

# Measuring the sensitivity of parameters estimates to evaluate the tractive force model

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**Abstract:** Sensitivity measured parameters were used to evaluate a better tractive efficiency and tractor forward speeds. The predictive models for tractive force were developed using dimensional analysis. A disc ploughing tillage operations in a loamy sand soil, at tractor forward speeds of 1.94 m/s, 2.22 m/s and 2.5 m/s were conducted, using trace tractor techniques. The dependent and independent variables involved in the models were measured. The wheel tractors tractive force models have sensitivity coefficients ranging from -0.004394 to 2.353, -6.25E.05 to 1.0877 and -1.32E-07 to 1.00 for ploughing, with all the independent variables at 1.94 m/s, 2.22 m/s and 2.5 m/s forward speeds respectively. The results obtained in this disc ploughing tillage operation showed that independent variable (U1) caused a negligible change in dependent variables (N). This shows that the sensitivity coefficients obtained in the developed predicted models as variables change are insignificant. Thus, sensitivity coefficients of the tractive force models developed fitted experimental data. Therefore the developed predicted models could be applied.

**Keywords:** Tractive force, predictive, disc ploughing, loamy, sand, soil, sensitivity measurement

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## 1 Introduction

Agricultural operations must be carried out within a time frame to achieve maximum yield and operational efficiency (Sale et al., 2013). In the report of Ajav and Adewoyin (2012), it was opined that profit making is critical to the success and sustainability of any business venture and it is pertinent that agricultural mechanization follow the same trend for a meaningful economic and environmental impact. Tillage is the mechanical manipulation of the soil, for crop production in agriculture. A good tillage operation provides suitable soil pulverization (Nwokedi, 1992). Agricultural tillage involves soil cutting, turning and pulverization, therefore requires high tractive force to pull the tillage tool through the soil during tillage operations (Nkakini and Akor, 2012). Tillage is a practice that is significant to the soil

quality, and has important effects on many soil characteristics (Ahaneku and Dada, 2013). The aim of tillage is to create a soil environment and good surface configuration for favourable plant growth (Nail et al., 2007; Nkakini and Akor, 2013; Nkakini et al., 2008). Primary (ploughing) tillage is the initial reduction of soils strength which cuts deeper into the soil, leaving a rougher surface relative to secondary tillage operation. Al-Suhaibani and Ghaly (2010) investigated the effect of ploughing depth and forward speed on the performance of a medium size chisel plow operating in a sandy soil and observed that ploughing depth and forward speeds affect the average fuel consumption for different kinds of farm tractors operating in the same zone. Traction is required to pull the tillage tool through the soil during tillage operations. It is another name for grip. Grip is the driving force developed by a wheel. A wheel tyre generates tractive force by reacting (pushing) against the soil. The traction of the tyre is achieved by its good grip with the surface over which tractor moves. When tractor wheels are in motion, slip occurs and causes

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reduction in speed, which can be achieved by increasing traction (Ghosal and Das, 2008). Soil strength is often represented by the cone index, CI, which is the average force per unit area required to force a cone –shape probe vertically into the soil at a steady rate. Cone index of agricultural soils is a very important factor that is measured in most tillage studies, and it indicates the resistance of soil to penetration. Fasinmirin and Olorunfemi (2013) reported from the findings of other researchers that cone index of an agricultural soil affects the penetration of plant roots. Cone index is extensively used in assessing soil compaction (Canillas and Solakhe, 2012), soil traffic ability (Goering et al., 2003) and the effectiveness of tillage operations (Canarache, 1990).

Drawbar pull is the total thrust minus the total resistance to vehicle motion. It is the lateral forward force a wheel can develop when moving and indicates the force the wheel or the tractor can generate over the main force resisting to movement consisting of rolling, obstacle, slope steering resistances (Saarilahti, 2002).

The rolling resistance increases as a function of sinkage, being lower on hard surface and higher on softer soils (Saarilahti, 2002; Georing et al., 2003).

The operational parameters such as depth and speed influence draft and energy requirements of tillage implements. The relationship between draft and speed has been reported as linear, second-order polynomial, parabolic and exponential by some researchers. Ale et al. (2013) reported a polynomial relationship between speed and draught and stated that draught increased in forward speed of the tractor, and later decreased with increase in forward speed at average moisture content of 4.9%.

Likewise according to the reports of Al-Janobi and Al-Suhaibani(1998) the effect of depth on draught, also varies linearly, while the effect of speed on draught shows increases with increase in speed in most of the tillage implements. This may be due to rapid acceleration of any soil that is moved appreciably.

According to Yousef et al. (2006) the draught force increased as the forward speed increased in all soil types. However, the tillage depth had the greatest effect on the draught and drawbar power than the tractor forward speed (Ahaneku et al., 2011). Tractive force, plays a vital role in tillage operations, therefore the need to determine the appropriate tractive force models becomes necessary in order to balance the variables in operations (Nkakin and Fubara-Manuel, 2012; Nkakini and Douglas, 2013). Tractive force is the force measured in the drawbar of a tractor. The tractive effort of a vehicle, otherwise known as tractive force refers to the pulling or pushing force that a vehicle holds. This is the force that allows a vehicle to move forward. Thrust (gross tractive effort, gross traction, traction and wheel torque) is a friction force between a tyre and soil or the grip a tyre can generate from the soil surface to overcome the forces resisting the movement. The prediction of tractive force is an essential part of the requirements for vehicle performance stimulation (Young et al., 1984). Thangaradivelu and Colvin (1990) and Nkakini and Douglas (2013) reported in their findings, that field operations is largely affected by the moisture content and traffic ability of the field in question.

If the soil moisture is too high, working the soil leads to paddling compaction, soil deterioration and reduction in soil quality. For mechanised operations, this leads to increasing wheel slip, traction and time spent in tillage. This research has much of its roots in the need for improving the energy- efficiency, and tractive performance of agricultural tractor in field operations. It is necessary to be able to predict how a machine and implements perform a given task under a given condition.

Thus, the need to improve on the efficiency of tillage operations in order to increase productivity in mechanized agriculture becomes obvious. Such positive effects can only be realized through the use of appropriate models that include all important compounding variables such as tractive force, drawbar pull, rolling resistance, wheel slip, moisture content, cone index, wheel numeric,

contact pressure and tractor operational speeds of the earlier tillage models which do not include variables such as, depth and width of cuts.

The assessment of developed models for field operations using sensitivity analysis is important in creating a condition that allows for the optimization of the involved variables (Nkakini, 2014). The developed tractive force models were verified using sensitivity of measured parameters. Hence, sensitivity analysis of any developed model makes the relative role of each variable to that model known. This is done by taking notice of the change of dependent variable in the model with respect to the change in each of the several independent variables (Simonyan, 2006). Sensitivity measured parameters, is defined as the rate of change in the output of the model with respect to the change in the value of the parameter while keeping other parameters constant (Nkakini, 2014). In recent years the sensitivity analysis of ordinary differential equations (ODE<sub>s</sub>) and differential-algebraic equations (DAE<sub>s</sub>) have attracted much research (Eberhard and Bishop, 1999; Li et al., 2000). The principal motivation comes from mathematical models used to investigate physical phenomena where some parameters may not be accurately known and, therefore, sensitivity analysis becomes a useful tool (Roberto, 2006).

Dynamic traction ratio (DTR) was predicted using the Brixius (1987). Gonzalez et al. (2004) used Brixius model to predict traction of heavy sugarcane machinery. Empirical methods for traction performance modeling are mainly based on the soil cone index (CI) as the single soil strength parameter to be measured. Semi-empirical model was used for predicting tractive and motion resistance forces, but has limited practical application (Georing et al., 2006). Canillas and Salokhe (2001) developed a soil compaction prediction equations using regression analysis. Dimensional analysis has been used for prediction of the traction driving force for a wheel moving on a soil. Kazam and Alper (2012) used artificial neural network (ANN) model and non-linear

regression analysis to investigate the relationship between input parameter (travel reduction) and output parameters (net traction ratio and tractive efficiency). It was found that the ANN model consistently gave better predictions compared to the non linear regression-based model.

The objective of the study is to evaluate the degree at which developed tractive force models change to varying independent variables in ploughing operation using sensitivity analysis, optimising the variables.

## 2 Materials and methods

### 2.1 Description of site

Experiments were conducted in the field (in-situ) using trace tractor technique. It was conducted at National Root Crops Research Institute (NRCRI) experimental farm, in Umudike, Umuahia, Abia State of Nigeria. Umudike is under the derived tropical humid ecological zone of Nigeria, and is 122 m above sea level and lies on latitude 05<sup>o</sup> 29<sup>1</sup>N and longitude 07<sup>o</sup> 33<sup>1</sup>E. Soil particle size distribution analysis showed the soil to be loamy sand (clay-11.04%, silt-4% and sand-84.96%).

### 2.2 Experimental procedure

The parameters: cone index, tractive force, drawbar pull, rolling resistance, wheel slip, moisture content, speed, width and depth of plough were determined using two tractors of the Massey Ferguson 435 is of model: P4000, Gross Power: 72 hp:52.7 kw, PTO power:62.5hp:46.6kw, operating weight : 2870 kg, tyre type/size : 8 ply rating, front 12.4-24 rear 18.4-30, cone penetrometer having an enclosed angle of 30<sup>o</sup>, with a base area of 323mm<sup>2</sup> mounted on a shaft of 2.03mm, EDjunior Dynamometer 0-10,000ibf/5000kgf ID No. OEDJ1510046, measuring tape, disc plough : 3furrows weight, 2950 kg, Model: N4D-26F, auger, stop watch and instrument for measuring weight of tractor (static hydraulic press). All field tests were conducted in a loamy sand soil for which the physical properties were determined. Prior to the starting of field experiment, experimental layout area of 90 m by 90 m was designed with three different blocks of 90 m x 27 m each. Each

block was divided into 9(nine) strips of 90 m by 2 m wide with a space of 3m between each strip. Disc ploughing operation was carried out on each of the blocks twenty four (24) hours after each rainfall event. Three replications of ploughing operations were conducted after every rainfall event. There were altogether 20 rainfall events. Therefore, the total treatments were 9x20 rainfall events. The order of tillage operation was:

Rainfall event 1, disc ploughing on block 1, strip 1; block 2, strip 1; and blocks 3, strips 1. Rainfall event 2, disc plough on block 1, strip 2; block 2 strip 2 and block 3 strip 2. Rainfall event 3, disc plough on block 1 strip 3; block 2 strip 3 and block 3, strip 3. This pattern was followed for the remaining days up to the last day when minimum moisture content was achieved.

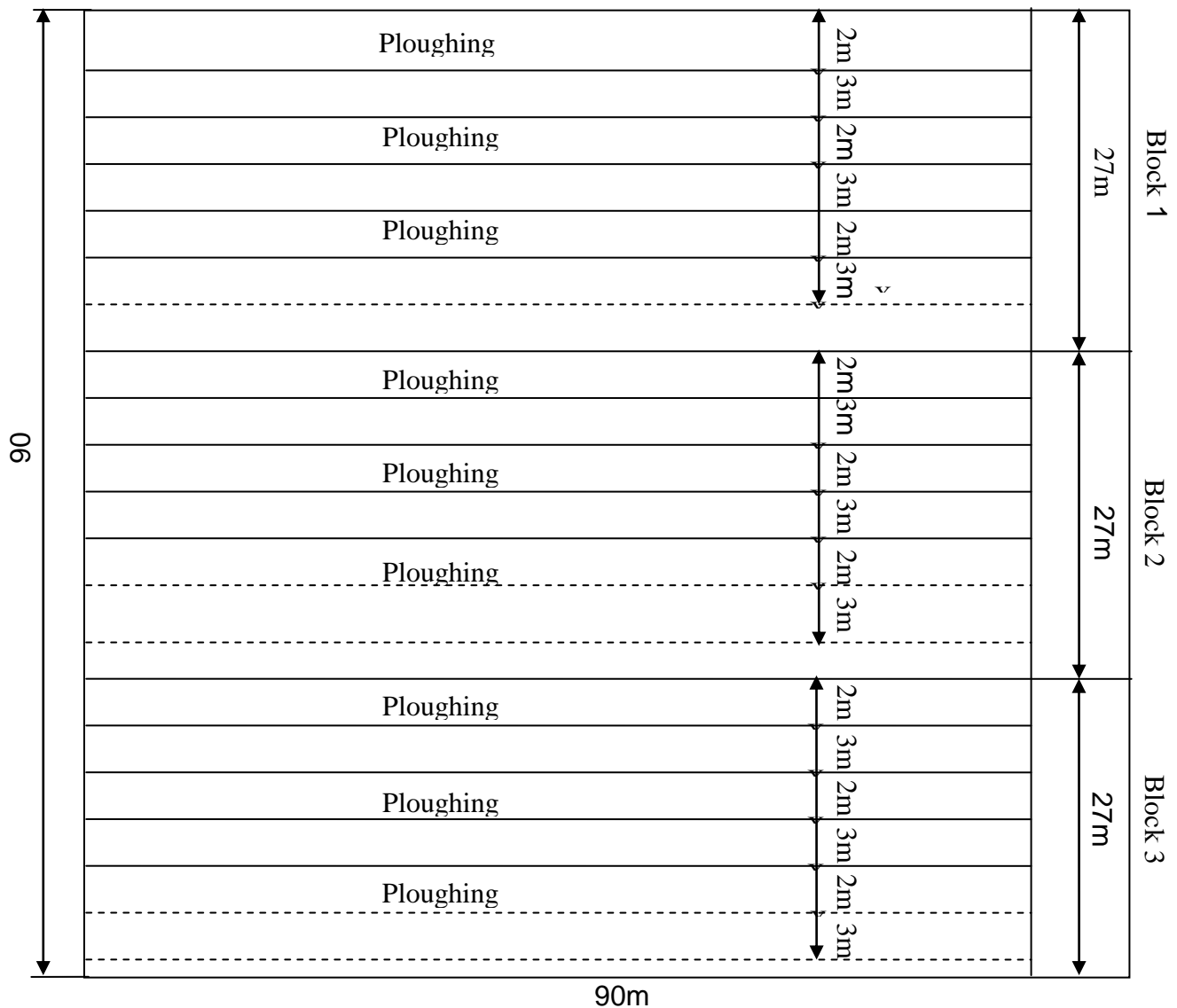


Figure 1 Experimental field layout

Soil data were collected to the depths of 50 mm, 150 mm and 200 mm respectively using soil auger, core sampler and a hand-operated soil cone penetrometer (ASAE, 2001) having an enclosed angle of  $30^{\circ}$ , with a base area of  $323 \text{ mm}^2$  mounted on a shaft of 2.03 mm for the determination of moisture content, bulk density and soil resistance before tillage operation. During the sampling process, cone penetrometer was positioned between the operator's two legs and pressed down the soil until the marked point on the shaft was reached, before readings were taken. The bulk density was measured using core sampler. Soil moisture content was determined using gravimetric method (oven dry method). The tractor wheel load was measured using static hydraulic press. The tractor with mounted disc plough implement was driven into station's weighing platform (single point load cell) positioned with its rear wheels and weighed. The record was taken and recorded. This is the static load. The dynamic load was measured as the tractor was put in motion with and without disc plough implement mounted on the tractor and the weight taken and recorded respectively. However, in this study, static load was used in computation for purpose of accuracy. The towing force and drawbar-pull forces were determined using trace- tractor technique. (Appendix E, shows the measured parameters and predicted tractive forces). That is, two Massey Ferguson 435 model tractors of 72-horse power (hp) were used. The tractor carrying the implement with its engine disengaged (neutral gear) was coupled to another tractor which towed it with the

dynamometer in between them. The first tractor pulled the second tractor coupled to the implement (disc plough). The dynamometer reading was taken to determine the towing force. The drawbar-pull force was the difference between the towing force in neutral gear without implements in tillage operation and towing force when the implement was engaged in tillage operations. Width and depth of cuts were measured with a steel tape. The speeds of operations were obtained by setting the tractor at a suitable gears of a gear reduction unit for targeted speed of 1.94 m/s a negligible change, 2.22 m/s and 2.5 m/s. Simultaneously, the time taken to cover a fixed distance of 90 m was recorded using a stopwatch to calculate the operating speed of the tractor and implement combination. Figure 2 shows the tractor-dynamometer, tractor-implement combination in action.



Figure 2 A photograph depicting tractor- dynamometer, implement mounted position during disc ploughing operations

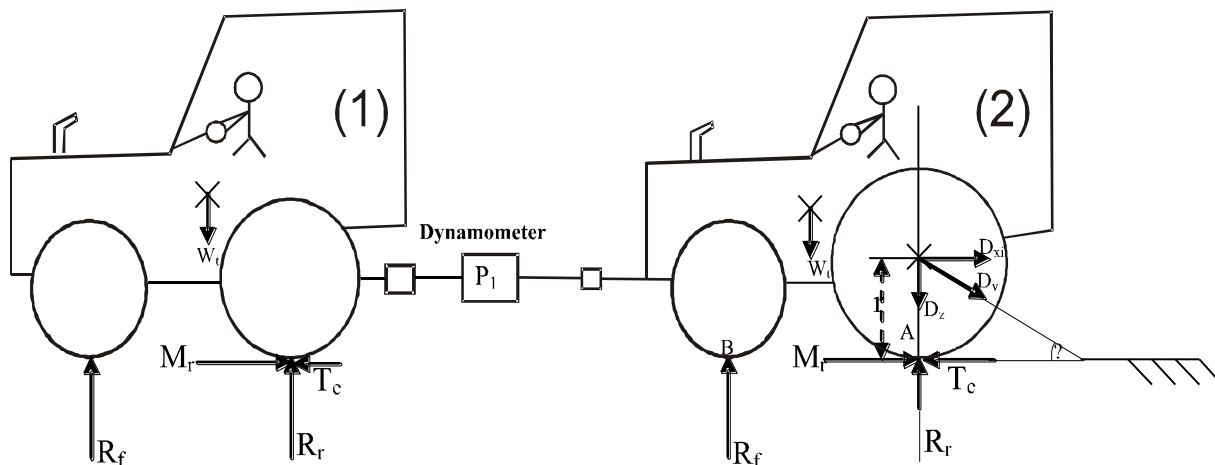


Figure 3a Depicts the force diagram of implement mounted position during disc ploughing operations

$D_v$  = Pull of the implement upon the tractor,  $D_x$  = Horizontal Component of  $D$   
 $D_z$  = Vertical Component of  $D$ ,  $R_f$  &  $R_r$  = Vertical support soil reactions upon the front and rear wheels

respectively,  $T_e$  = Tractive effort of the tractor,  $W_t$  = Force of gravity on the tractor contributed by the weight,  $r$  = Rolling radius,  $M$  = Motion Resistance

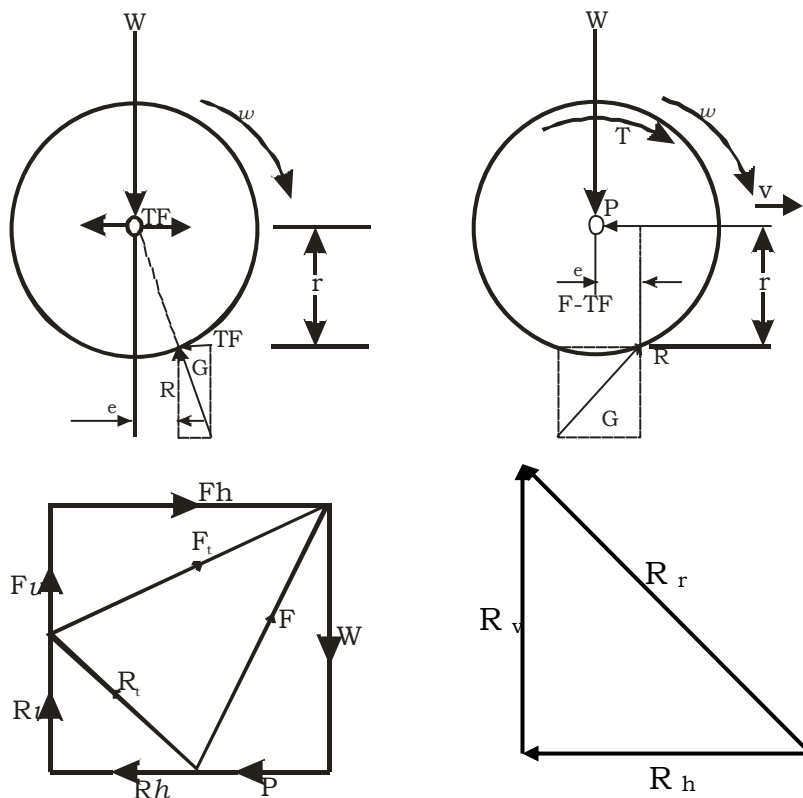


Figure 3b Force triangle analysis of traction wheel and towed wheel (Source: Ghosal and Das, 2008)

## 2 Theory

There is the need to identify accurate parameters that represent the soil and device factors to determine the functional relations between the parameters. Table 1

shows the dependent and independent variables involved in traction determination and their units, derivations and basic dimensions,

**Table 1 Variables affecting traction requirement**

Variables	Symbol	Unit	Derivation	Dimension
Dependent variable				
Tractive force	F	N	Mass x Acceleration $\text{Mass} \times \frac{m}{s^2}$	$ML/T^2$
Independent variables				
Drawbar pull	$F_p$	N	Mass x Acceleration $\text{Mass} \times \frac{m}{s^2}$	$ML/T^2$
Rolling resistance	$F_R$	N	Mass x Acceleration $\text{Mass} \times \frac{m}{s^2}$	$ML/T^2$
Wheel slip	S	%	(No derivation)	$M^0L^0T^0$
Moisture content	$\mu$	%	(No derivation)	$M^0L^0T^0$
Cone index	CI	$N/m^2$	$\frac{\text{stress}}{\text{strain}} \Rightarrow \frac{\text{Force}}{\text{Area}/\text{Number}} = \frac{MLT^{-2}}{L^2/1}$	$M/L/T^2$
Wheel numeric	$C_n$	-	$\left( \frac{CIbd}{W} \right)$	$M^0L^0T^0$
Contact pressure	$P_c$	kPa	$kN/m^2 = \frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2}$	$M/L/T^2$
Speed	V	m/s	$\frac{\text{Length}}{\text{Time}} = \frac{L}{T}$	L/T
Width of plough	a	m	-	L
Depth of plough	b	m	-	L

## 3 Model development

In evaluating the traction requirements of a driven wheel tractor, the soil-wheel interactions were considered. The mathematical tool employed in this work was dimensional analysis.

### 3.1 Theoretical background of dimensional analysis

Dimensional analysis is a mathematical tool used for balancing the fundamental dimensions, mass, length and time (Banga et al., 1991). Buckingham's theory is based on the knowledge that if there are n basic dimensions and m variables, then there are n-m dimensionless parameters. Basic dimensions are mass (M), length (L) and time (T)

$$S_n = n - b \quad (1)$$

where,  $S_n$  = the number of Pi terms, n = the total number of variables, b = the number of basic dimensions.

The pertinent variables that affect the traction performance of the wheel tractor in loamy sand soil during tillage are twelve variables, which are presented in Table 1 with their corresponding dimensions. Among which are three dimensionless parameters: moisture content  $\mu$ , wheel slip S and wheel numeric  $C_n$ . Thus, traction force F, may be expressed as a function of the other seven variables

$$F = f(F_p, F_R, CI, P_c, V, a, b) \quad (2)$$

This equation may be re-written as

$$f(F, F_p, F_R, CI, P_c, V, a, b) = 0 \quad (3)$$

Therefore the number of variables are,  $n = 8$ , but since the basic dimensions are

MLT, = 3. Then, number of dimensionless  $\pi$  - terms, will be

$$\begin{aligned} S_n &= n - b \\ S_n &= 9 - 3 = 5 \end{aligned}$$

Hence, five  $\rho_i$  - terms say,  $\pi_1, \pi_2, \pi_3, \pi_4$ , and  $\pi_5$  will be generated.

$$\text{Thus, } F = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) \quad (4)$$

where  $f$  = functional notation for traction force

$\therefore$  The Equation (4) may be written as  $f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0$

$$(5)$$

### 3.2 Determination of $\pi$ -Terms

From Equation (5) above, five  $\pi$ -terms ( $\pi_1, \pi_2, \pi_3, \pi_4$  and  $\pi_5$ ) are required to develop a general prediction equation for traction force in wheel-tractor tillage operation (ploughing), on loamy sand soil.

The repeating variables involved are cone index (CI), tractor forward speed (V), and width of plough (a). According to Tarham and Carman (2004) the repeating variables themselves should not form a dimensionless term. They must contain jointly all fundamental dimensions equal to “b” number of basic dimensions. And also no dimensionless group is formed by them (Barenblat, 1987).

$$CI = M/LT^2, V = L/T^1, a = L$$

Each  $\pi$  - term, is  $b + 1$  variables and is written as

$$\pi_1 = CI^{a^1} \cdot V^{b^1} \cdot a^{c^1} \cdot F \quad (6a)$$

$$\pi_2 = CI^{a^2} \cdot V^{b^2} \cdot a^{c^2} \cdot F_p \quad (6b)$$

$$\pi_3 = CI^{a^3} \cdot V^{b^3} \cdot a^{c^3} \cdot F_R \quad (6c)$$

$$\pi_4 = CI^{a^4} \cdot V^{b^4} \cdot a^{c^4} \cdot P_c \quad (6d)$$

$$\pi_5 = CI^{a^5} \cdot V^{b^5} \cdot a^{c^5} \cdot b \quad (6e)$$

Wheel slip (s), wheel numeric  $C_n$  and moisture content are already dimensionless hence excluded from

the dimensionless terms determination exercise, but to be added when the other dimensionless terms had been formed.

After transformation some existing  $\pi$  -terms were eliminated, then dimensionless pi terms obtained are as follows:

$$\pi_1 = \frac{F}{F_p}, \quad \pi_2 = \frac{F_R P_c b C_n S}{CI^2 a^3}, \quad \pi_3 = \mu S$$

The developed tractive force models for ploughing at different tractor forward speeds after substituted coefficients are:

For ploughing at 1.94 m/s

$$F = 4489.72 + 0.001457 F_p \frac{F_R P_c b C_n S}{CI^2 a^3} + 3.145 F_p \mu S + 2.334 F_p, \quad (7)$$

For ploughing at 2.22 m/s

$$F = 3322.1 + 2.309 F_p \frac{F_R P_c b C_n S}{CI^2 a^3} + 0.000543 F_p \mu S + 1.15259 F_p, \quad (8)$$

For ploughing at 2.5 m/s

$$F = 1151.419 + 0.000297 F_p \frac{F_R P_c b C_n S}{CI^2 a^3} + 0.8405 F_p \mu S + 2.433855 F_p, \quad (9)$$

The concept of sensitivity measured parameters, as applied generally according to the theories of Mc Cuen, (1973) as reported by Simonyan (2006), is defined as the rate of change in the output of the model with respect to the change in the value of the parameter while keeping other parameters constant. Sensitivity analysis was done by mathematically differentiating the developed models (7, 8 and 9) using error equation whereby Equation (10) was obtained as below.

$$N \xi = \left( \frac{\partial N}{\partial u_1} \cdot \frac{U_1}{N} \right) U_1 \xi \quad (10)$$

The bracketed terms become dimensionless coefficient. This is the sensitivity coefficient which shows the relative importance of each of the variables to



the models solutions. Calculation of sensitivity equation (also called error equation) is in Appendix A.

Analyses of variances were conducted to find out if there are significant differences between tractive forces and tractor forward speeds during ploughing operations.

#### 4 Results and discussion

**Table 2 Analysis of variance for combined measured forces on various ploughing forward speeds of 1.94m/s, 2.22m/s and 2.5m/s**

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	45335126	2	22667563	30.58463	9.46E-10	3.158843
Within Groups	42245105	57	741142.2			
Total	87580231	59				

$F_{cal} > F_{crit}$ :  $P < 0.05$

Table 2 shows existence of the relationship between the measured forces and various tractor forward speeds of 1.94 m/s, 2.22 m/s and 2.5 m/s during ploughing operations. There is highly significant difference ( $p \leq 0.05$ ) between the measured forces at different tractor forward speeds. That is an indication that different tillage speeds will produce different forces, thus force is the dependant of tractor forward speed.

**Table 3 Analysis of variance between tractive force and ploughing forward speed of 1.94 m/s**

Source of Variation	df	SS	MS	F	P-value	Significance F
Regression	1	0.080247	0.080247	25.31789	1.53E-05	8.67E-05 NS
Residual	18	0.057052	0.00317			
Total	19	0.137299				

$F_{cal} > F_{crit}$ :  $P < 0.05$

$R^2 = 0.5844$

**Table 4 Analysis of variance between tractive force and ploughing forward speed of 2.22 m/s**

Source of Variation	df	SS	MS	F	P-value	Significance F
Regression	1	0.081949	0.081949	24.60286	2.03E-05	0.000101 NS
Residual	18	0.059956	0.003331			
Total	19	0.141905				

$F_{cal} > F_{crit}$ :  $P < 0.05$

$R^2 = 0.5774$

**Table 5 Analysis of variance between tractive force and ploughing forward speed of 2.25 m/s**

Source of Variation	df	SS	MS	F	P-value	Significance F
Regression	1	0.087313	0.087313	24.43939	2.38E-05	0.000105 NS
Residual	18	0.064308	0.003573			
Total	19	0.151621				

$F_{cal} > F_{crit}$ :  $P < 0.05$

$R^2 = 0.575866$

These tables, shows the statistical significance comparison on forces at different tractor forward speeds of 1.94 m/s, 2.22 m/s, 1.94 m/s and 2.5 m/s respectively. Tables 3-5, depicted the regression of forces on the various tractor forward speeds during ploughing operations. The analysis of variance between tractive forces and their

respective three forward speeds indicated no significant differences ( $p < 0.05$ ) between them. However, coefficient of determinations of  $R^2 = 0.5845$  for ploughing at 1.94 m/s  $R^2 = 0.5774$ , ploughing at 2.22 m/s and  $R^2 = 0.5758$ , ploughing at 2.5 m/s became clear that the best forward speed for ploughing operation is 1.94 m/s

having the highest value of coefficient of determination  $R^2=0.5845$ .

The sensitivity coefficients of the field measured parameters to the wheel tractor tractive force model were determined. Tables 6-8, depicted the predicted tractive forces (F(N) Pred), independent variables and their respective sensitivity coefficients. The sensitivity coefficient showed their rate of changes in the predicted models with respect to changes in the parameters while

keeping other parameters constant. The Tables also shows the means values of sensitivity coefficients for each of the independent variables for ploughing at various tractor forward speeds. The results so far obtained, depicted consistence low sensitivity coefficient values in the three tractor forward speeds. This indicates that the tractive force models developed fitted experimental data. The differences in the mean values of sensitivity coefficient are though not significant

**Table 6 Sensitivity coefficients of independent variables to the rate of change to tractive force as other variables remains constant at 1.94 m/s**

F(N) Pred	Sent. Coef of Fp	Sent. Coef of FR	Sent. Coef of CI	Sent. Coef of Pc	Sent Coef of a	Sent. Coef of b	Sent. Coef of Cn	Sent. Coef of S	Sent. Coef of $\mu$
13743.47	0.8586	0.0344	-0.0171	0.0344	-0.011457	0.0344	0.0344	19.4152	0.0309
13106.02	0.8017	0.0171	-0.0085	0.0171	-0.005689	0.0171	0.0171	8.7276	0.0336
14136.72	0.8706	0.0372	-0.0186	0.0372	-0.012414	0.0372	0.0372	22.3943	0.0301
12750.28	0.8319	0.0227	-0.0113	0.0227	-0.007561	0.0227	0.0227	10.3629	0.039
14213.75	0.8607	0.0369	-0.0185	0.0370	-0.012329	0.037	0.037	23.5434	0.0269
12792.83	0.9747	0.0367	-0.0183	0.0366	-0.012216	0.0366	0.0366	13.1885	0.0635
14061.01	0.7204	0.0017	-0.0009	0.0017	-0.000572	0.0017	0.0017	2.1328	0.0141
14003.4	0.7088	0.0013	-0.0007	0.0013	-0.000434	0.0013	0.0013	1.8138	0.0105
12946.46	0.8517	0.0282	-0.0141	0.0282	-0.009399	0.0282	0.0282	13.2605	0.0384
13119.99	0.8579	0.0292	-0.0146	0.0292	-0.009729	0.0292	0.0292	13.9309	0.0385
13860.25	0.7519	0.0047	-0.0024	0.0047	-0.001581	0.0047	0.0047	3.4931	0.0243
13967.12	0.7295	0.0036	-0.0018	0.0036	-0.001191	0.0036	0.0036	3.8900	0.0164
13956.31	0.7318	0.0039	-0.0020	0.0039	-0.001325	0.004	0.004	4.1839	0.017
14111.42	0.6975	0.0008	-0.0004	0.0008	-0.000273	0.0008	0.0008	1.1927	0.0056
15468.23	0.7149	0.0008	-0.0004	0.0008	-0.000279	0.0008	0.0008	1.3536	0.0014
15371.73	0.7144	0.0008	-0.0004	0.0008	-0.000261	0.0008	0.0008	1.269	0.002
15304.42	0.7124	0.0007	-0.0004	0.0007	-0.000236	0.0007	0.0007	1.1631	0.0018
14073.09	0.6982	0.0008	-0.0004	0.0008	-0.000274	0.0008	0.0008	1.1675	0.0062
15392.27	0.7155	0.0008	-0.0004	0.0007	-0.000254	0.0008	0.0008	1.2751	0.0024
14072.42	0.712	0.0012	-0.0006	0.0012	-0.0004	0.0012	0.0012	1.6109	0.0113
<b>Mean</b>									
<b>14022.56</b>	<b>0.7758</b>	<b>0.0132</b>	<b>-0.0066</b>	<b>0.0132</b>	<b>-0.0044</b>	<b>0.0132</b>	<b>0.0132</b>	<b>2.3539</b>	<b>0.0207</b>

**Table 7 Sensitivity coefficients of variables to the rate of change to tractive force as other variables remains constant at 2.22 m/s**

F(N) Pred	Sent. Coef of Fp	Sent. Coef of FR	Sent. Coef of CI	Sent. Coef of Pc	Sent. Coef of a	Sent. Coef of b	Sent. Coef of Cn	Sent. Coef of S	Sent. Coef of $\mu$
14108.88	0.7951	0.00039	-0.0002	0.000394	-0.000131	0.0004	0.0004	22.54808	0.0143
13743.57	0.7914	0.00023	-0.00012	0.00023	-7.67E-05	0.0002	0.0002	12.18837	0.0155
14485.09	0.7999	0.00035	-0.00017	0.000349	-0.000116	0.0003	0.0003	21.20892	0.0139
13342.23	0.7912	0.00042	-0.00021	0.000425	-0.000142	0.0004	0.0004	20.14012	0.0182
14630.05	0.7988	0.00032	-0.00016	0.000316	-0.000105	0.0003	0.0003	20.43567	0.0124
13012.1	0.8155	0.00083	-0.00042	0.000832	-0.000277	0.0008	0.0008	30.25029	0.0308
14994.18	0.7907	2.6E-05	-1.3E-05	2.59E-05	-8.65E-06	3E-05	3E-05	3.413255	0.0061
15074.57	0.7886	1.8E-05	-9.2E-06	1.85E-05	-6.16E-06	2E-05	2E-05	2.751812	0.0045
13442.39	0.7933	0.00049	-0.00025	0.000491	-0.000164	0.0005	0.0005	23.30543	0.0183
13587.83	0.7951	0.00046	-0.00023	0.00046	-0.000153	0.0005	0.0005	22.37387	0.0181
14707.02	0.796	8.8E-05	-4.4E-05	8.75E-05	-2.92E-05	9E-05	9E-05	6.786779	0.0107
14882.84	0.7913	5.8E-05	-2.9E-05	5.78E-05	-1.93E-05	6E-05	6E-05	6.63769	0.0072
14863.43	0.7915	6.1E-05	-3.1E-05	6.12E-05	-2.04E-05	6E-05	6E-05	6.799479	0.0074
15142.26	0.7853	6.4E-06	-3.2E-06	6.43E-06	-2.14E-06	6E-06	6E-06	0.998688	0.0024
16512.64	0.7999	5.5E-06	-2.7E-06	5.48E-06	-1.83E-06	5E-06	5E-06	0.93724	0.0006
16412.97	0.7992	5.2E-06	-2.6E-06	5.24E-06	-1.75E-06	5E-06	5E-06	0.900777	0.0009
16349.72	0.7982	6.3E-06	-3.1E-06	6.25E-06	-2.08E-06	6E-06	6E-06	1.087907	0.0008
15100.00	0.7853	1.1E-05	-5.5E-06	1.1E-05	-3.68E-06	1E-05	1E-05	1.670244	0.0027
16430.84	0.7997	6.5E-06	-3.2E-06	6.47E-06	-2.16E-06	6E-06	6E-06	1.143585	0.001
15056.59	0.7891	1.8E-05	-8.9E-06	1.78E-05	-5.95E-06	2E-05	2E-05	2.541234	0.0049
<b>Mean</b> <b>14793.96</b>	<b>0.7948</b>	<b>0.0002</b>	<b>-9.5E-05</b>	<b>0.0099</b>	<b>-6.35E-05</b>	<b>0.0002</b>	<b>0.0002</b>	<b>1.0877</b>	<b>0.0095</b>

**Table 8 Sensitivity coefficients of independent variables to the rate of change to tractive force as other variables remains constant at 2.5 m/s**

F(N) Pred	Sent. Coef of Fp	Sent. Coef of FR	Sent. Coef of CI	Sent. Coef of Pc	Sent. Coef of a	Sent. Coef of b	Sent. Coef of Cn	Sent. Coef of S	Sent. Coef of $\mu$
15062.62	1.0322	2.7E-12	-0.00013	0.000267	-2.14E-07	0.0003	0.0003	7.5065	0.0027
14670.09	1.0533	1.6E-12	-8.1E-05	0.000161	-7.04E-08	0.0002	0.0002	7.4650	0.003
15456.67	1.0323	2.3E-12	-0.00011	0.000228	-1.61E-07	0.0002	0.0002	7.6594	0.0028
14205.21	1.0485	2.8E-12	-0.00014	0.000276	-3.01E-07	0.0003	0.0003	7.9934	0.0033
15601.67	1.0337	2.2E-12	-0.00011	0.000223	-1.4E-07	0.0002	0.0002	7.2638	0.0023
13880.36	1.0418	5.8E-12	-0.00029	0.000577	-9.64E-07	0.0006	0.0006	10.5036	0.006
16001.4	1.0643	1.9E-13	-9.3E-06	1.85E-05	-1.59E-08	2E-05	2E-05	7.1242	0.0012
16084.83	1.0721	1.3E-13	-6.3E-06	1.25E-05	-1.08E-08	1E-05	1E-05	5.9699	0.0008
14336.7	1.0437	3E-12	-0.00015	0.000295	-2.76E-07	0.0003	0.0003	8.0554	0.0034
14494.44	1.0424	3.1E-12	-0.00015	0.000309	-2.64E-07	0.0003	0.0003	8.1394	0.0034
15685.06	1.0626	1.1E-12	-5.6E-05	0.000113	-1.06E-07	0.0001	0.0001	7.4645	0.002
15877.22	1.0636	3.9E-13	-1.9E-05	3.85E-05	-3.2E-08	4E-05	4E-05	7.3447	0.0013
15856.18	1.0632	4.2E-13	-2.1E-05	4.18E-05	-3.38E-08	4E-05	4E-05	7.3550	0.0014
16161.59	1.0667	2.9E-13	-1.5E-05	2.95E-05	-2.51E-08	3E-05	3E-05	3.3133	0.0004
17609.81	1.0647	3.1E-14	-1.9E-06	3.79E-06	-2.75E-09	4E-06	4E-06	0.8764	0.0001
17504.03	1.0648	3E-14	-1.8E-06	3.62E-06	-2.56E-09	4E-06	4E-06	1.2712	0.0002
17437.59	1.0651	1.3E-14	-7.6E-07	1.51E-06	-1.16E-09	2E-06	2E-06	1.1331	0.0001
16116.01	1.0667	5.1E-14	-2.5E-06	5.08E-06	-3.85E-09	5E-06	5E-06	3.5465	0.0005
17522.46	1.0648	3.7E-14	-2.2E-06	4.4E-06	-3.08E-09	4E-06	4E-06	1.5340	0.0002
<b>Mean</b> <b>16065.29</b>	<b>1.0556</b>	<b>1.3E-12</b>	<b>-6.5E-05</b>	<b>0.000131</b>	<b>-1.32E-07</b>	<b>0.0001</b>	<b>0.0001</b>	<b>5.8831</b>	<b>0.0018</b>

The results obtained in this disc plough tillage operations, indicated sensitivity coefficients within the range of -0.004394 to 2.353, -6.35E-05 to 1.0877 and -1.32E-07 to 1.00 at forward speeds of 1.94 m/s, 2.22 m/s and 2.5 m/s respectively. It becomes clear that changes

in independent variables ( $U_i$ ) caused negligible changes in predicted tractive forces (F(N) Pred). These indicate that the sensitivity of developed predicted model to changes in the constituent independent variables is negligible and insignificant.

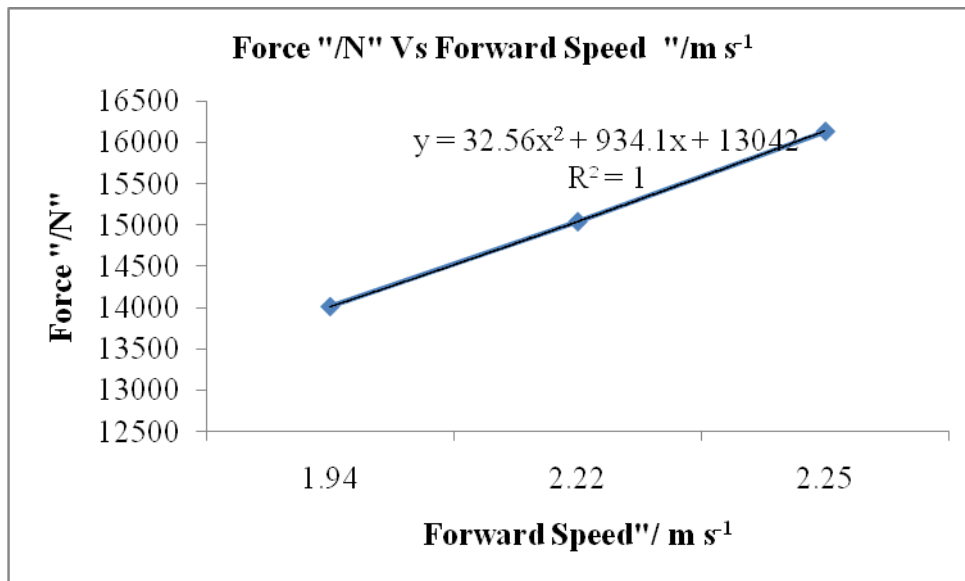


Figure 4 A plot of relationship between mean sensitivity of forces and tractor forward speeds during ploughing operations

**Conclusions**

In conclusion, the sensitivity coefficients of the predicted tractive forces models parameters for disc ploughing at forward speeds of 1.94 m/s, 2.2 m/s and 2.5 m/s showed negligible insignificant changes. Deducing from the insignificant values of sensitivity coefficients, is an indication that the best forward speed for disc ploughing is 1.94 m/s in loamy sand soil. From the discussion above it is conclusive that the tractive force models developed in this work are reasonably valid to characterise the tillage tractive efficiency of disc ploughing operations under various tractor forward speeds.

**Appendix A**

Below is the detailed calculation of sensitivity equation (also called error equation) is developed for a function using

$$N = f(U_1, U_2 \dots U_n) \tag{10}$$

where

N = dependent variable

$(U_1, U_2, \dots, U_n)$  = Independent variable

Using Taylor's Theorem

We differentiate the independent function

$$N + \Delta N = f(U_1 + \Delta U_1, U_2 + \Delta U_2, \dots, U_n + \Delta U_n) \tag{11}$$

Taylor's theorem says

$$f(a) = f(a) + hf^1(a)$$

$$f(N) = f(U_1, U_2 \dots U_n) + hf^1(U_1, U_2 \dots U_n) \tag{12}$$

Using function of functions method equation (13) is obtained.

$$\Delta N = \frac{\partial N}{\partial u_1} \Delta U_1 + \frac{\partial N}{\partial u_2} \Delta U_2 + \frac{\partial N}{\partial u_n} \Delta U_n \tag{13}$$

$$\Delta N = \sum_{i=1}^n \Delta Ni$$

Then applying Taylor's theorem and neglecting squares, products, higher powers and also not considering other variables when discussing one variable causes other variables to be zero, hence,

$$\Delta N = \frac{\partial N}{\partial u_1} \Delta U_1 + \frac{\partial N}{\partial u_2} \Delta U_2 \tag{14}$$

Relative changes or error was defined as follows

$$N \xi = \frac{\Delta N}{N} \tag{15}$$

$$u \xi = \frac{\Delta u}{U} \tag{16}$$

Putting Equation (14) into Equation (15) we obtained equation below.

$$N\xi = \frac{\partial N}{\partial u_1} \cdot \frac{\Delta U_1}{N} + \frac{\partial N}{\partial U_2} \cdot \frac{\Delta U_2}{N} \quad (17)$$

$$N \xi = \left( \frac{\partial N}{\partial u_1} \cdot \frac{U_1}{N} \right) U_1 \xi + \left( \frac{\partial N}{\partial u_2} \cdot \frac{U_2}{N} \right) U_2 \xi \quad (18)$$

This expresses relative changes of each independent variables  $U_n$ , with respect to the relative dependent variable. If the error or change that occurs in only one variable is considered, all the other terms would be zero i.e. only one variable causes others to become zero.

$$\text{Therefore, } N \xi = \left( \frac{\partial N}{\partial u_1} \cdot \frac{U_1}{N} \right) U_1 \xi \quad (19)$$

The bracketed terms become dimensionless coefficient, which expresses the percentage of the relative change of each independent variable, transmitted to the relative dependent variable. This is the sensitivity coefficient which shows the relative importance of each of the variables to the models solutions.

### Appendix B

The developed and the calculated sensitivity equations for the tractive force developed models are shown as:

For ploughing at 1.94 m/s

$$F = 4489.715 + 0.00145708 F_p \frac{F_R Pcb Cns}{CI^2 a^3}$$

$$+ 3.145007 F_p \mu_s + 2.3343 F_p$$

$$(1) \frac{dF}{dF_p} = 0.00145708 \frac{F_R Pcb Cns}{CI^2 a^3}$$

$$+ 3.145007 \mu_s + 2.3343$$

$$(2) \frac{dF}{dF_R} = 0.00145708 F_p \frac{Pcb Cns}{CI^2 a^3}$$

$$(3) \frac{dF}{dCI} = -0.00145708 F_p \frac{F_R Pcb Cns}{2CI^3 a^3}$$

$$(4) \frac{dF}{dP_c} = 0.00145708 F_p \frac{F_R b Cns}{CI^2 a^3}$$

$$(5) \frac{dF}{da} = -0.00145708 F_p \frac{F_R Pcb Cns}{CI^2 3a^4}$$

$$(6) \frac{dF}{db} = 0.00145708 F_p \frac{F_R Pc Cns}{CI^2 a^3}$$

$$(7) \frac{dF}{dCn} = 0.00145708 F_p \frac{F_R Pcb S}{CI^2 a^3}$$

(8)

$$\frac{dF}{ds} = 0.00145708 F_p \frac{F_R Pcb Cn}{CI^2 a^3} + 3.145007 F_{p\mu}$$

$$(9) \frac{dF}{d\mu} = 3.145007 F_p S$$

### Appendix C

For ploughing at 2.2 m/s

$$F = 3322.1 + 0.000543 F_p \frac{F_R Pcb Cns}{CI^2 a^3}$$

$$+ 1.152594 F_p \mu_s + 2.309034 F_p$$

$$(1) \frac{dF}{dF_p} = 0.000543 \frac{F_R Pcb Cns}{CI^2 a^3} + 1.152594 \mu_s$$

$$+ 2.309034$$

$$(2) \frac{dF}{dF_R} = 0.000543 \frac{F_p Pcb Cns}{CI^2 a^3}$$

$$(3) \frac{dF}{dCI} = -0.000543 F_p \frac{F_R Pcb Cns}{2CI^3 a^3}$$

$$(4) \frac{dF}{dP_c} = 0.000543 F_p \frac{F_R b Cns}{CI^2 a^3}$$

$$(5) \frac{dF}{da} = -0.000543 F_p \frac{F_R Pcb Cns}{CI^2 3a^4}$$

$$(6) \frac{dF}{db} = 0.000543 F_p \frac{F_R Pc Cns}{CI^2 a^3}$$

$$(7) \frac{dF}{dCn} = 0.000543 F_p \frac{F_R Pcb S}{CI^2 a^3}$$

(8)

$$\frac{dF}{ds} = 0.000543 F_p \frac{F_R Pcb Cn}{CI^2 a^3} + 1.152594 F_{p\mu}$$

$$(9) \quad \frac{dF}{d\mu} = 1.152594F_p S$$

$$(4) \quad \frac{dF}{dP_c} = 0.000297F_p \frac{F_R b CnS}{CI^2 a^3}$$

$$(5) \quad \frac{dF}{da} = -0.000297F_p \frac{F_R Pcb CnS}{CI^2 3a^4}$$

**Appendix D**

For ploughing at 2.5 m/s

$$F = 1151.42 + 0.000297F_p \frac{F_R Pcb CnS}{CI^2 a^3} + 0.841F_p \mu S + 2.434F_p$$

$$(6) \quad \frac{dF}{db} = 0.000297F_p \frac{F_R Pc CnS}{CI^2 a^3}$$

$$(7) \quad \frac{dF}{dCn} = 0.000297F_p \frac{F_R Pcb S}{CI^2 a^3}$$

(1)

$$\frac{dF}{dF_p} = 0.000297 \frac{F_R Pcb CnS}{CI^2 a^3} + 0.841\mu S + 2.434$$

(8)

$$\frac{dF}{ds} = 0.000297F_p \frac{F_R Pcb Cn}{CI^2 a^3} + 0.841 F_{p\mu}$$

$$(2) \quad \frac{dF}{dF_R} = 0.000297F_p \frac{Pcb CnS}{CI^2 a^3}$$

$$(9) \quad \frac{dF}{d\mu} = 0.184F_p S$$

$$(3) \quad \frac{dF}{dCI} = -0.000297F_p \frac{F_R Pcb CnS}{2CI^3 a^3}$$

**Appendix E**

**Table 9** shows the measured parameters for Ploughing at tractor forward speed of 1.94 m/s

F(N)	Fp(N)	FR(N)	Pc, N	b(m)	Cn	S	CI(N)	a(m)	μ
13505.49	3580	9925	9092.72	0.158	0.07322	0.2433	320	0.487	0.155
13331.7	3406.66	9925	8762.72	0.169	0.137288	0.2563	600	0.502	0.1604
13650	3725	9925	9375.34	0.146	0.080153	0.2351	350.3	0.464	0.1544
13101.7	3201.66	9900	8713.85	0.195	0.062535	0.2791	273.3	0.576	0.1771
13696.7	3776.66	9920	9782.72	0.136	0.08656	0.2233	378.3	0.451	0.1442
12908.3	3008.33	9900	8133.18	0.214	0.041942	0.3555	183.3	0.59	0.2414
13910	4005	9905	13275.00	0.176	0.106387	0.1131	464.95	0.792	0.1392
13950	4005	9900	13834.48	0.174	0.110963	0.1005	484.95	0.816	0.1158
13153.3	3253.33	9900	9070.40	0.174	0.066722	0.2753	291.6	0.535	0.1766
13226.7	3316.66	9910	8634.55	0.178	0.072065	0.2749	314.95	0.521	0.1762
13761.7	3841.66	9920	10667.68	0.198	0.093425	0.1882	408.3	0.716	0.1483
13855.3	3940.33	9915	10991.13	0.186	0.098001	0.1283	428.3	0.693	0.1444
13845	3930	9915	11066.57	0.181	0.098607	0.1326	430.95	0.679	0.1448
14033.3	4083.33	9950	20534.77	0.128	0.121637	0.0968	531.6	0.891	0.0631
15603.3	4688.33	10915	20440.94	0.127	0.141864	0.0957	620	0.88	0.0153
15553.3	4643.33	10910	22125.00	0.118	0.146441	0.0949	640	0.885	0.0224
15516.7	4616.66	10900	19732.22	0.135	0.146441	0.0932	640	0.903	0.0201
14050	4063.33	9950	15151.48	0.169	0.137288	0.099	600	0.868	0.0686
15600	4650	10950	20790.48	0.126	0.144519	0.092	631.6	0.888	0.027
13930	4030	9900	13537.22	0.18	0.119738	0.1048	523.3	0.826	0.1194

**Table 10** shows the measured parameters for Ploughing at tractor forward speed of 2.2m/s

F(N)	Fp(N)	FR(N)	Pc (N)	b(m)	Cn	S	CI(N)	a(m)	$\mu$
14561.66	4581.66	9975	8938.855	0.166	0.07322	0.2468	320	0.503	0.155
14395	4420	9975	8503.911	0.179	0.137288	0.2599	600	0.516	0.1604
14705	4745	9960	9290.584	0.154	0.080153	0.2387	350.3	0.485	0.1544
14141.66	4231.66	9920	8406.068	0.206	0.062535	0.2818	273.3	0.587	0.1771
14736.66	4816.66	9920	9572.028	0.143	0.08656	0.2267	378.3	0.464	0.1442
13928.33	4018.33	9910	7941.295	0.224	0.041942	0.3584	183.3	0.603	0.2414
14930	5015	9915	12665.76	0.184	0.106387	0.1142	464.95	0.79	0.1392
14985	5060	9925	13258.97	0.184	0.110963	0.1014	484.95	0.827	0.1158
14183.33	4273.33	9910	8479.235	0.183	0.066722	0.2833	291.6	0.526	0.1766
14266.66	4336.66	9920	8345.187	0.187	0.072065	0.2793	314.95	0.529	0.1762
14776.66	4861.66	9915	10282.45	0.208	0.093425	0.19	408.3	0.725	0.1483
14880	4960	9920	10604.87	0.195	0.098001	0.1297	428.3	0.701	0.1444
14870	4950	9920	10601.56	0.192	0.098607	0.134	430.95	0.69	0.1448
15058.33	5103.33	9955	19478.68	0.136	0.121637	0.0978	531.6	0.898	0.0631
16628.33	5708.33	10920	20813.89	0.126	0.141864	0.0966	620	0.889	0.0153
16578.33	5663.33	10915	20650	0.128	0.146441	0.0955	640	0.896	0.0224
16566.66	5636.66	10930	18427.4	0.146	0.146441	0.094	640	0.912	0.0201
15043.33	5083.33	9960	14469.83	0.179	0.137288	0.0999	600	0.878	0.0686
16625	5670	10955	18958.09	0.136	0.144519	0.093	631.6	0.874	0.027
14960	5050	9910	13118.09	0.188	0.119738	0.1059	523.3	0.836	0.1194

**Table 11** shows the measured parameters for Ploughing at tractor forward speed of 2.5m/s

F(N)	Fp(N)	FR(N)	Pc(N)	b(m)	Cn	S	CI(N)	a(m)	$\mu$
15661.66	5,637	10025	8598.58	0.176	0.07322	0.2541	320	0.513	0.155
15495	5,470	10025	8194.444	0.189	0.137288	0.2742	600	0.525	0.1604
15805	5,795	10010	9360.577	0.156	0.080153	0.2614	350.3	0.495	0.1544
15241.66	5,272	9970	8177.674	0.215	0.062535	0.2774	273.3	0.596	0.1771
15836.66	5,867	9970	9002.258	0.155	0.08656	0.234	378.3	0.473	0.1442
15028.33	5,068.33	9960	7727.991	0.234	0.041942	0.3738	183.3	0.613	0.2414
16030	6,065.00	9965	12149.74	0.194	0.106387	0.1232	464.95	0.799	0.1392
16085	6,110	9975	12662.31	0.195	0.110963	0.1033	484.95	0.837	0.1158
15283.33	5,323.33	9960	8461.842	0.19	0.066722	0.2834	291.6	0.545	0.1766
15356.66	5,386.66	9970	8071.32	0.197	0.072065	0.2849	314.95	0.539	0.1762
15876.66	5,911.66	9965	6859.422	0.329	0.093425	0.1938	408.3	0.765	0.1483
15980	6,010	9970	10231.46	0.205	0.098001	0.1327	428.3	0.711	0.1444
15970	6,000	9970	10164.36	0.202	0.098607	0.137	430.95	0.696	0.1448
16158.33	6,153.33	10005	7176.747	0.372	0.121637	0.0995	531.6	0.905	0.0631
17728.33	6,758.33	10970	19500.37	0.136	0.141864	0.0983	620	0.899	0.0153
17678.33	6,713.33	10965	19367.39	0.138	0.146441	0.0973	640	0.906	0.0224
17666.66	6,686.66	10980	27505.24	0.105	0.146441	0.0957	640	0.979	0.0201
16143.33	6,133.33	10010	16368.9	0.164	0.137288	0.1017	600	0.91	0.0686
17675	6,720	10955	17881.85	0.146	0.144519	0.0949	631.6	0.885	0.027
16060	6,100	9960	12604.55	0.198	0.119738	0.1082	523.3	0.846	0.1194

**Table 12** shows the predicted tractive forces in ploughing operations at forward speeds of **1.94 m/s, 2.2 m/s, and 2.5 m/s**

Predicted tractive force at 1.94m/s	Predicted tractive force at 2.2m/s	Predicted tractive force at 2.5m/s
13383.14	14154.38	15092.78
12938.71	13766.70	14685.96
13723.59	14530.21	15488.57
12557.01	13382.60	14231.40
13794.96	14674.16	15630.61
12487.20	13079.76	13927.28
14052.02	15000.97	16006.13
13997.31	15079.92	16088.34
12685.90	13490.29	14367.25
12840.99	13631.55	14523.25
13828.21	14721.37	15692.87
13941.18	14893.09	15883.97
13926.41	14874.28	15863.39
14109.04	15146.41	16163.94
15465.31	16517.06	17612.66
15369.12	16417.10	17506.70
15302.42	16353.60	17439.86
14070.24	15103.84	16118.45
15389.79	16435.12	17525.21
14067.06	15061.63	16068.61
<b>Mean</b>	<b>13896.48</b>	<b>14815.70</b>

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