

# Realization of the stiffness matrix of statically indeterminate trusses and 20 bar sensitivity penalty method using Mathcad

J.H. Lee

(Department of Infrastructure Civil Engineering, Chonbuk National University, JeonJu, South Korea)

**Abstract:** This paper examines the stiffness matrix of two-dimensional statically indeterminate truss structures. Using Mathcad, a stiffness matrix analysis of a truss problem is completed as a template for other sorts of problems. This allows learners to understand the stiffness matrix theory and rapidly acquire the basic concepts of analysis theory. As is shown in the example, the transformation of the matrix is apparent and the overall structure of the stiffness matrix can be easily constructed. Unknown reactions and member forces caused by unknown displacement were calculated as well. In addition, the problem of the 20 bar boundary condition sensitivity penalty method explained in the example 2 for structure analysis. Although this process is difficult to acquire due to complex procedures, it is beneficial for students because they can check for errors using Mathcad.

**Keywords:** truss, stiffness matrix, Mathcad, penalty method, structure analysis

**Citation:** J. H. Lee. 2012. The realization of the stiffness matrix of statically indeterminate trusses and 20 bar sensitivity penalty method using Mathcad. *Agric Eng Int: CIGR Journal*, 14(3): 14–23.

## 1 Introduction

Civil engineering students spend a considerable amount of time learning to understand structures. One way to analyze a structure is the stiffness matrix method. Some students have a hard time understanding the theoretical concept of the stiffness matrix method. Developing a structural analysis solution, even for a trivial frame problem, requires much time and effort. An educational structure analysis package using Mathcad was developed in order to analyze a truss structure using the stiffness matrix method. With this program, students can algebraically construct the stiffness matrix of a structure to solve a suggested problem. The algebraic capacity of Mathcad helps them solve a problem easily and conveniently without going through laborious computation. Using the programmed Mathcad file proposed in this article, students can produce their own

spreadsheet and solve a problem by making some changes. Many packages have been developed focusing on spreading abilities in students to manage the structural behavior in different kinds of structures by solving quickly a large number of problems with different load conditions or supports. However, students still have problems to understand the matrix structural analysis theory underneath the structural analysis packages they use.

Al-Ansari and Senouci (1999) demonstrated the use of Mathcad to supplement and enhance traditional teaching and learning methods for reinforced concrete design. The article, which focuses on the topic of foundation design, demonstrates a Mathcad program for the design of combined footings and shows its impact as a teaching and learning tool. Mtenga and Spainhour (2000) presented that in an example, a series of formulas are used to demonstrate the transparency of Mathcad, including the ability of using real engineering units in the calculations, formulas as they may appear in textbooks or in codes, and ability to hide and password protects some areas. Mathcad procedures, in the form of worksheets,

**Received date:** 2009-05-14 **Accepted date:** 2012-08-07

**Corresponding author:** J. H. Lee, Chonbuk National University, JeonJu, South Korea, Tel.: +82 63 270 4789. Email: [jinhyu@jbnu.ac.kr](mailto:jinhyu@jbnu.ac.kr).

are presented and discussed for problems associated with piping systems in Hodge and Taylor (2002). Bianco and Pezzo (2002) observed a slight discrepancy of the approximate model with the non-approximate numerical simulation at short distances (less than 10 km) and this mainly for  $E_1(r)$ , the energy integral relative to the early S-wave coda. They adapted this method to a Mathcad2000 professional worksheet. Using this worksheet, non-expert programmers can easily modify the formulas in order to use models different from the 3-D multiple scattering adopted in the present report. Yuan and Teng (2002) presented an innovative web-based package named CALSB for computer-aided learning of structural behavior. The package includes an analysis engine based on the matrix stiffness method, so the response of any two-dimensional skeletal structure can be predicted and graphically displayed. Almeida et al. (2003) described an educational software developed to be helpful for undergraduate engineering students when learning with structural analysis. The software enables the user to analyze plane-framed structures, under static or dynamic loading, and it was fully implemented by using web technology. An algorithm for modeling the strain and rotation of deformable ellipsoidal objects in viscous flows based on Eshelby's theory was presented and was implemented in a fully graphic mathematics application (Mathcad) in Jiang (2007). With the Mathcad worksheets presented in the supplementary data associated with this paper, one could model the strain and rotation paths of individual deformable objects and the development of preferred orientation and shape fabrics for a population of deformable objects in any homogeneous viscous flow. Cedeno-Rosete (2007) presented an educational program designed to understand in a simple form, the stiffness structural analysis theory of two-dimensional framed structures. The program is written in spreadsheet form using the software Mathcad. Charney (2008) demonstrated a "Transformational" approach to teaching the theory of matrix structural analysis, and a methodology for implementing the procedure in Mathcad. Consolazio (2008) explained that Mathcad possesses a number of unique features not generally available in more traditional programming environments as an educational programming tool.

Features such as integrated algorithm coding, commenting, and data visualization make Mathcad an ideal tool for presenting engineering concepts in the classroom. Yildirim and Singh (2010) presented a design procedure which provides accurate solutions for three types of pipe design problems (Types A–C) taking into consideration the effect of local losses. For the solution of the general case of a Type B problem (sloping pipe with pumping power), a user-friendly MathCAD procedure, which produces a consistent framework for analyzing and solving common piping-system applications problem, is also developed.

Analyzing the stiffness matrix structure is difficult for students. In order to solve structural problems, the basic theoretical concept was included so that it can be applied to trusses and frame structures. Many package programs have been developed to handle serious problems with different loading conditions and points. However, these programs focus on improving students' abilities to deal with structural problems. Students have to depend on the structure analysis package they are using to understand the matrix structure analysis theory. This paper described a Mathcad spreadsheet with the stiffness matrix method to solve secondary truss structure problems. It contained an example so that it could be used as a template for other problems. This spreadsheet should make it easier for students to understand stiffness matrix structure analysis theory.

## 2 Materials and methods

Programming languages such as Fortran, C, C++, Basic, and Java require a text-based source code file. They can be used more flexibly and can be more powerful than spreadsheets. However, they are error-prone, difficult to share and time consuming to learn. Since Mathcad uses actual mathematical signals, it is easy to identify data, methods, or hypotheses used in engineering calculations. Numerical formulas are displayed on the screen, and the process of calculation can be explained by adding text as needed. It is possible to use units in formulas, so it is easy to correct analysis on the units frequently used in engineering calculation. Since the calculation process and design variables can be

shared without any difficulty, it is easy to determine how a particular change in a part of a project affects the overall process. Because the process or method of analysis is visually expressed on the worksheet with ease, it is quite efficient to provide or present the data to others.

**2.1 Computational example 1**

Figure 1 explains how to figure out the displacement of node 4 and the statically indeterminate truss for calculating each member force. The sectional area  $A$  and the elastic modulus  $E$  of all the members are consistent. The local coordinate was set up as the nodal number is directed from the smaller to the larger number. The angles from the global coordinate system  $X$  to each local coordinate axis  $x$  in a clockwise direction were set as  $\theta_{14}$ ,  $\theta_{24}$ , and  $\theta_{34}$  respectively.

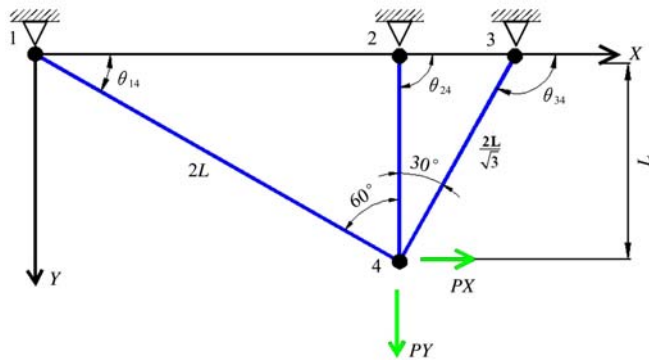


Figure 1 Example of a statically indeterminate truss

Figure 2 shows the calculating process using the stiffness matrix for the local coordinates of members 1-4. The coordinate transformation matrix is  $\cos \theta_{14} = \sqrt{3}/2$ ,  $\sin \theta_{14} = 1/2$  by  $u = T^*U$ , so matrix  $K_{14}$  can be calculated from  $T_{14}$  and the global coordinate system. In Mathcad, the  $K_{14}$  formula is formed with the signal  $:=$ , and the result can be obtained using the signal  $\rightarrow$ .

In a similar way, the stiffness matrix for the local coordinates of members 2-4 and the coordinate transformation matrix are  $\cos \theta_{24} = 0$ ,  $\sin \theta_{24} = 1$ , as is shown in Figure 3. Therefore, the stiffness matrix  $K_{24}$  of the global coordinate system is as follows.

Next, the stiffness matrix for the local coordinates of members 3-4 and the coordinate transformation matrix are shown in Figure 4 when  $\cos \theta_{34} = -1/2$ ,  $\sin \theta_{34} = \sqrt{3}/2$  is used. Therefore, the stiffness matrix of the global coordinate system is as follows.

$$\begin{pmatrix} x_1 \\ y_1 \\ x_4 \\ y_4 \end{pmatrix} := \frac{E \cdot A}{2L} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_4 \\ v_4 \end{pmatrix} := k_{14} \cdot u$$

$$k_{14} := \frac{E \cdot A}{2L} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{A \cdot E}{2L} & 0 & -\frac{A \cdot E}{2L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{A \cdot E}{2L} & 0 & \frac{A \cdot E}{2L} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_{14} := \begin{pmatrix} \cos \theta_{14} & \sin \theta_{14} & 0 & 0 \\ -\sin \theta_{14} & \cos \theta_{14} & 0 & 0 \\ 0 & 0 & \cos \theta_{14} & \sin \theta_{14} \\ 0 & 0 & -\sin \theta_{14} & \cos \theta_{14} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$K_{14} := T_{14}^T \cdot k_{14} \cdot T_{14}$$

$$K_{14} \rightarrow \begin{pmatrix} \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} & -\frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} & -\frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} \\ \frac{3 \cdot A \cdot E}{8 \cdot L} & -\frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ -\frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} & -\frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} \end{pmatrix}$$

Figure 2 Stiffness matrix to member 1-4 in global coordinate system

$$\begin{pmatrix} x_2 \\ y_2 \\ x_4 \\ y_4 \end{pmatrix} := \frac{E \cdot A}{L} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{pmatrix} := k_{24} \cdot u$$

$$k_{24} := \frac{E \cdot A}{L} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{A \cdot E}{L} & 0 & -\frac{A \cdot E}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{A \cdot E}{L} & 0 & \frac{A \cdot E}{L} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_{24} := \begin{pmatrix} \cos \theta_{24} & \sin \theta_{24} & 0 & 0 \\ -\sin \theta_{24} & \cos \theta_{24} & 0 & 0 \\ 0 & 0 & \cos \theta_{24} & \sin \theta_{24} \\ 0 & 0 & -\sin \theta_{24} & \cos \theta_{24} \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$K_{24} := T_{24}^T \cdot k_{24} \cdot T_{24}$$

$$K_{24} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{A \cdot E}{L} & 0 & \frac{A \cdot E}{L} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{A \cdot E}{L} & 0 & \frac{A \cdot E}{L} \end{pmatrix}$$

Figure 3 Stiffness matrix to member 2-4 in global coordinate system

In order to make the stiffness matrix of the global structure, instructions to augment or stack in Mathcad can be used as in Figure 5. When  $K_{14}$ ,  $K_{24}$ , and  $K_{34}$  are inserted in the proper locations, the result will be similar to Figure 6.

If the loading conditions,  $X_4=PX$  and  $Y_4=PY$ , and the boundary condition,  $U_1=V_1=U_2=V_2=U_3=V_3=0$ , in Figure 6 are considered, Figure 7 results. Next, calculate the unknown displacement of  $U_4$  and  $V_4$ . For this, submatrix instructions can be used in Mathcad.

$$\begin{pmatrix} x_3 \\ y_3 \\ x_4 \\ y_4 \end{pmatrix} := \frac{E \cdot A}{\begin{pmatrix} 2L \\ \sqrt{3} \end{pmatrix}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} := k_{34} \cdot u$$

$$k_{34} := \frac{E \cdot A}{\begin{pmatrix} 2L \\ \sqrt{3} \end{pmatrix}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{3} \cdot A \cdot E}{2L} & 0 & -\frac{\sqrt{3} \cdot A \cdot E}{2L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\sqrt{3} \cdot A \cdot E}{2L} & 0 & \frac{\sqrt{3} \cdot A \cdot E}{2L} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_{34} := \begin{pmatrix} \cos\theta_{34} & \sin\theta_{34} & 0 & 0 \\ -\sin\theta_{34} & -\cos\theta_{34} & 0 & 0 \\ 0 & 0 & \cos\theta_{34} & \sin\theta_{34} \\ 0 & 0 & -\sin\theta_{34} & -\cos\theta_{34} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$K_{34} := T_{34}^T \cdot k_{34} \cdot T_{34}$$

$$K_{34} \rightarrow \begin{pmatrix} \frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} & -\frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} \\ \frac{3 \cdot A \cdot E}{8L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} & \frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} \\ \frac{3 \cdot A \cdot E}{8L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} \end{pmatrix}$$

Figure 4 Stiffness matrix to member 3-4 in global coordinate system

$$KX_1 := \text{augment}[(K_{14})_{0,0}, (K_{14})_{0,1}, 0, 0, 0, 0, (K_{14})_{0,2}, (K_{14})_{0,3}]$$

$$KY_1 := \text{augment}[(K_{14})_{1,0}, (K_{14})_{1,1}, 0, 0, 0, 0, (K_{14})_{1,2}, (K_{14})_{1,3}]$$

$$KX_2 := \text{augment}[0, 0, (K_{24})_{0,0}, (K_{24})_{0,1}, 0, 0, (K_{24})_{0,2}, (K_{24})_{0,3}]$$

$$KY_2 := \text{augment}[0, 0, (K_{24})_{1,0}, (K_{24})_{1,1}, 0, 0, (K_{24})_{1,2}, (K_{24})_{1,3}]$$

$$KX_3 := \text{augment}[0, 0, 0, 0, (K_{34})_{0,0}, (K_{34})_{0,1}, (K_{34})_{0,2}, (K_{34})_{0,3}]$$

$$KY_3 := \text{augment}[0, 0, 0, 0, (K_{34})_{1,0}, (K_{34})_{1,1}, (K_{34})_{1,2}, (K_{34})_{1,3}]$$

$$KX_4 := \text{augment}[(K_{14})_{2,0}, (K_{14})_{2,1}, (K_{24})_{2,0}, (K_{24})_{2,1}, (K_{34})_{2,0}, (K_{34})_{2,1}, (K_{14})_{2,2} + (K_{24})_{2,2} + (K_{34})_{2,2}, (K_{14})_{2,3} + (K_{24})_{2,3} + (K_{34})_{2,3}]$$

$$KY_4 := \text{augment}[(K_{14})_{3,0}, (K_{14})_{3,1}, (K_{24})_{3,0}, (K_{24})_{3,1}, (K_{34})_{3,0}, (K_{34})_{3,1}, (K_{14})_{3,2} + (K_{24})_{3,2} + (K_{34})_{3,2}, (K_{14})_{3,3} + (K_{24})_{3,3} + (K_{34})_{3,3}]$$

$$GKXY := \text{stack}(KX_1, KY_1, KX_2, KY_2, KX_3, KY_3, KX_4, KY_4)$$

$$GKXY \rightarrow \begin{pmatrix} \frac{3 \cdot A \cdot E}{8L} & \frac{\sqrt{3} \cdot A \cdot E}{8L} & 0 & 0 & 0 & 0 & \frac{3 \cdot A \cdot E}{8L} & -\frac{\sqrt{3} \cdot A \cdot E}{8L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{A \cdot E}{8L} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{A \cdot E}{8L} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{A \cdot E}{L} & 0 & 0 & 0 & -\frac{A \cdot E}{L} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3} \cdot A \cdot E}{8L} & -\frac{3 \cdot A \cdot E}{8L} & -\frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} \\ 0 & 0 & 0 & 0 & -\frac{3 \cdot A \cdot E}{8L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} & -\frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} \\ \frac{3 \cdot A \cdot E}{8L} & -\frac{\sqrt{3} \cdot A \cdot E}{8L} & 0 & 0 & -\frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} & \frac{3 \cdot A \cdot E}{8L} + \frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{\sqrt{3} \cdot A \cdot E}{8L} - \frac{3 \cdot A \cdot E}{8L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8L} & \frac{A \cdot E}{8L} & 0 & -\frac{A \cdot E}{L} & \frac{3 \cdot A \cdot E}{8L} & -\frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} & \frac{\sqrt{3} \cdot A \cdot E}{8L} - \frac{3 \cdot A \cdot E}{8L} & \frac{9 \cdot A \cdot E}{8L} + \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8L} \end{pmatrix}$$

Figure 5 Matrix assembled using the augment and stack commands of Mathcad

$$\begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{pmatrix} = \begin{pmatrix} \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & 0 & 0 & 0 & 0 & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} & 0 & 0 & 0 & 0 & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{A \cdot E}{L} & 0 & 0 & 0 & \frac{A \cdot E}{L} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} \\ 0 & 0 & 0 & 0 & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & 0 & 0 & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} + \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} - \frac{3 \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} & 0 & \frac{A \cdot E}{L} & \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} - \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{9 \cdot A \cdot E}{8 \cdot L} + \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} \end{pmatrix} \begin{pmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \\ U_4 \\ V_4 \end{pmatrix}$$

Figure 6 Stiffness matrix in global coordinates

$$\text{submatrix}(\text{GKXY}, 6, 7, 6, 7) \rightarrow \begin{pmatrix} \frac{3 \cdot A \cdot E}{8 \cdot L} + \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} - \frac{3 \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} - \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{9 \cdot A \cdot E}{8 \cdot L} + \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} \end{pmatrix}$$

$$\begin{pmatrix} \text{PX} \\ \text{PY} \end{pmatrix} = \frac{E \cdot A}{L} \begin{pmatrix} \frac{3}{8} + \frac{\sqrt{3}}{8} & \frac{\sqrt{3}}{8} - \frac{3}{8} \\ \frac{\sqrt{3}}{8} - \frac{3}{8} & \frac{9}{8} + \frac{3 \cdot \sqrt{3}}{8} \end{pmatrix} \begin{pmatrix} U_4 \\ V_4 \end{pmatrix}$$

$$\begin{pmatrix} U_4 \\ V_4 \end{pmatrix} := \text{submatrix}(\text{GKXY}, 6, 7, 6, 7)^{-1} \cdot \begin{pmatrix} \text{PX} \\ \text{PY} \end{pmatrix}$$

$$\text{submatrix}(\text{GKXY}, 6, 7, 6, 7)^{-1} \rightarrow \begin{bmatrix} \frac{3 \cdot L + \sqrt{3} \cdot L}{A \cdot E + \sqrt{3} \cdot A \cdot E} & - \\ \frac{3 \cdot L - \sqrt{3} \cdot L}{3 \cdot A \cdot E + 3 \cdot \sqrt{3} \cdot A \cdot E} & \\ & \frac{3 \cdot L - 2 \cdot \sqrt{3} \cdot L}{3 \cdot A \cdot E} \\ & \frac{(\sqrt{3} + 3) \cdot (3 \cdot L - \sqrt{3} \cdot L)}{(3 \cdot A \cdot E + 3 \cdot \sqrt{3} \cdot A \cdot E) \cdot (\sqrt{3} - 3)} \end{bmatrix}$$

$$\text{recGKXY}(\text{GKXY}, \text{imax}, \text{jmax}) \rightarrow \begin{pmatrix} \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} \\ 0 & 0 \\ 0 & \frac{A \cdot E}{L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} \\ \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} \end{pmatrix}$$

$$U_4 \rightarrow \frac{\text{PX} \cdot (3 \cdot L + \sqrt{3} \cdot L)}{A \cdot E + \sqrt{3} \cdot A \cdot E} - \frac{\text{PY} \cdot (3 \cdot L - 2 \cdot \sqrt{3} \cdot L)}{3 \cdot A \cdot E}$$

$$V_4 \rightarrow \frac{\text{PX} \cdot (3 \cdot L - \sqrt{3} \cdot L)}{3 \cdot A \cdot E + 3 \cdot \sqrt{3} \cdot A \cdot E} - \frac{\text{PY} \cdot (\sqrt{3} + 3) \cdot (3 \cdot L - \sqrt{3} \cdot L)}{(3 \cdot A \cdot E + 3 \cdot \sqrt{3} \cdot A \cdot E) \cdot (\sqrt{3} - 3)}$$

$$\begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \end{pmatrix} = \begin{pmatrix} \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} \\ 0 & 0 \\ 0 & \frac{A \cdot E}{L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} \\ \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} \end{pmatrix} \begin{pmatrix} U_4 \\ V_4 \end{pmatrix}$$

Figure 7 Solution of unknown displacement

The unknown reactions can be calculated as is shown in Figure 8. A simple program can be carried out using instructions in Mathcad. The result is shown in Figure 9.

Figure 10 shows the member forces for each member.

$$\begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \end{pmatrix} = \begin{pmatrix} \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{A \cdot E}{8 \cdot L} \\ 0 & 0 \\ 0 & \frac{A \cdot E}{L} \\ \frac{\sqrt{3} \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot A \cdot E}{8 \cdot L} \\ \frac{3 \cdot A \cdot E}{8 \cdot L} & \frac{3 \cdot \sqrt{3} \cdot A \cdot E}{8 \cdot L} \end{pmatrix} \begin{pmatrix} U_4 \\ V_4 \end{pmatrix}$$

Figure 8 Solution flow of unknown reactions

$$\begin{aligned}
 X_1 &\rightarrow \frac{PY \cdot (3L - 2\sqrt{3}L)}{8AE} - \frac{3PX \cdot (3L + \sqrt{3}L)}{8AE + 8\sqrt{3}AE} - \frac{\sqrt{3} \left[ \frac{PX(3L - \sqrt{3}L)}{3AE + 3\sqrt{3}AE} - \frac{PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} \right]}{8} \\
 Y_1 &\rightarrow \frac{PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{8(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} - \frac{PX(3L - \sqrt{3}L)}{24AE + 24\sqrt{3}AE} - \frac{\sqrt{3} \left[ \frac{PX(3L + \sqrt{3}L)}{AE + \sqrt{3}AE} - \frac{PY(3L - 2\sqrt{3}L)}{3AE} \right]}{8} \\
 X_2 &\rightarrow 0 \\
 Y_2 &\rightarrow \frac{PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} - \frac{PX(3L - \sqrt{3}L)}{3AE + 3\sqrt{3}AE} \\
 X_3 &\rightarrow \frac{3PX(3L - \sqrt{3}L)}{24AE + 24\sqrt{3}AE} - \frac{\sqrt{3} \left[ \frac{PX(3L + \sqrt{3}L)}{AE + \sqrt{3}AE} - \frac{PY(3L - 2\sqrt{3}L)}{3AE} \right]}{8} - \frac{3PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{8(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} \\
 Y_3 &\rightarrow \frac{3PX(3L + \sqrt{3}L)}{8AE + 8\sqrt{3}AE} - \frac{3\sqrt{3} \left[ \frac{PX(3L - \sqrt{3}L)}{3AE + 3\sqrt{3}AE} - \frac{PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} \right]}{8} - \frac{PY(3L - 2\sqrt{3}L)}{8AE}
 \end{aligned}$$

Figure 9 Solution of unknown reactions

$$\begin{aligned}
 N_{14} &= \frac{E \cdot A}{2L} \begin{pmatrix} -\sqrt{3} & -1 & \sqrt{3} & 1 \\ 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} U_1 \\ V_1 \\ U_4 \\ V_4 \end{pmatrix} \\
 N_{14} &= \begin{pmatrix} 3 + \sqrt{3} & 3 - \sqrt{3} \\ 6 & 6 \end{pmatrix} \begin{pmatrix} PX \\ PY \end{pmatrix} \\
 N_{14} &\rightarrow \frac{\sqrt{3} \left[ \frac{PX(3L + \sqrt{3}L)}{AE + \sqrt{3}AE} - \frac{PY(3L - 2\sqrt{3}L)}{3AE} \right]}{4} + \frac{PX(3L - \sqrt{3}L)}{12AE + 12\sqrt{3}AE} - \frac{PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{4(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} \\
 N_{24} &= \frac{E \cdot A}{L} \begin{pmatrix} -0 & -1 & 0 & 1 \\ 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} U_2 \\ V_2 \\ U_4 \\ V_4 \end{pmatrix} \\
 N_{24} &= \begin{pmatrix} 2\sqrt{3} - 3 & \sqrt{3} \\ 3 & 3 \end{pmatrix} \begin{pmatrix} PX \\ PY \end{pmatrix} \\
 N_{24} &\rightarrow \frac{PX(3L - \sqrt{3}L)}{3AE + 3\sqrt{3}AE} - \frac{PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)} \\
 N_{34} &= \frac{E \cdot A}{2\sqrt{3}L} \begin{pmatrix} 1 & -\sqrt{3} & -1 & \sqrt{3} \\ 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} U_3 \\ V_3 \\ U_4 \\ V_4 \end{pmatrix} \\
 N_{34} &= \begin{pmatrix} \sqrt{3} - 4 & \sqrt{3} - 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} PX \\ PY \end{pmatrix} \\
 N_{34} &\rightarrow \frac{3PX(3L - \sqrt{3}L)}{12AE + 12\sqrt{3}AE} - \frac{\sqrt{3} \left[ \frac{PX(3L + \sqrt{3}L)}{AE + \sqrt{3}AE} - \frac{PY(3L - 2\sqrt{3}L)}{3AE} \right]}{4} - \frac{3PY(\sqrt{3} + 3)(3L - \sqrt{3}L)}{4(3AE + 3\sqrt{3}AE)(\sqrt{3} - 3)}
 \end{aligned}$$

Figure 10 Forces of each member

## 2.2 Computational example 2

Figure 11 shows the global displacements with the

number of 12 nodes and 21 elements. The element displacements  $u_i$  and element nodal forces  $F_i$  and total







U := K <sup>-1</sup> · P		React := KT · U	
	0		0
0	1.528 · 10 <sup>-12</sup>	0	0
1	2.495 · 10 <sup>4</sup>	1	0
2	6.507 · 10 <sup>3</sup>	2	0
3	0	3	-2.495 · 10 <sup>-6</sup>
4	7.276 · 10 <sup>-12</sup>	4	5.719 · 10 <sup>-6</sup>
5	-4.093 · 10 <sup>-12</sup>	5	-1.172 · 10 <sup>-5</sup>
6	-1.819 · 10 <sup>-11</sup>	6	1.86 · 10 <sup>-5</sup>
7	-5 · 10 <sup>4</sup>	7	-2.879 · 10 <sup>-5</sup>
8	-1.819 · 10 <sup>-11</sup>	8	2.653 · 10 <sup>-5</sup>
9	3.638 · 10 <sup>-12</sup>	9	-2.272 · 10 <sup>-5</sup>
10	2.728 · 10 <sup>-12</sup>	10	6.647 · 10 <sup>-6</sup>
11	-1.182 · 10 <sup>-11</sup>	11	-1.834 · 10 <sup>-5</sup>
12	5.864 · 10 <sup>3</sup>	12	0
13	7.276 · 10 <sup>-12</sup>	13	-5.01 · 10 <sup>-6</sup>
14	9.203 · 10 <sup>-12</sup>	14	0
15	5.01 · 10 <sup>4</sup>	15	0
16	6.972 · 10 <sup>-12</sup>	16	3.7 · 10 <sup>-5</sup>
17	-1 · 10 <sup>5</sup>	17	-7.831 · 10 <sup>-5</sup>
18	-3.537 · 10 <sup>-12</sup>	18	1.306 · 10 <sup>-5</sup>
19	1.455 · 10 <sup>-11</sup>	19	-3.878 · 10 <sup>-5</sup>
20	-1.237 · 10 <sup>4</sup>	20	0
21	5.121 · 10 <sup>-12</sup>	21	-7.495 · 10 <sup>-6</sup>
22	1.377 · 10 <sup>-11</sup>	22	0
23	7.495 · 10 <sup>4</sup>	23	0

Figure 17 Global displacement vector U and reactions vector React

### 3 Results and discussion

As is shown in the examples, matrix transformation and stiffness calculations of the global structure are straightforward. Unknown reactions and member forces from unknown displacements can be calculated. The students' learning progress of the structural basis is useful because it provides a very easy expression.

The main focus of this study is to improve whether students using Mathcad, a computer-supported application program for structure analysis comprehension, will increase their comprehension scores for structure analysis in comparison to students who learned under traditional instruction.

The evidence showed that direct instruction with Mathcad increased students' comprehension score for structure analysis more than traditional instruction. Specifically, analyses of pretest to posttest gain scores and posttest scores adjusted for pretest score demonstrated that overall Mathcad students significantly improved their comprehension level for structure analysis in contrast with traditional instruction students.

This study has additionally offered evidence that such instruction can successfully be embedded into a software application, facilitating the creation of active classroom environments between teachers and their students by teaching easy to use functionalities that make it possible to concentrate on the comprehension process for structure analysis.

### 4 Conclusions

Mathcad is an educational programming tool which can process various unique features that other programs have not been able to deal with. It has integrative algorithm coding, annotating, and data visualization capabilities. It does not require tedious low-level programming in languages like Fortran, C, or C++. Mathcad allows the simple realization of advanced algorithms. This paper has demonstrated a convenient approach to teaching the theory of matrix structural analysis for the stiffness matrix of statically indeterminate truss and 20 bar sensitivity penalty method, and a methodology for implementing the procedure in Mathcad.

Calculate element displacements  $u_i$  and calculate element nodal forces  $F_i$  and total forces along element  $R_i$  and element stresses  $\sigma_i$ .

$$i := 0..nel - 1$$

$$u_i := \begin{bmatrix} U_{(Glob,0)} \\ U_{(Glob,1)} \\ U_{(Glob,2)} \\ U_{(Glob,3)} \end{bmatrix} \quad u_1 = \begin{pmatrix} 0 \\ -2.495 \times 10^{-6} \\ 5.719 \times 10^{-6} \\ -1.172 \times 10^{-5} \end{pmatrix} \quad u_4 = \begin{pmatrix} 1.86 \times 10^{-5} \\ -2.879 \times 10^{-5} \\ 6.647 \times 10^{-6} \\ -1.834 \times 10^{-5} \end{pmatrix} \quad u_{10} = \begin{pmatrix} 6.647 \times 10^{-6} \\ -1.834 \times 10^{-5} \\ 0 \\ -5.01 \times 10^{-6} \end{pmatrix}$$

$$F_i := K e l_i \cdot u_i \quad F_0 = \begin{pmatrix} 1.528 \times 10^{-12} \\ 2.495 \times 10^4 \\ -1.528 \times 10^{-12} \\ -2.495 \times 10^4 \end{pmatrix} \quad F_1 = \begin{pmatrix} 1.129 \times 10^{-12} \\ 1.844 \times 10^4 \\ -1.129 \times 10^{-12} \\ -1.844 \times 10^4 \end{pmatrix}$$

$$R_i := \sqrt{[(F_i)_0]^2 + [(F_i)_1]^2} \quad F_4 = \begin{pmatrix} 1.584 \times 10^4 \\ -1.584 \times 10^4 \\ -1.584 \times 10^4 \\ 1.584 \times 10^4 \end{pmatrix} \quad F_{10} = \begin{pmatrix} -4.899 \times 10^{-12} \\ -2.667 \times 10^4 \\ 4.899 \times 10^{-12} \\ 2.667 \times 10^4 \end{pmatrix}$$

$$\sigma_i := \frac{R_i}{d_i}$$

$$R_0 = 2.495 \times 10^4 \quad R_1 = 1.844 \times 10^4 \quad R_4 = 2.241 \times 10^4 \quad R_{10} = 2.667 \times 10^4$$

$$\sigma_0 = 2.495 \times 10^5 \quad \sigma_1 = 1.844 \times 10^5 \quad \sigma_4 = 2.241 \times 10^4 \quad \sigma_{10} = 6.258 \times 10^5$$

Figure 18 Element displacements and nodal forces and total forces along element and stresses

By integrating the Mathcad programming into the course, the students not only can understand the concepts, but experience the implementation of the theory. While one might suggest that there is little need for the engineer to know how a commercial program works to use it properly. Understanding how the program work is a requirement for fluency in the computer-based structural analysis. The best one can expect from the student that is simply exposed to commercial programs is familiarity with that program and usefulness for learning.

The characteristics of Mathcad allow students, teachers, and engineers to develop sophisticated computer models that solve engineering problems in scientific

notation. Complete solutions to engineering problems can be developed to simply check the results of other programs. Mathcad allows students and teachers to focus on the engineering theory and procedures while letting the software handle the calculations along with the units of measure. As the engineering profession continues to move toward automated design software, engineers must find better tools that they can use to verify engineering calculations.

### Acknowledgments

This paper was supported by the research funds of Chonbuk National University, 2011.

### References

- Al-Ansari, M. S., and A. B. Senouci. 1999. Use of Mathcad as a teaching and learning tool for reinforced concrete design of footings. *Computer Applications in Engineering Education*, 7 (3): 146-154.
- Almeida S. F., R. Piazzalunga, and V. G. Ribero. 2003. A web-based 2D structural analysis educational software. *Computer Applications in Engineering Education*, 11(2): 83-92.
- Bianco, F., E. D. Pezzo. 2002. Scat-Cad: a Mathcad 2000 professional package to model the energy decay due to seismic attenuation. *Computers & Geosciences*, 28(7): 851-855.
- Cedeno-Rosete, R. 2007. Stiffness matrix structural analysis educational package by Mathcad, Wiley Periodicals, Inc. *Computer Applications in Engineering Education*, 15(2): 107-112.
- Charney, F. A. 2008. A transformational approach to teaching matrix structural analysis, and visual implementation using Mathcad, 18th Analysis and Computation Specialty Conference, 2008 ASCE.
- Consolazio, G. 2008. Leveraging the integrated programming and visualization features of Mathcad in teaching advanced structural analysis. 18th Analysis and Computation Specialty Conference, 2008 ASCE.
- Dilly, R. L. 2001. Application of commercial software features for optimizing construction operations. *Civil Engineering Education Issues* 2001, ASCE, 39-46.
- Hodge, B. K., and R. P. Taylor. 2002. Piping-system solutions using Mathcad. *Computer Applications in Engineering Education*, 10 (2): 59-78.
- Jiang, D. 2007. Numerical modeling of the motion of deformable ellipsoidal objects in slow viscous flows. *Journal of Structural Geology*, 29 (3): 435-452.
- Mtenga, P. V., and L. K. Spainhour. 2000. Applications of mathematical software packages in structural engineering education and practice. *Journal of Computing Civil Engineering*, 14 (4): 273-278.
- Yildirim, G., and V. P. Singh. 2010. A MathCAD procedure for commercial pipeline hydraulic design considering local energy losses. *Advances in Engineering Software*, 41 (3): 489-496.
- Yuan, X. F., and J. G. Teng. 2002. Interactive web-based package for computer aided learning of structural behavior. *Computer Applications in Engineering Education*, 10 (3): 121-136.