad (18)$$

If Q_O is the available water supply depth at the beginning of the season and Xi the water allocated to each growth stage, then Equation (19) and Equation (20) as below

$$Q_0 \geq \sum_{i=1}^{N} X_i \qquad (19)$$

$$X_i = \sum_{j=1}^{M_i} X_{ij} \qquad (20)$$

Equation (10) is maximized by dynamic programming using the recursive Equation (21) and Equation (22).

$$f_i(Q,W) = max\left[1 - k_i\left(1 - \frac{AET}{PET}\right)i\right] f_{i+1}(Q - X_{i.}W_{i+1}) \qquad (21)$$

$$for\ [0 \leq x_i \leq Q, 0 \leq Q \leq Q_0, i = N-1, N-2, N-3 \ldots\ldots\ldots..1]$$

$$F_N(Q,W) = max\left[1 - k_N\left(1 - \frac{AET}{PET}\right)N\right] \qquad (22)$$

$$for\ [0 \leq x_N \leq Q, 0 \leq Q \leq Q_0\ ]$$

Equation (21) and Equation (22) are maximized subject to Equation (11), Equation (12), Equation (13), Equation (14), Equation (15), Equation (16), Equation (17), Equation (18), Equation (19) and Equation (20) to obtain optimal water allocation X_i^* (i =1, N) to growth stage 'i' for specified 'Qo' and 'Wo'. The allocation to weekly intervals u_{ij} weekly intervals u_{ij} (j =1, M_i; i = 1, N) are obtained by running the water balance model Equation (11), Equation (12), Equation (13), Equation (14), Equation (15), Equation (16), Equation (17), Equation (18) and Equation (19) for the whole season with ($X_i = X_i^*$). X_i^* is the optimal water to be allocated to a particular stage.

3.3 Model application

3.3.1 Application of seasonal allocation model (module I)

The computer programs for seasonal allocation model were formulated to take into account the optimal allocation of area and water resources to a multiple crop situation. The data of canal water releases was fitted into gamma distribution and the allocation was done corresponding to 10%, 20%, 30%, 40% and 50% risk levels of canal water. The allocation at 10% and 50% risk levels corresponding to different combinations of available water is shown in Table 2 and Table 3 respectively.

Table 2 Water allocation at 10% risk level of canal water (ha–m)

Available water (ha – m)	Crop			
	Wheat	**Barley**	**Mustard**	**Gram**
Canal water (S=0.4lds/m)	3666.41	0.0	100.0	0.0
Canal water +10% G.W (S=0.606 ds/m)	3890.72	0.0	100.0	0.0
Canal water +20% G.W (S=0.785 ds/m)	4115.03	0.0	100.0	0.0
Canal water +30% G.W (S=0.940 ds/m)	4339.35	0.0	100.0	0.0

Table 3 Water allocation at 50% risk level of canal water (ha-m)

Available water (ha – m)	Crop			
	Wheat	**Barley**	**Mustard**	**Gram**
Canal water (S=0.4lds/m)	4818.0	0.0	100.0	0.0
Canal water +10% G.W (S=0.563 ds/m)	5043.27	0.0	100.0	0.0
Canal water +20% G.W (S=0.704 ds/m)	5267.57	0.0	100.0	0.0
Canal water +30% G.W (S=0.834 ds/m)	5491.89	0.0	100.0	0.0

From the tables it is clear that a limited amount of groundwater can be used in combination with canal water due to poor quality of groundwater. It can also be interpreted from the Table 2 and Table 3 that major allocation of water was to wheat crop. The results revealed that 97% of total available water was allocated to wheat crop and the rest 3% to other crops. The area allocation to different crops was based on the fact that maximum area is allocated to each crop out of total available area with respect to the constraints that have been fixed on each crop. Table 4 and Table 5 give the net returns corresponding to 10% and 50% risk levels of canal water. However, there was a decreasing trend in the increase of the net returns. The net returns corresponding to Barley, Gram and mustard were found to be decreasing with the increase in the proportion of groundwater used, while in the case of wheat the net returns were found to be increasing with the increase in the proportion of ground water use. The expected net returns obtained for different combinations of canal water and ground water are depicted in Figure 3. It was observed that net returns decreased with the decrease in the risk level. The maximum net returns were observed for canal water and 30% ground water application. The net return corresponding to 90%, 80%, 70%, 60% and 50% probability of exceedance are 814.73, 879.99, 934.24, 960.6 and 992.3 million rupees respectively.

Table 4 Net returns (million Rs) at 10% risk level of canal water

Available water (ha – m)	Crop			
	Wheat	Barley	Mustard	**Gram**
Canal water	690.16	1.49	25.31	4.92
Canal water +10% G.W	730.95	1.38	24.91	4.2
Canal water +20% G.W	769.71	1.28	24.9	3.52
Canal water +30% G.W	785.98	1.17	24.69	2.875

Table 5 Net returns (million Rs) at 50% risk level of canal water

Available water (ha – m)	Crop			
	Wheat	Barley	Mustard	**Gram**
Canal water	859.06	1.49	25.31	4.92
Canal water +10% G.W	897.72	1.38	25.10	4.2
Canal water +20% G.W	933.59	1.28	24.9	3.52
Canal water +30% G.W	963.53	1.17	24.69	2.87

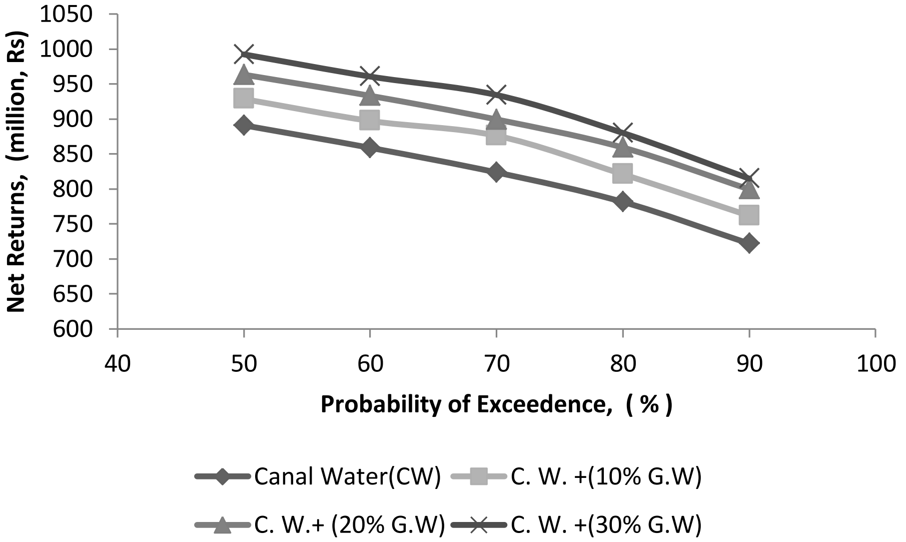


Figure 3 Expected value of total net returns for different combinations of canal water and ground water (G. W.)

3.3.2 Application of intraseasonal scheduling model (module II)

The weekly allocation for wheat and mustard crop was done at the different risk levels of potential evapotranspiration (Rhenals, 1981) and at different initial soil moisture content. No intraseasonal scheduling was done in case of Gram and Barley as no allocation was done on the basis of seasonal model, i.e. module I. The weekly allocation for wheat and mustard at 10%, 20%, 30%, 40%, 50% risk levels of PET and 0.4 mm/cm initial soil moisture content for wheat and 0.6 mm/cm mustard at the beginning of the season are given in Table 6 and Table 7 respectively. The uncertainty in PET was taken into account by making use of normal distribution and lognormal distribution depending upon the skewness of the data (Hann, 1979).

Table 6 Weekly optimal irrigation water allocation (mm) per unit area for wheat crop

(I.S.M.C =0.4 mm/cm)

Interval	**Calendar week**	**Probability of exceedence level of ETp**				
		90%	**80%**	**70%**	**60%**	**50%**
1	44	3.07	3.07	3.07	3.07	3.07
2	45	3.42	3.42	3.42	3.42	3.42
3	46	12.63	12.84	13	13.13	13.25
4	47	0	0	0	0	0
5	48	20.96	21.88	22.12	22.29	22.57
6	49	0	0	0	0	0
7	50	19.53	18.8	18.4	19.0	17.7
8	51	0	0	0	0	0
9	52	0	0	0	0	0
10	1	21.7	21.9	22.1	22.5	22.5
11	2	0	0	20.9	22.05	22.5
12	3	22.5	22.5	22.5	21.2	22.5
13	4	22.5	22.5	22.5	22.5	22.5
14	5	21.4	21.5	21.7	22.5	22.5
15	6	22.5	22.5	22.5	22.5	22.5
16	7	5.2	22.5	5.8	6.6	5.0
17	8	22.5	6	0	0	0
18	9	0	0	0	0	0
19	10	21.2	21.4	21.7	21.9	22.5
20	11	22.5	22.5	22.5	22.5	22.5
21	12	22.5	22.5	22.5	22.5	22.5
22	13	22.5	22.5	22.5	22.5	22.5
*X		278.08	278.41	279.01	279.21	279.61
**y/ymax		0.901	0.883	0.859	0.854	0.845

Note: *X= water allocation (mm), **y/y_{max}= relative yield.

Table 7 Weekly optimal irrigation water allocation (mm) per unit area for mustard crop

(I.S.M.C = 0.6 mm/cm)

Interval	**Calendar week**	***Probability of exceedence level of ETp***				
		90%	**80%**	**70%**	**60%**	**50%**
1	44	3.27	3.29	3.34	3.36	3.41
2	45	4.42	4.47	4.51	4.56	4.59
3	46	7.31	7.52	7.68	7.81	7.93
4	47	18.99	19.1	19.17	19.23	19.28
5	48	15.61	15.82	15.96	16.09	16.21
6	49	2.5	22.5	4.68	4.68	4.51
7	50	0	0	0	0	0
8	51	2.89	2.58	0	0	0
9	52	0	0	0	0	0
10	1	22.5	22.5	22.5	22.5	22.5
11	2	0	0	0	0	0
12	3	22.5	22.5	22.5	22.5	22.5
13	4	0	0	0	0	0
14	5	22.48	22.5	22.5	22.5	22.5
15	6	0	22.5	22.5	22.5	22.5
16	7	22.5	0	0	10.23	10.26
17	8	10.79	10.16	10.19	0	0
18	9	0	0	0	0	0
19	10	22.5	22.5	22.5	22.5	22.5
20	11	17.53	17.56	21.73	21.89	21.97
21	12	0	0	15.9	15.53	15.67
22	13	0	0	0	0	0
*X		195.79	215.5	215.84	215.88	216.33
**y/ymax		0.975	0.967	0.966	0.961	0.957

Note: *X = Water allocation (mm); ** y/y_{max} = relative yield

The allocation corresponding to initial soil moisture content of 0.4 mm/cm and seasonal water of 280 mm for wheat crop revealed that for 90% probability of exceedence of PET the water allocation was 278.08 mm and for 50% probability of exceedence of PET, the allocation was 279.61 mm which shows a very small increase in the allocated water with the increase in risk level of PET. The allocation is done in such a way that the stress suffered by the crops is minimal. The allocation at different risk levels of PET showed almost a similar pattern with slight increase in the amount of water with the increase in the risk level from 10% to 50%.

The allocation corresponding to initial soil moisture content of 0.6 mm/cm and seasonal water of 223 mm is given in Table 7. The table revealed that for 90% probability of exceedence of PET the water allocation was 195.79 mm and for 50% probability of exceedence of PET the water allocation was 216.33 mm. The allocation showed a considerable increase in the allocated water with the increase in the risk level of PET i.e. mustard crop is less resistant to weather changes than wheat crop. The limited capacity of the water delivery system restricts the maximum feasible irrigation depths to 22.5 mm for both wheat and mustard.

4 Conclusions

A certain portion of poor quality ground water can be used in conjunction with available canal water without much adverse effect on the yield of crops. From the seasonal water model it was observed that major portion of water allocation was done to wheat crop. The allocation of water as well as the area for each crop was found to depend upon the factors like net profit per yield, maximum yield obtainable per unit area and minimum water application needed for getting the maximum yield. The net returns obtained corresponding to the given method of allocation was higher as compared to the existing net returns in the area. It was observed that for canal water alongwith 30% ground water the net returns obtained at 90% and 50% probability of exceedence were 8.51% and 32.42% higher than the existing net returns. For canal water along with 20% groundwater the net returns at 90% and 50% probability of exceedence were 6.95% and 28.47% higher than the existing net returns. For canal water along with 10% ground water the net returns at 90% and 50% probability of exceedence were 1.99% and 24.12% higher than the existing returns. The conjunctive use of groundwater beyond 30% level was adversely affecting the yield and net returns in the area due to higher levels of salinity of ground water.

Notation

AWC_{ij}	Maximum available irrigation water depth in interval (i, j), mm.
N	Number of physiological growth stages of a crop.
A_k	Area of particular crop, ha.
W_o	Initial available soil moisture at the beginning of the season, mm/cm.
i	Crop physiological growth stage (i=1,N).
I	Standard week of the year (I=1,52).
J	Weekly interval of growth stage (j= 1,Mi).
k	Crop (k=1,n).
M_i	Number of weekly intervals in i^{th} growth stage.
N	Number of crops.
W_c	Field capacity of soil, mm/cm.
P	Soil moisture depletion factor.
X_i^*	Optimal water allocated to a particular stage.
A_k	Area allocated to a particular crop.
NP	Number of days in a particular interval.
P	Soil moisture depletion factor.
R_{ij}	Rainfall in that particular interval.
W_o	Initial soil moisture content.
ΔZ_{ij}	Change in root depth in that particular interval.
U_{it}	Available water supply in that particular interval.

AWC_{ij} Irrigation channel capacity in that particular interval.

X_{ij} Irrigation depth in that particular interval.

I.S.M. C Initial soil moisture content.

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