Comparative Evaluation of Batch and Continuous Anaerobic Digesters in Biogas Production from Municipal Solid Waste using Mathematical Models

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ABSTRACT

An investigation was conducted into the suitability of either of the batch or continuous (CSTR) digesters for anaerobic degradation of municipal solid waste (MSW) in the production of biogas. Mathematical models were developed for the design and evaluation of the two systems. The development of the models was based upon a material balance analysis of the digesters' operation. A Microsoft Visual Basic Version 6.0 Programme was developed for the solution of the model equations, and the digesters' operations simulated over a range of percentage total solids (PTS) concentration of 4-10% for the CSTR and 4-30% for the batch digester, and fractional conversion of 0.2-0.8. The results of the simulation show that although the amount of methane ($0.0764m^3$) produced per unit volume of the batch digester is about 4 times less than the amount ($0.284m^3$) per unit volume of the CSTR, the cost per unit volume of the batch digester (₦5.98) is 6 times less than that of the CSTR (₦33.8), suggesting that the overall cost of producing gas with the batch digester would be more economical. So, it was deduced that the batch digester is better suited for the digestion of MSW for biogas production, compared to the CSTR.

Key words: Batch, continuous, anaerobic, digesters, biogas, municipal solid waste, total solids concentration, Nigeria

Nomenclature

ϑ_c	-	mean cell residence time, days
ϑ_h	-	hydraulic retention time, days
μ_{max}	-	maximum growth rate of microorganisms, mg/l
μ_{net}	-	net rate of microorganisms growth, mass mlvss/unit volume time
k	-	maximum rate of substrate utilization per unit mass of cells produced (mass/mass, time)
k_d	-	decay rate coefficient (lysis constant)
K_s	-	half saturation constant, mg/l
N	-	range of integration
Q	-	flowrate, volume / time

r_s - rate of substrate utilization
S - concentration of limiting substrate, mg/l
S_e - effluent substrate concentration, mg/l
S_o - influent substrate concentration, mg/l
V_{bd} - volume of batch digester, m^3
V_c - Volume of CSTR, m^3
V_r - volume of reactor
V_{mb} - volume of methane in batch digester
V_{mc} - volume of methane in CSTR
X_e - concentration of microorganism in reactor, mass mlvss/unit volume
X_o - concentration of microorganisms in influent, mass mlvss/unit volume
Y - yield coefficient, mg/l

1. INTRODUCTION

The anaerobic digestion of municipal solid waste (MSW) can be carried out at different moisture contents, determining the level of percentage total solids. The moisture content of the waste is highly essential for enhanced microbial activity in the waste decomposition. Mattocks (1984) says sufficient amount of moisture is required for effective anaerobic digestion.

The anaerobic digestion of wastes is essentially the treatment of the waste in which microorganisms cause the decomposition of the organic component of the waste in the absence of oxygen. During the decomposition of the waste, in this case the organic component of MSW, biogas is generated as one of the by-products of the process. Figure 1 shows the end-products of organic decay as presented by Steadman (1976).

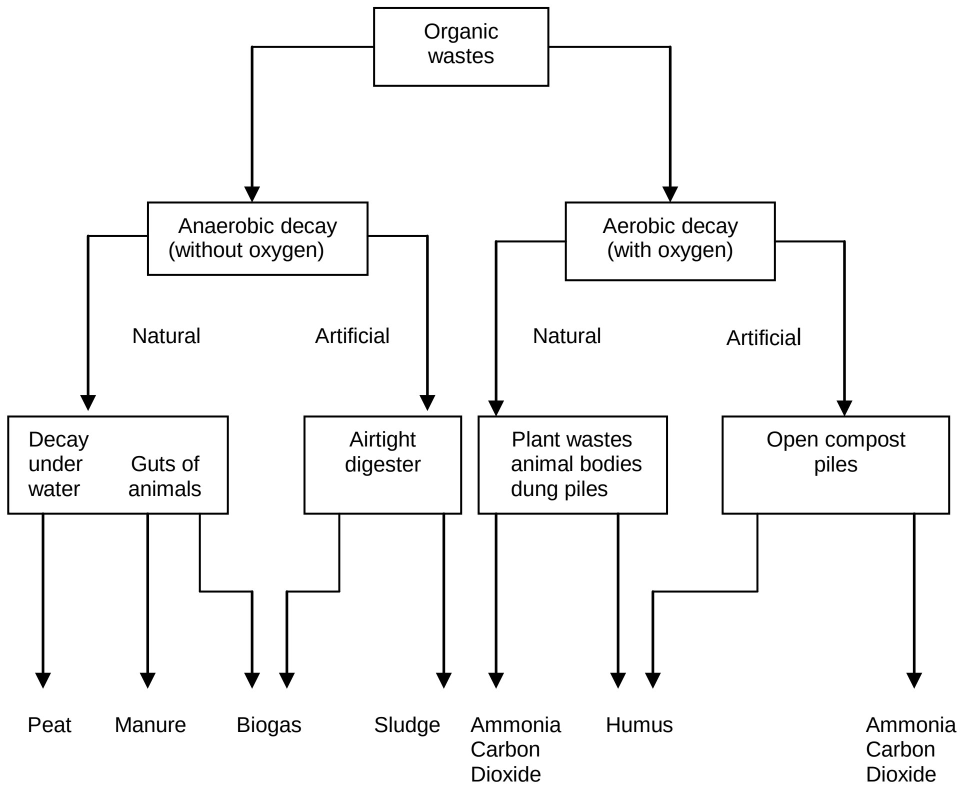

Figure 1. End-products of organic decay

Biogas is a colourless, relatively odourless and inflammable gas, with the composition shown in Table 1 (adapted from Madu and Sodeinde, 2001).

Table 1. Composition of Biogas

Constituents	% Composition
Methane (CH_4)	55 – 75%
Carbon dioxide (CO_2)	30 – 45%
Hydrogen sulphide (H_2S)	1 – 2%
Nitrogen (N_2)	0 – 1%
Hydrogen (H_2)	0 – 1%
Carbon monoxide (CO)	Traces
Oxygen (O_2)	Traces

They explain that biogas burns with a blue flame and has a heat value of 4500 – 5000 kcal/m^2 when its methane content is in the range of 60 – 70%. It is also stable and non-toxic.

The Figure 1 shows that biogas could be produced naturally from decay under water or in the guts of animals, or artificially in an air tight digester. As a result, biogas has been described as “a methane-rich gas that is produced from the anaerobic digestion of organic materials in a digester” (Itodo and Philips, 2001); and GEMET (2000) says biogas is “gas rich in methane, which is produced by the fermentation of animal dung, human sewage or crop residues in an air-tight container”.

In Nigeria there are abundant supplies of biomass resources, particularly agro-forestry residues and municipal solid waste, which potentials are yet to be fully tapped for energy generation (Igoni et al 2005; Olorunnisola, 2007). MSW is a relatively high solids waste for which high solids digesters have general been prescribed for its digestion. Kayhanian et al (1991) in Kiely (1998) state that high solids digesters are particularly suited to the treatment of municipal solid waste; even also as Hobson et al (1981) state that "the time course and the kinetic model of the 'dry digester' is essentially that of the batch culture". However, a lot more possibilities exist for the treatment of the waste, including its been processed in a continuous-flow digester, where the waste would be shredded to fine particles and diluted with so much water to meet the desired total solids concentration for CSTRs operation; and CSTRs have also been designed for the processing of MSW (Bitrus, 2001).

However, whereas the batch digester can handle substantial amount of the waste with little amount of water, and therefore reduced microbial activity and low product yield, the CSTR is capable of utilizing a little quantity of the waste with a large amount of water and high microbial activity and product yield (Bailey and Ollis, 1986). Generally, the batch system is usually associated with high solids wastes of low volume, while the CSTR considers low solids wastes of high volume; but the MSW in Port Harcourt metropolis is indeed of high solids concentration and also of high volume (Igoni et al., 2007).

Therefore, this paper evaluates the operations in both the batch digester and the CSTR at various levels of total solids concentration to ascertain the system and level of PTS where biogas production can be optimized.

2. METHODOLOGY

Design models for the digesters were developed using the fundamental principles of material balance analysis (Igoni, 2006). With the kinetic parameters of the MSW determined by Igoni et al (2006), and the established relationship between microbial and substrate concentrations in Monod kinetics (Reynolds and Richard, 1996) both the batch digester and the CSTR were designed. The systems were then simulated over a range of total solids concentrations of 4 - 30% for the batch digester and 4 - 10% for the CSTR, and fractional conversion of 0.2 - 0.8, with a computer programme using the Microsoft Visual Basic Version 6.0 Software. The results of the simulation were analysed to obtain the operational performance of the two systems digesting MSW.

The model formulation for the anaerobic digestion of MSW is achieved with the following material balance expression, thus:

$$\begin{matrix} Rate\ of \\ Accumulation \\ of\ material\ in \\ \mathrm{Re}actor \end{matrix} = \begin{matrix} Rate\ of \\ material\ flow \\ \mathrm{int}o\ \mathrm{Re}actor \end{matrix} + \begin{matrix} Rate\ of \\ Appearance\ or \\ Disapperarance \\ of\ material\ due \\ \mathrm{Re}action \end{matrix} - \begin{matrix} Rate\ of\ material \\ flow\ out\ of \\ \mathrm{Re}actor \end{matrix}$$

This expression can be symbolically represented as:

$$\frac{d[X]}{dt}V_r = Q[X_o] + V_r\mu_{net} - Q[X] \qquad (1)$$

Where $\frac{d[X]}{dt}$ = rate of change of microorganism concentration in the reactor measured in terms of mass (mixed liquor volatile suspended solids), mass MLVSS/unit volume. time

V_r = volume of reactor

Q = flowrate, volume / time

X_o = concentration of microorganisms in influent, mass mlvss/unit volume

X = concentration of microorganism in reactor, mass mlvss/unit volume

μ_{net} = net rate of microorganisms growth, mass mlvss/unit volume time

and $$\mu_{net} = \mu_{\max}\frac{[S]}{K_s + [S]}[X] - k_d[X] \qquad (2)$$

Then, $$\frac{d[X]}{dt}V_r = Q[X_o] + V_r\left(\mu_{\max}\frac{[S]}{K_s + [S]}[X] - k_d[X]\right) - Q[X] \qquad (3a)$$

or $$\frac{d[X]}{dt}V_r = Q\left([X_o] - [X]\right) + V_r\left(\mu_{\max}\frac{[S]}{K_s + [S]}[X] - k_d[X]\right) \qquad (3b)$$

2.1 Application of the model to batch reactor processes

Applying the general form of the material balance expression to a batch process where there is no flow (i.e. Q = 0), the first term on the right hand side of the equation becomes zero;

i.e. $$Q\left([X_o] - [X]\right) = 0 \qquad (4)$$

(a) Material balance for mass of microorganism in the batch digester

$$\therefore \quad \frac{d[X]}{dt}V_{bd} = \frac{\mu_m [S][X]}{K_s + [S]}V_{bd} - k_d[X]V_{bd} \tag{5a}$$

or (eliminating V_{bd})
$$\frac{d[X]}{dt} = \frac{\mu_m [S][X]}{K_s + [S]} - k_d[X] \tag{5b}$$

And this represents the mass balance for the mass of microorganisms in the batch reactor.

(b) Material balance for total substrate utilization in the batch digester

The material balance for the total substrate utilization in a batch process is equally given as;

$$\frac{d[S]}{dt}V_{bd} = -\frac{k[S][X]}{K_s + [S]}V_{bd} \tag{6a}$$

or (eliminating V_{bd})
$$\frac{d[S]}{dt} = -\frac{k[S][X]}{K_S + [S]} \tag{6b}$$

where: k - maximum rate of substrate utilization per unit mass of cells produced (mass/mass, time)

and
$$k = \frac{\mu_m}{Y} \tag{7}$$

The solution of equation (6b) is as follows:

$$\frac{dt}{d[S]} = -\frac{K_s + [S]}{k[S][X]} \tag{8a}$$

$$\frac{dt}{d[S]} = -\frac{K_s}{k[S][X]} - \frac{1}{k[X]} \tag{8b}$$

$$dt = -\frac{K_s dS}{k[S][X]} - \frac{dS}{k[X]} \tag{8c}$$

$$\int_{t=0}^{t=t} dt = \int_{S_o}^{S_e} \frac{K_s dS}{k[S][X]} - \int_{S_o}^{S_e} \frac{dS}{k[X]} \tag{8d}$$

so that
$$t = \frac{K_s}{k[X]}\ln\left(\frac{S_o}{S_e}\right) + \frac{[S_o]-[S_e]}{k[X]} \tag{8e}$$

This equation expresses the time, t, required to degrade the substrate from S_o to S_e, which is also called the time for batch digestion, obtained from a computer solution of the equation using Simpson's numerical approximation, thus:

Let 'N' represent the range of integration, such that N = 10

Then interval
$$h = \frac{(S_e - S_o)}{N} \tag{9a}$$

Such that
$$S(N) = S_o + (N \times h) \tag{9b}$$

$$f(N) = -\frac{K_s + S(N)}{k[X]S(N) - k_d[K_s - S(N)]} \tag{9c}$$

$$\left.\begin{aligned} &S(0) = f(0) + f(10) \\ &S(1) = f(1) + f(3) + f(5) + f(7) + f(9) \\ &S(2) = f(2) + f(4) + f(6) + f(8) \end{aligned}\right\} \quad (9d)$$

So that
$$t = \frac{h}{3}\left[S(0) + 4S(1) + 2S(2)\right] \quad (10)$$

2.2 Application of the Model to CSTR Processes

To apply the general form of the material balance expression to a CSTR will actually be to adopt the form of the general model because the overall characteristics of the CSTR including its flow regime were considered in the development of the model.

(a) Material balance for mass of microorganism in the CSTR

The mass balance for the mass of microorganisms in a complete-mix reactor, will, therefore be:

$$\frac{d[X]}{dt}V_c = Q([X_o] - [X]) + V_c\left(\mu_{\max}\frac{[S]}{K_S + [S]}[X] - k_d[X]\right) \quad (11)$$

Now, considering steady state condition, i.e. $\frac{d[X]}{dt} = 0$, and assuming that x_o is negligible at the commencement of the process, then equation (11) becomes

$$Q[X] = V_c\left(\mu_{\max}\frac{[S]}{K_S + [S]}[X] - k_d[X]\right) \quad (12a)$$

or (eliminating [X])

$$Q = V_c\left(\mu_{\max}\frac{[S]}{K_S + [S]} - k_d\right) \quad (12b)$$

But $\frac{V_c}{Q}$ is defined as the mean cell residence time (θ_c). Therefore

$$\frac{1}{\theta_C} = \mu_{\max}\frac{[S]}{K_S + [S]} - k_d \quad (13a)$$

or
$$\frac{1}{\vartheta_c} = \frac{\mu_{\max}[S] - k_d(K_S + [S])}{K_S + [S]} \quad (13b)$$

(b) Material balance for total substrate utilization in the CSTR

The mass balance for substrate utilization in a CSTR will be given as

$$V_c\frac{d[S]}{dt} = Q[S_o] + r_S V_c - Q[S] \quad (14a)$$

$$V_c\frac{d[S]}{dt} = Q([S_o] - [S]) + r_s V_c \quad (14b)$$

Where r_s - rate of substrate utilization, and defined mathematically as

$$r_s = -\frac{k[S]}{K_S + [S]}[X] \tag{15}$$

So, substituting for ‘r_s’ in equation (14b) from equation (15) and assuming steady – state condition $\left(i.e.\ \frac{d[S]}{dt} = 0 \right)$ gives

$$Q([S_O] - [S]) - \frac{k[S]}{K_S + [S]}[X]V_c = 0 \tag{16a}$$

or

$$[S_o] - [S] = \left(\frac{V_c}{Q} \right) \left(\frac{k[S]}{K_S + [S]}[X] \right) \tag{16b}$$

$$\therefore \quad [S_o] - [S] = \theta_h \frac{k[S]}{K_S + [S]}[X] \tag{16c}$$

Where θ_h = hydraulic retention time which is the same as the mean cell residence time (θ_c) for no cell recycle anaerobic system, and describes the digestion time for the CSTR, such that

$$\therefore \quad \vartheta_h = \frac{([S_o] - [S_e])(K_s + [S_e])}{k[S_e][X]} \tag{17}$$

3. RESULTS AND DISCUSSION

3.1 Analysis of the Batch and Continuous Digesters’ Performances

The summary of the results of the simulation is presented in Table 1 and analysed graphically using Figures 1 – 4.

Table 1. Results for the Batch and Continuous Digesters’ Simulation at 10% TS

α	T (days)		S_e (mg/l)		X_e (mg/l)		Vol. of Digester (m^3)		Vol. of Methane (m^3)		Cost ‘000 000 ₦	
	Batch	CSTR	Batch	CSTR	Batch	CSTR	Batch	CSTR	Batch	CSTR	Batch	CSTR
0.2	8.45	7.40	5.70	5.70	0.39	0.39	13867.30	184.55	250.11	12.31	659	49
0.3	8.69	7.62	5.15	5.15	0.57	0.57	14243.89	189.99	394.49	19.45	669	50
0.4	8.82	7.74	4.60	4.60	0.75	0.75	14444.77	193.03	539.99	26.69	675	51
0.5	8.90	7.82	4.06	4.06	0.94	0.94	14574.41	195.12	686.30	34.01	679	51
0.6	8.96	7.89	3.51	3.51	1.12	1.12	14668.43	196.83	833.41	41.44	681	51
0.7	9.01	7.96	2.96	2.96	1.30	1.30	14743.59	198.44	981.48	49.04	683	52
0.8	9.05	8.03	2.41	2.41	1.48	1.48	14809.47	200.22	1130.8	56.91	685	52

Bailey and Ollis (1986) state that the relationship between batch and continuous biological reactors is usually assessed on the bases of biomass production, substrate utilization and product yield. In this study, for a successful evaluation of the processes, the batch and continuous models were analyzed for the same level of total solids concentration (i.e. 10%TS), which is the reported upper limit for the capacity of pumps in the operation of CSTRs (Hobson et al, 1981).

Figure 2 shows a perfectly linear relationship between the fractional conversion and microbial growth in both the batch and continuous digesters, guided by the equation $X_e = 1.8214\alpha + 0.025$, indicating that microbial growth during the process is directly proportional to the fractional conversion.

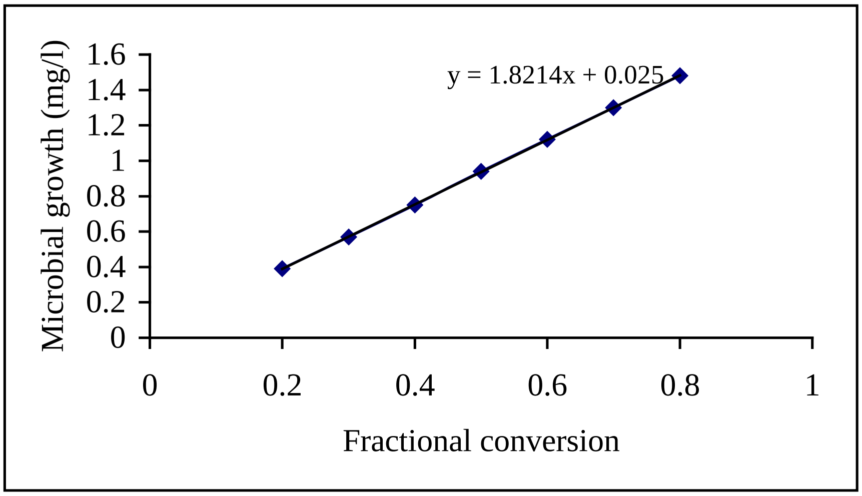

Figure 2. Relationship between fractional conversion and microbial growth

From the data in Table 1, at a fractional conversion of 0.8, the biomass production per unit volume in the batch digester is $9.994x10^{-5}$ kg/m^3 (0.09994 mg/l) and is greatly lower than that of 0.007392 kg/m^3 (7.392 mg/l) for the CSTR. So, for the same degree of operation, with the fractional conversion as index, the CSTR has a greater biomass yield than the batch digester. This is also the case for the amount of methane produced per unit volume of digester, which is 0.0764 m^3 of CH_4 / m^3 of digester for the batch, and 0.284 m^3 of CH_4 / m^3 of CSTR. Figures 3 and 4 depict an exponential relationship between the volume of digester and that of the methane produced, as $V_{mb} = 5E\text{-}08e^{0.0016Vbd}$ for the batch digester and $V_{mc} = 1E\text{-}07e^{0.1005Vc}$ for the CSTR.

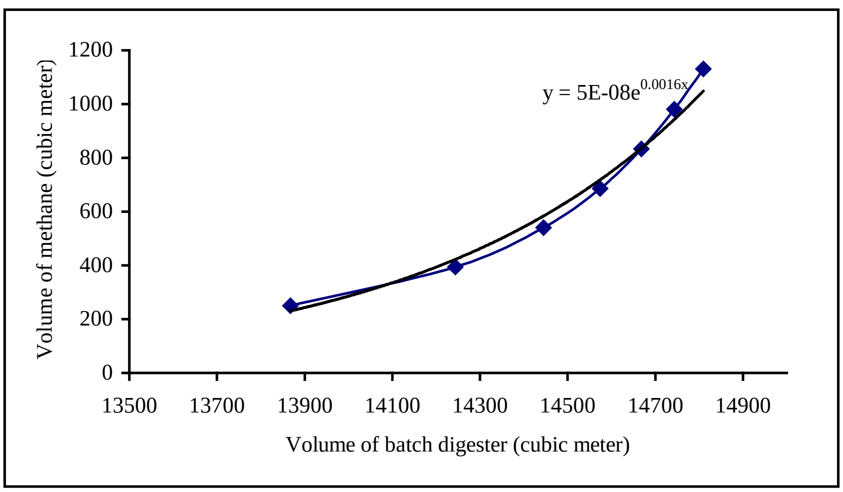

Figure 3. Relationship between volume of batch digester and the volume of methane produced

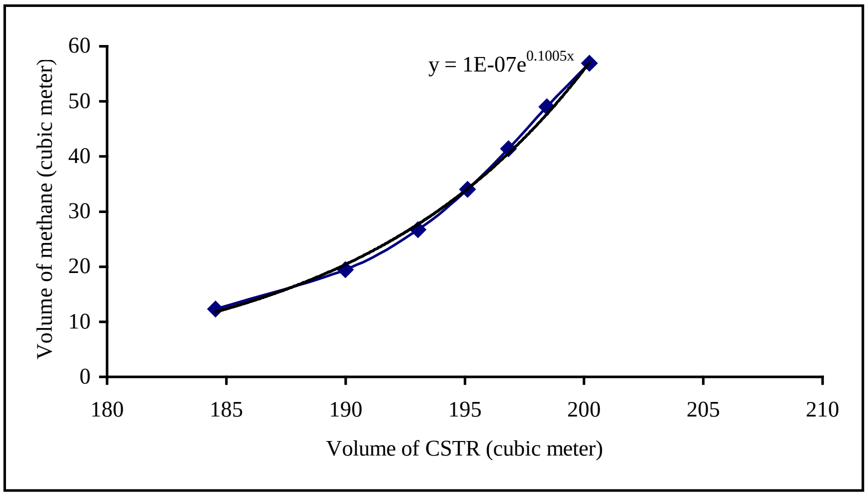

Figure 4. Relationship between volume of CSTR and the volume of methane produced

These relationships show that towards the upper limit of the fractional conversion, the increasing volume of methane produced will no longer be dependent on increasing the volume of the digester.

Also in the Table 1, the cost of the digester follows this trend, as the cost per unit volume of the digesters at the upper limit of the fractional conversion of 0.8 is ₦5.98 for the batch and ₦33.8 for the CSTR. This shows that the cost of a unit volume of CSTR is about 6 times more than that of a batch digester; and when this is related to the amount of methane/biogas per unit volume of digester, it shows that the cost of a unit volume of gas in a batch digester is ₦78.26 against ₦118.95 for the CSTR, indicating that it cost more to produce gas in the CSTR than in the batch digester. Figures 5 and 6 show the variation of the cost of the digesters with their volumes, as being characterized by a power function relationship, indicating that at higher operational limits, marginal increases in the volume of the digesters will have little or no effect on their costs.

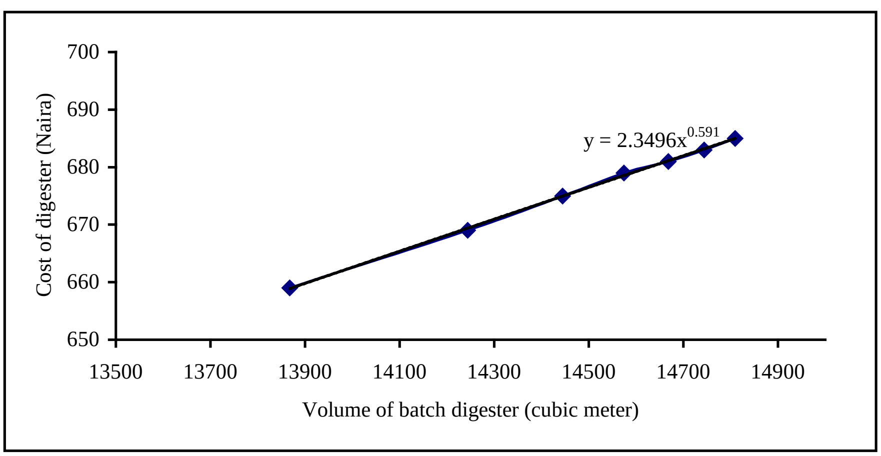

Figure 5. Variation of cost of digester with the volume of batch digester

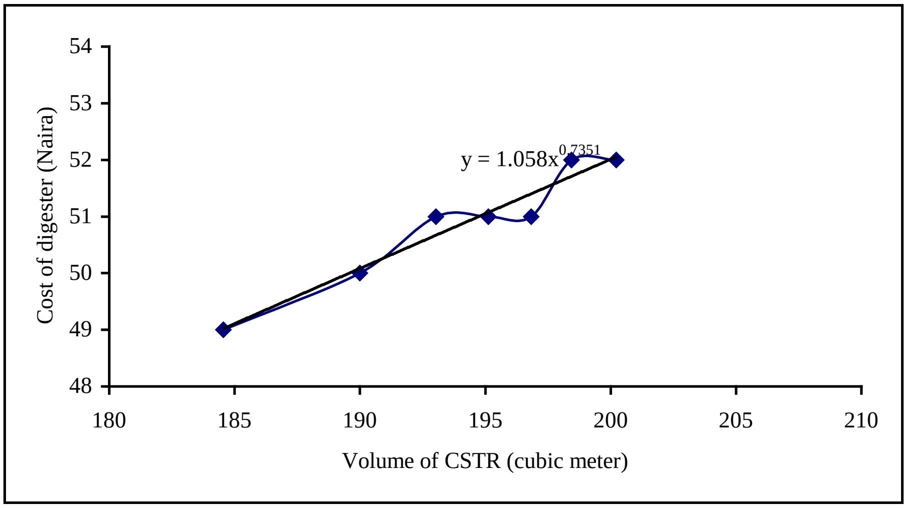

Figure 6. Variation of cost of digester with volume of the CSTR

Considering the time of digestion (Table 1), it was observed that the time required to achieve the same level of microbial growth in the CSTR is lower than that of the batch, which is an indication that "the continuous process always provides a greater yield of cells per unit volume of cultivator vessel than a batch process does" (Bailey and Ollis, 1986).

It is therefore evident that, although the initial cost investment in a batch digester is higher than that of a CSTR, the overall amount of gas produced is higher in the batch compared to the CSTR.

It is important to state here that this comparison of the batch and continuous digesters is based on the same level of total solids concentration of 10%. However, it has been noted earlier that one of the major distinctions between the two systems occasioned by their respective flow regimes is that, whereas the batch digester is suitable for high solids processing, the CSTR is suitable for low solids processing with an upper limit of 10%TS. Therefore, the batch digester can handle higher levels of total solids concentration.

4. CONCLUSION

There are a variety of options for the anaerobic digestion of municipal solid waste in the production of biogas. The two principal modes of operation are the batch and continuous processing. Whereas a larger volume of the batch digester is required to handle the same amount of waste as the CSTR, and the initial cost of building a batch digester is also higher than that of the CSTR, the cost of a unit volume of CSTR is about six (6) times more than that of a batch digester, and the cost of producing the same quantity of gas in a CSTR is about two (2) times that of a batch digester. Therefore, it is possible to assert that the batch digester is better suited for the digestion of municipal solid waste than the CSTR.

5. REFERENCES

Bailey, J. E. and D.F. Ollis. 1986. *Biochemical Engineering Fundamentals.* New York: McGraw-Hill. 753p.

Bitrus, R. 2001. Design of an anaerobic bio-reactor for the production of methane from solid waste organic matter. M.Tech. Thesis, Rivers State University of Science and Technology, Port Harcourt, Nigeria.

General Environmental Multilingual Thesaurus (GEMET), 2000. Biomass Resources. http://glossary.eea.eu.int./EEAGlossary/B/Biogas

Hobson, P. N., S. Bousfield and R. Summers. 1981. *Methane Production from Agricultural and Domestic Wastes*. London: Applied Science Publishers Ltd. 269p.

Igoni, A. H., C. L. Eze, M. J. Ayotamuno and M. F. N. Abowei. 2005. Potentials of biogas generation from municipal solid-waste in the Port Harcourt metropolis. In *Proc. 1st Annual Conference of Science and Technology Forum*, 1(2): 67 – 72. *Uyo, Nigeria*.

Igoni, A. H. 2006. Design of anaerobic bio-reactors for the simulation of biogas production from municipal solid waste. Ph.D Thesis, Rivers State University of Science and Technology, Port Harcourt, Nigeria.

Igoni, A. H., M. F. N. Abowei, M. J. Ayotamuno and C. L. Eze. 2006. Biokinetics of anaerobic digestion of municipal solid waste. *Newviews Engineering Analysis Model* 1 (1): 98 – 112.

Igoni, A. H., M. J. Ayotamuno, S. O. T. Ogaji and S. D. Probert. 2007. Municipal Solid Waste in Port Harcourt, Nigeria. *Applied Energy*. (84), 664-670. www.sciencedirect.com

Itodo, I. N. and Phillips, T. K. (2001). Determination of suitable material for anaerobic biogas digesters. *Proceedings of the 2nd international conference and 23rd Annual General Meeting of the Nigerian Institution of Agricultural Engineers* (23), pp 437-441

Kiely, G. 1998. *Environmental Engineering*, International Edition. Boston: Irwin McGraw-Hill. 979p.

Kahaynian, M., K. Lindenauer, S. Hardy and G. Tchobanoglous, 1991. Two-stage process combines anaerobic and aerobic methods. *Biocycle*, March Issue.

Madu, C. and Sodeinde, O. A. 2001. Relevance of biomass in the sustainable energy development in Nigeria. *Proceedings of the National Engineering Conference and Annual General Meeting of the Nigerian Society of Engineers.* Pp 220-227.

Mattocks, R. 1984. Understanding biogas generation. Technical Paper No. 4. Virginia, USA: *Volunteer in Technical Assistance*. 13p.

Olorunnisola A. 2007. Production of fuel briquettes from waste paper and coconut husk admixtures. *Agricultural Engineering International: the CIGR Ejournal* Vol. IX, Manuscript EE 06 006.

Reynolds, T. D. and P. A. Richards. 1996. *Unit operations and processes in Environmental Engineering*, 2nd ed., Boston: PWS Publishing Company. 798p.

Steadman, P. 1975. *Energy, Environment and Building*. A report to the Academy of natural Sciences of Philadelphia, Cambridge University Press, Cambridge, London. 287p