

A new concept of modelling water infiltration in a sandy loam soil

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Abstract: Infiltration is the process of water entering the soil from the surface into its layers, within the pores, and between the particles. This process determines the soil's ability to absorb water and significantly impacts the selection of the appropriate irrigation method. This research aims to apply a new approach to modeling the infiltration process in sandy loam soil, to calculate cumulative infiltration, and then develop an equation that can be used to estimate wetting front depth based on the cumulative infiltration and the physical properties of the soil. Nine laboratory experiments were conducted on a homogeneous sandy loam soil, involving three initial moisture contents and three bulk densities. The infiltration process was then modeled using six popular infiltration models (Kostiakov, modified Kostiakov, revised modified Kostiakov, Philip, Horten, and Soil Conservation Service "SCS"). The Microsoft Excel Solver tool was used to find the parameters of these models. The performance of these models was compared using three statistical criteria: sum of square errors SSE, determination coefficient R^2 , and root mean square error RMSE. Results showed that the revised modified Kostiakov model achieved the best performance, with statistical values of 152.49 (SSE), 0.9995 (R^2), and 0.882 (RMSE). The Horten model performed weakest, with values of 978.8, 0.9967, and 2.2347, respectively. Analyses indicated an inverse relationship between initial moisture content, bulk density, and cumulative infiltration depth. In contrast, a direct relationship was observed between initial moisture content and wetting front advance.

Keywords: cumulative infiltration, infiltration models, Kostiakov, bulk density, Solver.

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1 Introduction

Infiltration is the mechanism by which water penetrates the soil profile through its surface vertically downward. Water infiltration occurs in soil due to two main forces: the downward force of gravity and capillary action resulting from the interaction of surface tension between air, water, and the solid components of the soil (Nimmo and Shillito, 2023). Infiltration plays a critically important role in the

practical design of irrigation systems. The soil's water infiltration characteristics determine the appropriate irrigation rate to prevent surface runoff under sprinkler irrigation systems and deep percolation losses under surface irrigation systems. Furthermore, the optimal length of the furrows or strips depends on the soil's water infiltration characteristics. In addition, soils with high infiltration rates are often incompatible with surface irrigation systems, while they are more suitable for drip or sprinkler irrigation. Many factors that

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influence infiltration also influence the movement and distribution of water in the soil during and after infiltration. The most prominent of these factors are the physical properties of the soil, moisture content, and the irrigation rate, which depends on the irrigation method used (Hachum and Yasin, 1992).

Soil water represents a small percentage of the Earth's overall water resources; it is critically essential for plant life. It is also important in water resources engineering, as it requires practical rainfall accounting, groundwater recharge, drought and flood control, runoff estimation, irrigation system design and scheduling, and drainage design (Zakwan et al., 2016; Angelaki et al., 2013). Therefore, researchers and scientists have resorted to developing many models of infiltration, including Kostiakov (Kostiakov, 1932), Philip (Philip, 1958), Green and Ampt (Green and Ampt, 1911), Horten (Horten, 1940), Soil Conservation Service "SCS" (Jury et al., 1991). Infiltration models are divided into three basic categories: physical models based on Darcy's law and the law of mass conservation; empirical models derived from experimental data; and semi-empirical models based on the continuity equation and the relationship between infiltration rate and cumulative infiltration depth (Mishra et al., 2003; Al-Layla and Al-Ogaidi, 2025).

Researchers have conducted numerous studies to evaluate and compare various infiltration models to predict their suitability for soil types and properties such as texture, bulk density, moisture content, and other influential field conditions. To evaluate the effect of changing the bulk density of a two-layered soil, Al-Ogaidi et al. (2023) performed 10 laboratory experiments using two soils with different textures (clay and sandy loam), with three bulk densities. The results revealed that an increase was achieved in the infiltration and wetting front depths with time and reduced bulk density, regardless of whether the soil was situated in the top or bottom layer of the soil profile. The study carried out by Dahak et al. (2022) aimed to evaluate the performance of three infiltration

models: Kostiakov, Philip, and Horten in predicting the infiltration rate in the study area. The performance of these models was assessed using statistical criteria such as the Nash-Sutcliffe efficiency (NSE) and the coefficient of determination (R^2). The findings revealed that the Horten model had the best performance in estimating the infiltration rate when the effect of initial humidity was taken into account. Yasin and Ghazal (2021) studied the influence of some soil physical traits such as bulk density, the percentages of its components (sand, silt, clay), and initial soil moisture on the cumulative infiltration, infiltration rate, and basic infiltration rate, using data from 31 laboratory infiltration experiments with 516 values of cumulative infiltration depth and infiltration time with wide changes in the soil physical characteristics. The study revealed a significant impact of physical properties on the infiltration process, as the infiltration rate reduced with increasing bulk density and initial moisture content, and increasing the sand content resulted in an increase in the infiltration rate.

Patle et al. (2019) used a double-ring infiltrometer to measure the infiltration on agricultural land, where 25 points were identified with a spacing of 10 meters. The investigations demonstrated that the soil texture in the area was coarse, sandy loam, and loamy sand. The range of bulk density was between 1.412 - 1.716 g cm⁻³. The results showed that the bulk density reversely correlated with the infiltration rate, while the sand percentage had a direct relationship with the infiltration rate. Based on the study of Ahaneku (2011), the highest cumulative infiltration of the sandy loam soil was 12.2 cm after 240 minutes from the beginning of the test using a double-ring infiltrometer. The infiltration rate was also steady at 30 mm h⁻¹ after 180 minutes. Additionally, Abdel Rahman (2010) attained an infiltration rate of 31.99 mm h⁻¹ for a sandy loam soil and concluded that water movement could be accelerated by increasing the sand content in the soil due to the relatively larger soil pores. Mesele et al. (2024) presented a study on assessing and comparing infiltration models to predict the infiltration rate. The

infiltration rate was measured in the field using a double-ring infiltrometer across five soil textures (clay, sandy loam, sandy clay loam, clay loam, and loam). The performance of six infiltration models (Kostiakov, modified Kostiakov, revised modified Kostiakov, Philip, Horten, and Novel) was assessed using numerous statistical criteria including RMSE, R^2 , PAE, MAE, and BIAS. The results showed that the capability of the infiltration models to forecast infiltration rate differed based on the soil texture. The Noval model was the most precise for the clay loam soils. The modified Kostiakov model provided accurate estimations for the loam soils, while the revised modified Kostiakov model was the best for the sandy loam soils. In addition, Horten model outperformed all other models for both the sandy loam and clay soils.

Zakwan (2017) assessed the precision of the Kostiakov model in predicting the infiltration rate for five soil kinds, including sandy soils. He used the Excel solver which is a nonlinear optimization technique available in Microsoft Office Excel. This technique presented more accurate results in computing cumulative infiltration than the graphical method, which requires a long time to fit curves. Sharief et al. (2023) utilized an Excel solver tool to predict infiltration rates in a region of India experiencing severe water shortage. They considered published infiltration data from the literature, applying the most common infiltration models such as Green-Ampt, Horten, Kostiakov, Philip, and Soil Conservation Services (SCS) to predict infiltration rates. Both Excel solver and traditional graphical methods were used to derive the models' parameters. Some statistical indices were used to evaluate the performance of the infiltration models, such as MAE (maximum absolute error), SSE (sum of squared error), MARE (mean absolute relative error), WI (Wilmette index), and NSE (Nash-Sutcliffe efficiency). It was noted that the Horten model had the best performance in the study area, while the Green-Ampt model had the worst performance. Moreover, compared to the

graphical method, SSE was reduced by 39.55%, 70.48%, and 87.39% for the Horten, Kostiakov, and SCS models, respectively, when using the Excel solver tool.

Traditional infiltration models face significant challenges in representing soil properties that impact water movement. Therefore, the current research aims to introduce a new concept for modeling water infiltration in sandy loam soils. This concept integrates physical soil properties such as bulk density and initial moisture content with infiltration model parameters to find the cumulative infiltration based on laboratory data from infiltration experiments. Furthermore, from the knowledge of the cumulative infiltration, an equation is proposed to estimate the wetting front depth.

2 Materials and methods

2.1 General methodology of laboratory experiments

The laboratory experiments for this research were carried out at the Soil Water Physics and Management Laboratory, Department of Dams and Water Resources Engineering, College of Engineering, University of Mosul, Mosul, Iraq. Coarse-textured soil was selected and conditioned through air-drying, crushing, and sifting using a 2 mm sieve. The soil was then collected and mixed well to obtain a homogeneous soil and divided into three parts: the first part was left to air-dry, the second part was moistened to the maximum possible moisture without impeding monitoring of the wetting front's progress, and the final part was moistened to a moisture level intermediate between the first and second parts. The initial moisture content of soil samples was monitored by choosing random samples of known weight, drying them in an electric oven for 24 hours at 105°C, and then reweighing them. Moisture content was calculated from the difference in weight before and after drying. Once achieving minimal variations in moisture content values across the samples, each soil part was packed in airtight plastic bags to maintain a constant and homogeneous

initial moisture content. Soil texture was determined, and the percentages of sand, silt, and clay were identified using a hydrometer, and the soil was

Table 1 Physical properties of the soil used in the experiments

Soil Texture	Sand%	Silt%	Clay%	Gravimetric Initial Moisture%
Sandy Loam	75	20	5	1.5
				3.44
				4.95

To perform laboratory experiments and observe the wetting front advance and cumulative infiltration, two transparent cylinders with a height of 63 cm and an internal diameter of 11 cm were used. Each cylinder contained three vertical rows of holes, each 2 mm in

classified according to the USDA Texture Triangle. Table 1 presents the percentages of soil fractions and the initial moisture content of each part.

diameter, spaced 2 cm apart along the soil column. There is also a series of holes of the same diameter randomly distributed at the base of the cylinder to allow soil aeration and facilitate air movement during testing. A graduated ruler made of adhesive paper was attached vertically to the side of the cylinder to measure the progress of the wetting front. The water application system consists of two transparent cylinders with an internal diameter of 4.4 cm and a total capacity of 1 liter. A graduated ruler is attached to the cylinder wall to measure changes in water level (Figure 1).



Figure 1 The transparent cylinders used in the laboratory experiments

2.2 Infiltration tests

A soil profile was constructed with a height of 52 cm, where the soil was compressed into cylinders in layers, each 5 cm thick, based on the weight of soil

determined by the target bulk density and initial soil moisture content. Nine laboratory experiments were conducted with three different bulk densities for each moisture content, as shown in Table 2.

Table 2 Details of laboratory experiments

Experiment no.	Initial moisture content %	Bulk density g cm ⁻³
1	1.5	1.49
2	1.5	1.6
3	1.5	1.7
4	3.44	1.49
5	3.44	1.6
6	3.44	1.7
7	4.95	1.49
8	4.95	1.6
9	4.95	1.7



Figure 2 Wetting front progress

2.3 Summarized overview of infiltration models adopted for the current study

Six infiltration models (Horten, Philip, Kostiakov standard, modified Kostiakov, revised modified Kostiakov, Soil Conservation Services SCS) were used. A brief description of these models is given below:

Kostiakov model (Kostiakov, 1932):

$$D = ct^m \quad (1)$$

Where,

D = cumulative infiltration depth (mm);

t = infiltration time (min);

m, c = empirical constants.

The value of power m is usually greater than zero but less than one.

Modified Kostiakov model (Smith, 1972):

$$D = ft^m + Pt \quad (2)$$

Where,

P = basic infiltration rate (mm min⁻¹);

f and m = empirical constants.

Revised modified Kostiakov model (Parhi et al., 2007):

$$D = \alpha_1 t^{\beta_1} + \alpha_2 t^{-\beta_2} \quad (3)$$

Where,

$\alpha_1, \beta_1, \alpha_2, \beta_2$ = empirical constants.

Philip's model (Philip, 1958):

$$D = At^{0.5} + Ft \quad (4)$$

Where,

A, F = constants;

A represents the sorptivity;

F is the transmissivity coefficient, which depends on soil properties as well as initial and saturated moisture content.

Horton model (Horton, 1940):

$$D = i_f t + i_o (1 - e^{-\beta t}) \quad (5)$$

Where,

i_0 = initial infiltration rate (mm min^{-1});

i_f = final or basic (steady-state) infiltration rate (mm min^{-1});

β = decay constant, which controls the rate at which the infiltration capacity decreases over time. Or,

$$D = at + b(1 - e^{-ct}) \quad (6)$$

SCS Infiltration model (Jury et al., 1991):

$$D = ct^m + 0.6985 \quad (7)$$

Where,

m and c = empirical constants.

2.4 Determination of infiltration parameters

In order to determine the values of parameters for various infiltration models, Excel Solver was used. It is a built-in tool in Microsoft Excel used to solve optimization problems related to complex engineering issues. Using the Excel Analyzer enables the user to find the optimal value of an objective function in a specific target cell in a worksheet. It can be used to obtain optimal parameters for linear and nonlinear equations in a specified problem. Solver operates on a set of cells called variable cells (decision variables) directly or indirectly linked to the objective function. By adjusting the values of these cells to achieve the best result in the target cell, while adhering to the restrictions imposed on the changeable cells, it automatically calculates the required values to adjust the equations and reach the optimal solution (Barati, 2013).

Based on this, the objective function can be represented as the SSE between the estimated and observed infiltration rates. The infiltration model parameters were considered as decision variables, and a solver was used to find their values. The best infiltration model was identified using three statistical criteria (SSE, R^2 , and RMSE).

$$\text{Objective function SSE} = \sum_{i=1}^n (Y_{obsD} - Y_{evalD})^2 \quad (8)$$

Where,

Y_{obsD} = observed cumulative infiltration depth;

Y_{evalD} = evaluated cumulative infiltration depth;

i = index of observed/ evaluated depth from 1 to n ;

n = total number of observations.

$$R^2 = 1 - \frac{\sum_{i=1}^n (Y_{obsD} - Y_{evalD})^2}{\sum_{i=1}^n (Y_{obsD} - \bar{Y}_{obsD})^2} \quad (9)$$

$\bar{Y}_{obsD}$ = mean of the observed values.

$$\text{RMSE} = \sqrt{\sum_{i=1}^n \frac{1}{n} (Y_{obsD} - Y_{evalD})^2} \quad (10)$$

3 Results and discussion

This study investigates the possibility of developing and proposing an equation to calculate cumulative infiltration based on six infiltration models (Kostiakov, modified Kostiakov, revised modified Kostiakov, Philip, Horten, and SCS). Using the solver tool built into Excel, the results were compared using statistical indicators based on the values of the soil physical properties (moisture content and bulk density) shown in Table 2.

3.1 Influence of bulk density variation on infiltration and wetting depth

Figure 3 presents the variation of cumulative infiltration and depth of wetting front over time under the effect of changes in bulk density at constant moisture content for a sandy loam soil. Figure 3 shows an increase in cumulative infiltration depth and wetting front depth with a decrease in bulk density at the same moisture content for all cases. It is noted from Figure 3A that the cumulative infiltration increases with time when the bulk density decreases for each moisture content. Figure 3B shows an increase in the depth of the wetting front with time as the bulk density of the soil decreases for each moisture content. When the bulk density of the soil decreases, its resistance to water movement decreases, and its cohesion decreases. Thus, it contains a large percentage of pores among its particles, which allows water to pass through more quickly. On the contrary, increasing soil compaction decreases the continuity of the pores, making it difficult for water to infiltrate and move. This is in agreement with Al-Ogaidi et al. (2023), Liu et al. (2013), Guo and Liu (2019), Peng et al. (2019), Cleophas et al. (2022), and Mohammed et al. (2025).

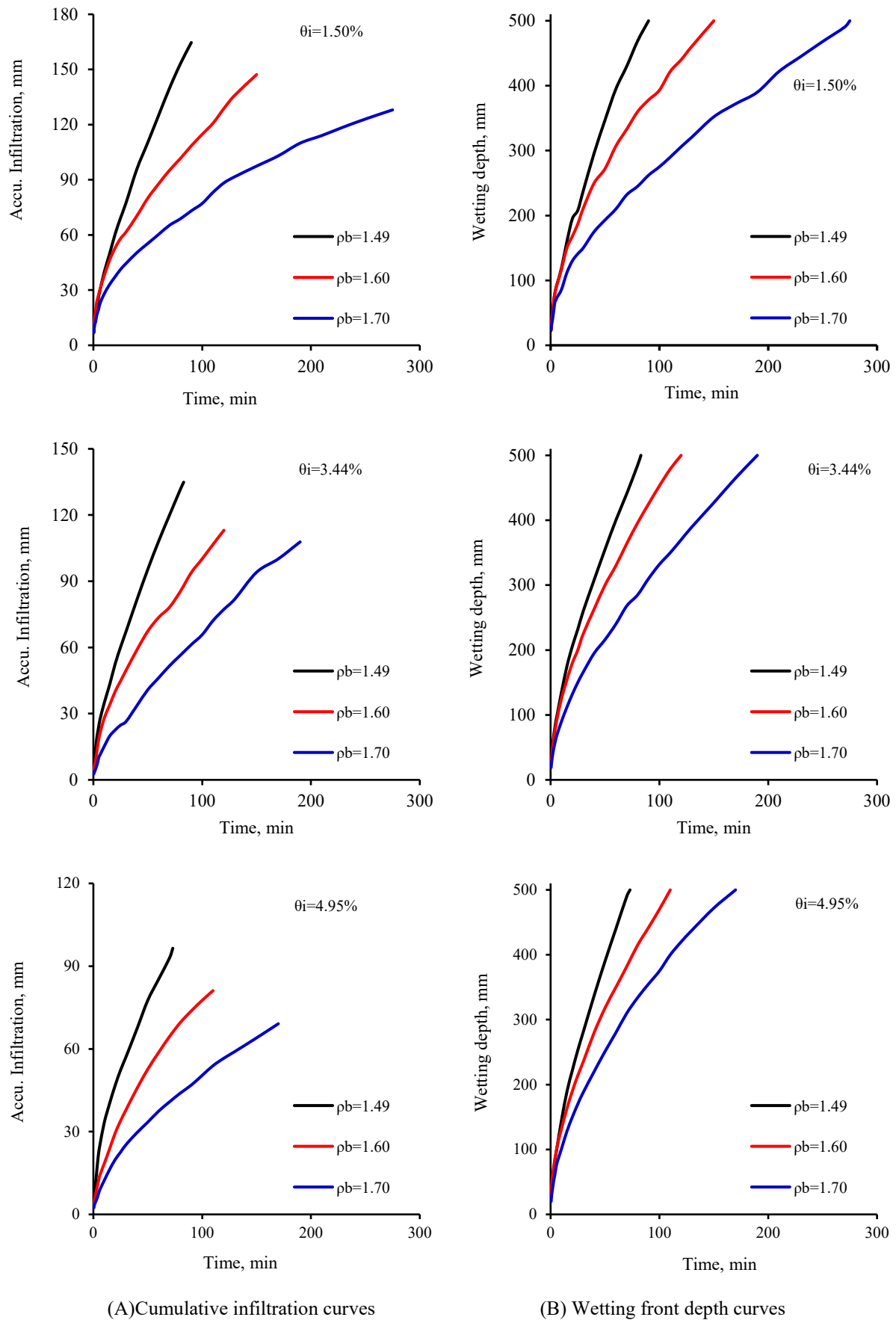
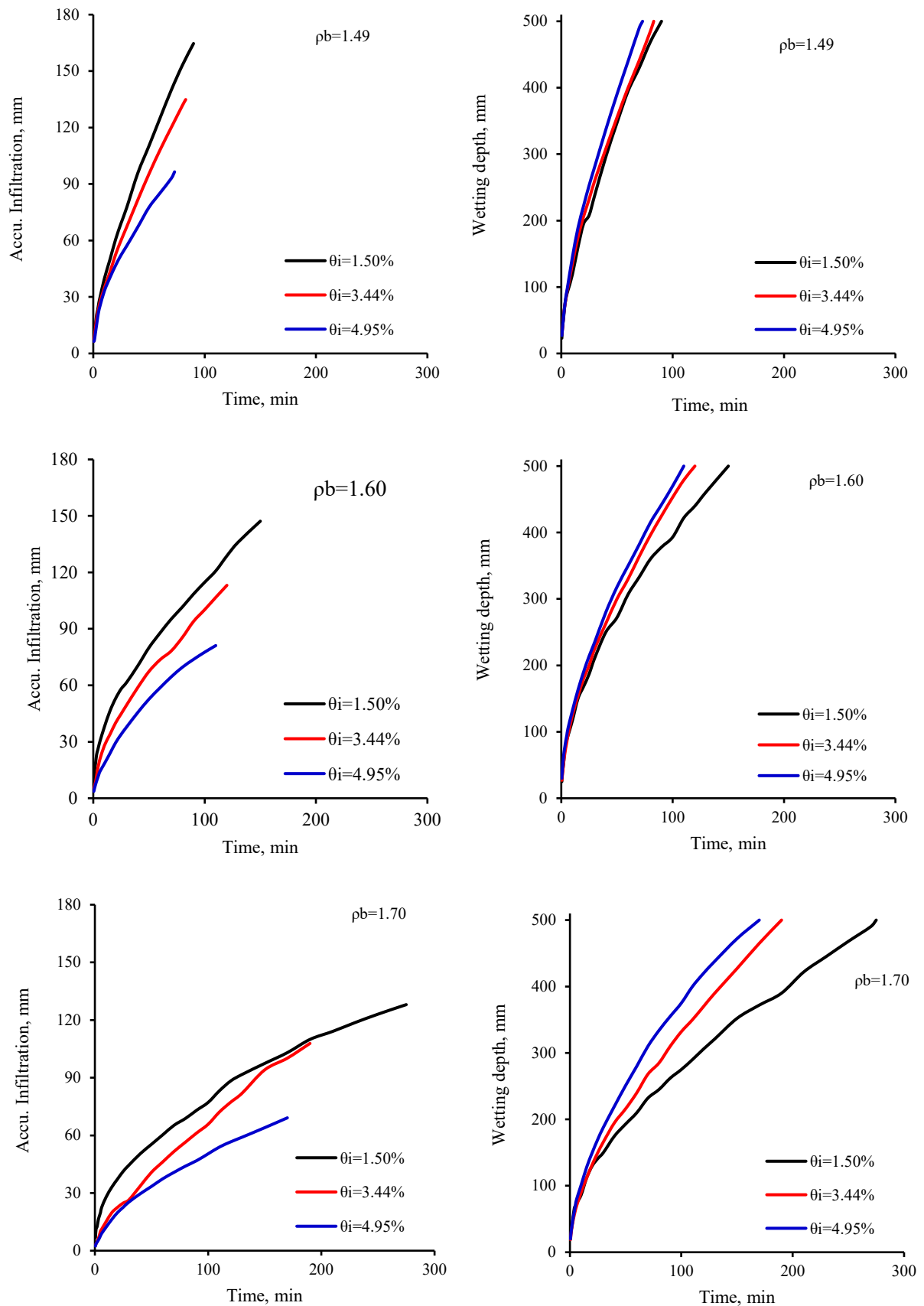


Figure 3 Variations in the wetting front depth and cumulative infiltration over time under different bulk density and initial moisture content



(A) Cumulative infiltration curves

(B) Wetting front depth curves

Figure 4 Variations in the wetting front depth and cumulative infiltration over time under different bulk density and initial moisture contents

3.2 Influence of initial moisture content variation on infiltration and wetting depth

The variation in the wetting front depth and the cumulative infiltration over time under the influence of the initial soil moisture content and at a constant bulk density is illustrated in Figure 4. As shown in Figure 4A, the cumulative infiltration declines with increasing moisture content at the same bulk density. In all cases, the increase is larger as the moisture reduces, as a result of the reduction in the hydraulic gradient at the soil surface, and consequently, the force causing water infiltration into the soil weakens. In contrast, Figure 4B shows an increase in wetting front depth with higher initial soil moisture content at the same bulk density. This is due to the low capillary force, which allows water to move easily downwards. As the moisture content increases, soil particles absorb water, expanding in volume. This contributes to decreased soil porosity, which reduces the soil's ability to absorb water. In addition, when the surface soil is saturated, it consumes a small amount of moisture, and thus a large part of it seeps deep downwards, as confirmed by researchers (Zhang et al., 2013; Liu et al., 2019; Liu et al., 2011; Al-Jwari and Al-Jubouri, 2024).

Table 3 illustrates the impacts of both initial moisture content and bulk density on the wetting front depth and cumulative infiltration at a common infiltration time of 70 min. It is obvious that bulk density has a more significant impact on decreasing both the wetting front depth and cumulative infiltration

compared to the effect of moisture content. The greatest decrease in cumulative infiltration occurred when the bulk density increased from 1.49 to 1.7 g cm⁻³, at a moisture content of 3.44%, with a decrease of 56.9%. On the other hand, the lowest decrease in cumulative infiltration depth was 30.6% at a moisture content of 4.95%, when the bulk density increased from 1.49 to 1.6 g cm⁻³. For the wetting front advance, the lowest decrease was recorded at 17.5% at a moisture content of 3.44%, with the bulk density increasing from 1.49 to 1.6 g cm⁻³. The highest decrease in wetting front advance was recorded at a moisture content of 1.5%, with the bulk density increasing from 1.49 to 1.7 g cm⁻³, with a value of 46%.

Furthermore, Table 3 shows that the effect of moisture content is less pronounced on wetting front and cumulative infiltration. The highest decrease in cumulative infiltration depth was 37.5% at bulk density 1.7 g cm⁻³ with increasing moisture content from 1.5% to 4.95%. The smallest decrease was 14.2% when increasing moisture content from 1.5% to 3.44% at a bulk density of 1.49 g cm⁻³. In contrast, an increase in the advance of the wetting front was observed with increasing initial moisture content. The highest increase was 33.6% at moisture content 1.7 g cm⁻³ with increasing bulk density from 1.5% to 4.95%. The smallest increase was 2.6% when increasing moisture content from 1.5% to 3.44% at bulk density 1.49 g cm⁻³.

Table 3 Influence of bulk density and moisture content on cumulative infiltration and wetting front depth

θ_i						ρ_b g cm ⁻³					
	ρ_b g cm ⁻³	D mm	Decr. %	Z mm	Decr. %		θ_i %	D mm	Decr. %	Z mm	Incr. %
1.5	1.49	139.84		430		1.49	1.5	139.84		430	
	1.6	95.2	31.9	333	22.6		3.44	120	14.2	441	2.6
	1.7	65.28	53.3	232	46		4.95	92.96	33.5	490	14
3.44	1.49	120		441		1.6	1.5	95.2		333	
	1.6	78.28	34.8	364	17.5		3.44	78.24	17.8	364	9.3
	1.7	51.68	56.9	268	39.2		4.95	64.48	32.3	382	14.7
4.95	1.49	92.96		490		1.7	1.5	65.28		232	
	1.6	64.48	30.6	382	22		3.44	51.68	20.8	268	15.5
	1.7	40.8	56.1	310	36.7		4.95	40.8	37.5	310	33.6

3.3 Modelling water infiltration

Table 4 presents the parameter values of the six infiltration models referred to in this study (Kostiakov, modified Kostiakov, revised modified Kostiakov, Philip, Horten, SCS), which were estimated using the solver tool of the Microsoft Excel program. This was based on the data of the nine experiments, which include three levels of moisture content and three bulk densities. This was done by combining measured infiltration data (time, cumulative infiltration) with infiltration models, and determining the optimal values for the parameters.

For Kostiakov's model, parameter c ranged from 2.47 to 11.65, while the parameter m 's value ranged from 0.476 to 0.719. According to Ogbe et al. (2011), parameter m has a positive value between 0 and 1. As for the modified Kostiakov model, coefficients p , m , and f ranged from 3.07 to 14.04, 0.387 to 0.696, and -

0.355 to 0.786, respectively. For the revised modified Kostiakov model, the four parameters α_1 , β_1 , α_2 , and β_2 ranged from 5.27E-08 to 5.48, 0.538 to 3.84, 0.73 to 13.65, and 0.227 to 0.604, respectively. Although these parameters have no physical significance, they help improve the model's consistency with experimental data.

In the Philip model, the sorptivity coefficient A ranged from 3.42 to 11.35, and the hydraulic conductivity coefficient F ranged from -0.0312 to 0.784. Similarly, the values of a , b , and c for the Horten model ranged from 0.3 to 1.53, 14.37 to 42.21, and -0.2415 to -0.0648, respectively. For the SCS model, the c ranged from 2.3 to 11.26, while the values of m ranged from 0.483 to 0.732. Note that the coefficients of the infiltration functions did not undergo a specific change, as they increase or decrease in a certain pattern.

Table 4 Values of the coefficients of the infiltration models considered

Models	Coefficients	$\theta_i = 1.50\%$			$\theta_i = 3.44\%$			$\theta_i = 4.95\%$		
		$\rho b = 1.49 \text{ g cm}^{-3}$	$\rho b = 1.6 \text{ g cm}^{-3}$	$\rho b = 1.7 \text{ g cm}^{-3}$	$\rho b = 1.49 \text{ g cm}^{-3}$	$\rho b = 1.6 \text{ g cm}^{-3}$	$\rho b = 1.7 \text{ g cm}^{-3}$	$\rho b = 1.49 \text{ g cm}^{-3}$	$\rho b = 1.6 \text{ g cm}^{-3}$	$\rho b = 1.7 \text{ g cm}^{-3}$
K	c	8.82	11.65	8.84	7.96	6.90	2.47	8.82	4.87	3.23
	m	0.6490	0.4996	0.4767	0.6375	0.5805	0.7189	0.5549	0.6037	0.5974
	f	10.05	14.04	9.65	9.34	7.18	3.55	8.81	4.59	3.07
MK	m	0.4995	0.3875	0.4395	0.4639	0.5462	0.4814	0.5570	0.6965	0.6365
	p	0.7859	0.3221	0.0575	0.7543	0.1123	0.3434	-0.0101	-0.3556	-0.0693
RMK	α_1	1.96	0.00	5.48	0.15	0.00	0.47	0.21	4.13	0.73
	β_1	0.8761	2.0101	0.5380	1.2543	3.8398	0.9594	0.5564	0.6036	0.5980
	α_2	9.04	13.65	4.78	9.73	7.32	3.48	8.62	0.73	2.50
	β_2	0.4366	0.4442	0.2271	0.5199	0.5620	0.4527	0.5549	0.6043	0.5972
P	A	10.04	11.35	8.27	8.93	7.68	3.42	9.31	5.58	3.82
	F	0.7842	0.0302	-0.0312	0.6440	0.2337	0.3282	0.2246	0.2305	0.1196
H	a	1.53	0.74	0.33	1.34	0.71	0.51	0.93	0.52	0.30
	b	31.50	40.34	42.21	26.19	29.54	14.37	29.12	26.19	19.88
SCS	c	-0.1986	-0.2019	-0.0904	-0.2415	-0.1305	-0.1286	-0.1901	-0.0778	-0.0648
	c	8.50	11.26	8.51	7.63	6.57	2.30	8.42	4.55	2.96
	m	0.6565	0.5058	0.4828	0.6463	0.5898	0.7324	0.5645	0.6165	0.6131

Table 5 shows the values resulting from evaluating the performance of the six infiltration models using three statistical criteria (SSE, R^2 , RMSE) to evaluate the accuracy of the models in representing the data for the nine experiments. To compare the performance of the infiltration models, a column was added to Table 5 that gives one value for each statistical criterion representing the nine experiments. The findings

indicate a strong agreement between the predicted and observed values from laboratory experiments for the cumulative infiltration. They also show that the revised modified Kostiakov model outperformed other models, as it achieved the lowest sum of squared errors with a value of 152.5 and the highest coefficient of determination with a value of 0.9995. It also achieved a clear decrease in the root mean square error (0.882),

which indicates a clear match between the observed and estimated values, due to its multiple parameters contributing to a more accurate representation of

laboratory data. The weakest performance was for the Horten model with high SSE and RMSE of (978.8), (2.2347), respectively, and R^2 of (0.9967).

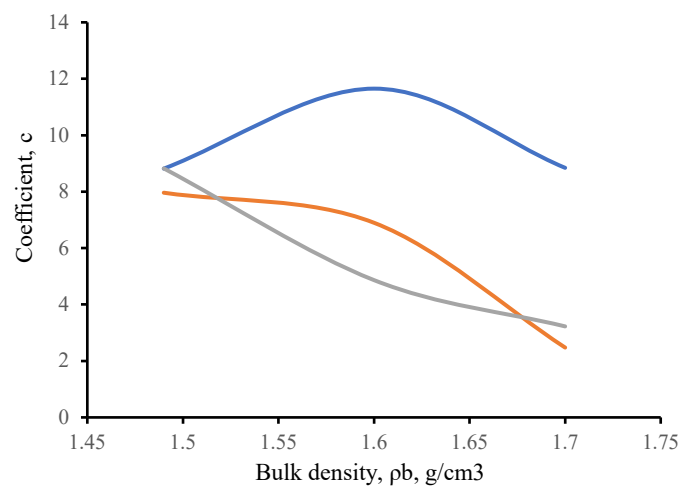
Table 5 Values of statistical criteria for the considered infiltration models

Models	Statistical criteria	$\theta_i = 1.50\%$			$\theta_i = 3.44\%$			$\theta_i = 4.95\%$			For all cases
		$\rho_b = 1.49 \text{ g cm}^{-3}$	$\rho_b = 1.6 \text{ g cm}^{-3}$	$\rho_b = 1.7 \text{ g cm}^{-3}$	$\rho_b = 1.49 \text{ g cm}^{-3}$	$\rho_b = 1.6 \text{ g cm}^{-3}$	$\rho_b = 1.7 \text{ g cm}^{-3}$	$\rho_b = 1.49 \text{ g cm}^{-3}$	$\rho_b = 1.6 \text{ g cm}^{-3}$	$\rho_b = 1.7 \text{ g cm}^{-3}$	
K	SSE	44.73	111.84	56.91	48.89	32.17	57.39	17.72	12.86	4.37	386.9
	R^2	0.9990	0.9973	0.9987	0.9985	0.9987	0.9978	0.9988	0.9991	0.9996	0.9987
	RMSE	1.5764	2.2051	1.3773	1.6481	1.2378	1.5151	1.0211	0.8017	0.4267	1.405
MK	SSE	6.86	26.39	45.56	7.79	30.10	27.38	17.72	8.04	3.46	173.3
	R^2	0.9999	0.9994	0.9990	0.9998	0.9988	0.9990	0.9988	0.9994	0.9997	0.9994
	RMSE	0.6172	1.0711	1.2324	0.6578	1.1972	1.0465	1.0209	0.6340	0.3798	0.940
RMK	SSE	6.60	13.53	41.80	6.99	21.29	27.33	17.72	12.86	4.37	152.5
	R^2	0.9999	0.9997	0.9991	0.9998	0.9992	0.9990	0.9988	0.9991	0.9996	0.9995
	RMSE	0.6056	0.7669	1.1804	0.6230	1.0068	1.0456	1.0211	0.8017	0.4267	0.882
P	SSE	6.86	105.87	72.21	9.77	34.03	27.61	21.91	31.21	15.00	324.5
	R^2	0.9999	0.9974	0.9984	0.9997	0.9987	0.9990	0.9985	0.9977	0.9986	0.9989
	RMSE	0.6172	2.1455	1.5514	0.7368	1.2729	1.0510	1.1352	1.2491	0.7907	1.287
H	SSE	92.40	155.84	495.74	45.98	33.22	56.01	20.89	58.37	20.35	351.3
	R^2	0.9980	0.9962	0.9889	0.9986	0.9987	0.9979	0.9986	0.9957	0.9981	0.9967
	RMSE	2.2657	2.6030	4.0650	1.5982	1.2577	1.4969	1.1086	1.7084	0.9207	2.2347
SCS	SSE	35.73	102.77	51.05	40.79	33.14	46.65	19.36	14.93	6.91	978.8
	R^2	0.9992	0.9975	0.9989	0.9987	0.9987	0.9982	0.9987	0.9989	0.9994	0.9988
	RMSE	1.4089	2.1138	1.3044	1.5053	1.2563	1.3659	1.0671	0.8639	0.5365	1.339

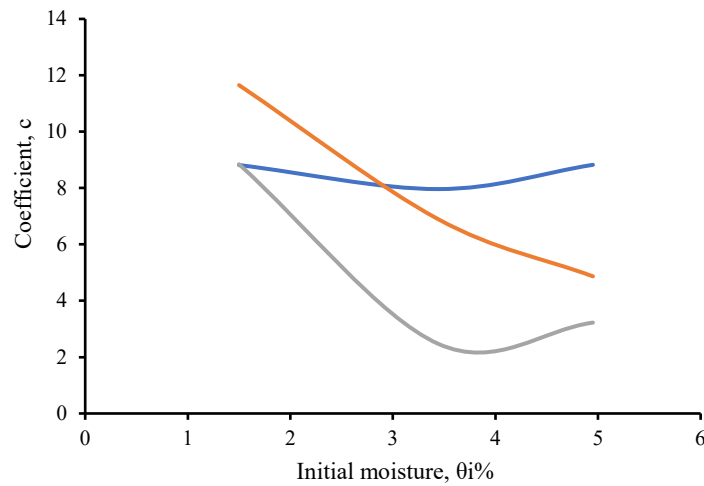
3.4 Proposed model for water infiltration and wetting front

Due to the variation of the infiltration function coefficients between increases and decreases, it was observed that they follow a pattern similar to a

quadratic function, as shown in Figure 5 for the coefficient c of the Kostakov function, taken here as an example. The figure shows the change of the coefficient c with variations in moisture content and bulk density.



(A) Bulk density



(B) Initial moisture

Figure 5 Change in the coefficient c of the Kostiakov function with the bulk density for all moisture contents and the change of c with the moisture content for all bulk densities

Table 6 Coefficients of the enhanced Kostiakov's model

$D = (a_1\rho_b^2 + a_2\rho_b + a_3)t^{(a_4\rho_b^2 + a_5\rho_b + a_6)}$							
$\theta_i = 1.50\%$	a ₁	-249.89	a ₄	6.09542	SSE	310.55	
	a ₂	798.683	a ₅	-20.322	R ²	0.9977	
	a ₃	-627.16	a ₆	17.419	RMSE	0.1185	
$\theta_i = 3.44\%$	a ₁	-15.014	a ₄	3.55746	SSE	217.52	
	a ₂	18.2622	a ₅	-10.899	R ²	0.9975	
	a ₃	14.9876	a ₆	8.95321	RMSE	0.1248	
$\theta_i = 4.95\%$	a ₁	99.259	a ₄	-1.808	SSE	47.14	
	a ₂	-346.43	a ₅	6.09497	R ²	0.9989	
	a ₃	305.098	a ₆	-4.5244	RMSE	0.1280	
$D = (b_1\theta_i^2 + b_2\theta_i + b_3)t^{(b_4\theta_i^2 + b_5\theta_i + b_6)}$							
$\rho_b=1.49 \text{ g cm}^{-3}$	b ₁	0.29276	b ₄	-0.0141	SSE	111.347	
	b ₂	-1.8861	b ₅	0.06395	R ²	0.99887	
	b ₃	10.9863	b ₆	0.58491	RMSE	1.44944	
$\rho_b=1.6 \text{ g cm}^{-3}$	b ₁	0.3201	b ₄	-0.0076	SSE	156.866	
	b ₂	-4.0314	b ₅	0.07949	R ²	0.9983	
	b ₃	16.9774	b ₆	0.39754	RMSE	1.56558	
$\rho_b=1.7 \text{ g cm}^{-3}$	b ₁	1.09568	b ₄	-0.0595	SSE	118.665	
	b ₂	-8.696	b ₅	0.41896	R ²	0.99879	
	b ₃	19.4231	b ₆	-0.0178	RMSE	1.2256	
$D = (c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b)t^{(c_7+c_8\theta_i^2+c_9\theta_i+c_{10}\rho_b^2+c_{11}\rho_b+c_{12}\theta_i\rho_b)}$							
c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	SSE	1085.38
4.3991	0.6459	4.8548	0.2788	6.9884	-6.3801	R ²	0.9963
c ₇	c ₈	c ₉	c ₁₀	c ₁₁	c ₁₂	RMSE	2.3532
3.0052	-0.0254	-0.2308	0.3335	-2.2061	0.2541		

On this basis, quadratic Equation 11 was proposed to express the change in coefficients with bulk density, and Equation 12 to express the change in coefficients with moisture content.

$$\text{coefficient} = a_1\rho_b^2 + a_2\rho_b + a_3 \quad (11)$$

$$\text{coefficient} = b_1\theta_i^2 + b_2\theta_i + b_3 \quad (12)$$

The equation below expresses the coefficient as a function of both moisture content and bulk density.

$$\text{coefficient} = c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b \quad (13)$$

An equation was developed to calculate the cumulative infiltration depth by integrating infiltration model parameters with physical soil properties such as bulk density and initial moisture content. Tables 6, 7,

8, 9, 10, and 11 present the representation of the parameters of these models in the form of quadratic mathematical functions, that depend first on the bulk density with constant moisture content, and second on the moisture content when the bulk density is constant. Finally, the model parameters were considered as quadratic functions of both bulk density and moisture content, and the resulting effect of their interaction. Statistical criteria were also found for each case. The revised and modified Kostiakov models outperformed the other models, achieving the highest R² value of 0.998 and the lowest SSE and RMSE of 577.4 and 1.716, respectively.

Table 7 Coefficients of the enhanced Modified Kostiakov Model

$D = (a_1\rho_b^2 + a_2\rho_b + a_3)t^{(a_4\rho_b^2 + a_5\rho_b + a_6)} + (a_7\rho_b^2 + a_8\rho_b + a_9)t$								
$\theta_i=1.50\%$	a ₁	-295.658	a ₄	6.1824	a ₇	5.6686	SSE	96.336
	a ₂	940.0632	a ₅	-19.9667	a ₈	-21.588	R ²	0.9993
	a ₃	-733.812	a ₆	16.5123	a ₉	20.3729	RMSE	1.1648
$\theta_i=3.44\%$	a ₁	-67.5712	a ₄	-5.3613	a ₇	32.6060	SSE	68.3694
	a ₂	187.4035	a ₅	17.1854	a ₈	-105.83	R ²	0.9992
	a ₃	-119.757	a ₆	-13.2343	a ₉	86.0128	RMSE	1.0336
$\theta_i=4.95\%$	a ₁	87.7710	a ₄	-7.0366	a ₇	23.7327	SSE	30.7217
	a ₂	-306.549	a ₅	22.7789	a ₈	-75.79	R ²	0.9993
	a ₃	270.5031	a ₆	-17.7485	a ₉	60.185	RMSE	0.7097
$D = (b_1\theta_i^2 + b_2\theta_i + b_3)t^{(b_4\theta_i^2 + b_5\theta_i + b_6)} + (b_7\theta_i^2 + b_8\theta_i + b_9)t$								
$\rho_b=1.49 \text{ g cm}^{-3}$	b ₁	0.0046	b ₄	0.0232	b ₇	-0.1420	SSE	32.3628
	b ₂	-0.3875	b ₅	-0.1330	b ₈	0.6853	R ²	0.9997
	b ₃	10.6201	b ₆	0.6469	b ₉	0.0773	RMSE	0.7814
$\rho_b=1.6 \text{ g cm}^{-3}$	b ₁	0.5296	b ₄	0.0051	b ₇	-0.0585	SSE	64.5258
	b ₂	-6.1541	b ₅	0.0569	b ₈	0.1808	R ²	0.9993
	b ₃	22.0787	b ₆	0.2909	b ₉	0.1823	RMSE	1.0041
$\rho_b=1.7 \text{ g cm}^{-3}$	b ₁	0.8205	b ₄	0.0235	b ₇	-0.122	SSE	76.4012
	b ₂	-7.199	b ₅	-0.0942	b ₈	0.7498	R ²	0.9992
	b ₃	18.5996	b ₆	0.5280	b ₉	-0.7928	RMSE	0.9834
$D = (c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b)t^{(c_7 + c_8\theta_i^2 + c_9\theta_i + c_{10}\rho_b^2 + c_{11}\rho_b + c_{12}\theta_i\rho_b)} + (c_{13} + c_{14}\theta_i^2 + c_{15}\theta_i + c_{16}\rho_b^2 + c_{17}\rho_b + c_{18}\theta_i\rho_b)t$								
c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	SSE	719.2689	
4.3601	0.7560	5.6984	-0.0954	8.6655	-7.3857			
c ₇	c ₈	c ₉	c ₁₀	c ₁₁	c ₁₂	R ²	0.9975	
0.8832	-0.0216	-0.0937	-0.4155	0.321	0.1474			
c ₁₃	c ₁₄	c ₁₅	c ₁₆	c ₁₇	c ₁₈	RMSE	1.9157	
-0.0722	-0.0266	-0.2656	-0.4673	0.7338	0.2872			

Table 8 Coefficients of the enhanced Revised Modified Kostiakov Model

$D = (a_1\rho_b^2 + a_2\rho_b + a_3)t^{(a_4\rho_b^2+a_5\rho_b+a_6)} + (a_7\rho_b^2 + a_8\rho_b + a_9)t^{(a_{10}\rho_b^2+a_{11}\rho_b+a_{12})}$										
0i=1.50%	a ₁	-0.1266	a ₄	7.6459	a ₇	-0.2413	a ₁₀	-1.2254	SSE	208.768
	a ₂	0.4143	a ₅	-26.456	a ₈	-4.0065	a ₁₁	3.334	R ²	0.9984
	a ₃	-0.3072	a ₆	24.029	a ₉	18.3339	a ₁₂	-1.7135	RMSE	1.7148
0i=3.44%	a ₁	-0.0005	a ₄	-0.8445	a ₇	-0.0005	a ₁₀	0.974	SSE	73.0745
	a ₂	-3.5459	a ₅	0.9378	a ₈	-25.376	a ₁₁	-0.0298	R ²	0.9992
	a ₃	9.3712	a ₆	1.2347	a ₉	43.8796	a ₁₂	-1.8629	RMSE	1.0685
0i=4.95%	a ₁	-0.0005	a ₄	-1.8278	a ₇	-0.0005	a ₁₀	-0.0595	SSE	40.4997
	a ₂	-7.2855	a ₅	1.4028	a ₈	-21.552	a ₁₁	0.0073	R ²	0.9990
	a ₃	10.6001	a ₆	2.9388	a ₉	40.5992	a ₁₂	0.7221	RMSE	0.8148
$D = (b_1\theta_i^2 + b_2\theta_i + b_3)t^{(b_4\theta_i^2+b_5\theta_i+b_6)} + (b_7\theta_i^2 + b_8\theta_i + b_9)t^{(b_{10}\theta_i^2+b_{11}\theta_i+b_{12})}$										
ρb=1.49 g cm ⁻³	b ₁	0.2815	b ₄	-0.190	b ₇	-0.3165	b ₁₀	-0.0057	SSE	31.3142
	b ₂	-2.3252	b ₅	1.1356	b ₈	1.9189	b ₁₁	0.073	R ²	0.9997
	b ₃	4.8182	b ₆	-0.399	b ₉	6.8736	b ₁₂	0.3425	RMSE	0.7687
ρb=1.6 g cm ⁻³	b ₁	0.7933	b ₄	-0.8945	b ₇	-0.3173	b ₁₃	-0.0094	SSE	54.9396
	b ₂	-3.9192	b ₅	5.362	b ₈	-1.6978	b ₁₄	0.1073	R ²	0.9994
	b ₃	4.0949	b ₆	-4.0200	b ₉	16.911	b ₁₅	0.3046	RMSE	0.9265
ρb=1.7 g cm ⁻³	b ₁	0.7983	b ₄	-0.1324	b ₇	0.0046	b ₁₆	-0.006	SSE	73.5090
	b ₂	-6.5252	b ₅	0.87112	b ₈	-0.6906	b ₁₇	0.1458	R ²	0.9925
	b ₃	13.4693	b ₆	-0.4709	b ₉	5.8023	b ₁₈	0.0218	RMSE	0.9646
$D = (c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b)t^{(c_7+c_8\theta_i^2+c_9\theta_i+c_{10}\rho_b^2+c_{11}\rho_b+c_{12}\theta_i\rho_b)} + (c_{13} + c_{14}\theta_i^2 + c_{15}\theta_i + c_{16}\rho_b^2 + c_{17}\rho_b + c_{18}\theta_i\rho_b)t^{(c_{19}+c_{20}\theta_i^2+c_{21}\theta_i+c_{22}\rho_b^2+c_{23}\rho_b+c_{24}\theta_i\rho_b)}$										
c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	SSE		577.3968		
4.9055	0.3671	6.5577	-7.8848	6.5577	-1.813					
c ₇	c ₈	c ₉	c ₁₀	c ₁₁	c ₁₂	R ²		0.9980		
-0.9314	-0.0269	1.3816	-1.1619	1.3816	0.1098					
c ₁₃	c ₁₄	c ₁₅	c ₁₆	c ₁₇	c ₁₈	RMSE		1.7164		
4.9795	0.5923	7.204	-4.9740	7.204	-2.9845					
c ₁₉	c ₂₀	c ₂₁	c ₂₂	c ₂₃	c ₂₄					
-7.5387	-0.0502	4.4462	-2.6287	4.4462	0.2260					

Table 9 Coefficients of the enhanced Philip's Model

$D = (a_1\rho_b^2 + a_2\rho_b + a_3)t^{0.5} + (a_4\rho_b^2 + a_5\rho_b + a_6)t$							
$\theta_i = 1.50\%$	a ₁	-9.0974	a ₄	11.5311	SSE	446.781	
	a ₂	14.1608	a ₅	-39.9181	R ²	0.9966	
	a ₃	10.7089	a ₆	34.4904	RMSE	2.5085	
$\theta_i = 3.44\%$	a ₁	-9.7909	a ₄	11.7021	SSE	332.9963	
	a ₂	13.1334	a ₅	-39.8025	R ²	0.9962	
	a ₃	10.3483	a ₆	34.0907	RMSE	2.2810	
$\theta_i = 4.95\%$	a ₁	-9.8441	a ₄	11.9462	SSE	265.0567	
	a ₂	13.0533	a ₅	-39.8233	R ²	0.9936	
	a ₃	10.3093	a ₆	33.2802	RMSE	2.0845	
$D = (b_1\theta_i^2 + b_2\theta_i + b_3)t^{0.5} + (b_4\theta_i^2 + b_5\theta_i + b_6)t$							
$\rho_b=1.49\text{ g cm}^{-3}$	b ₁	0.2399	b ₄	-0.05957	SSE	38.5358	
	b ₂	-1.76	b ₅	0.2220	R ²	0.9996	
	b ₃	12.1442	b ₆	0.5852	RMSE	0.8527	
$\rho_b=1.6\text{ g cm}^{-3}$	b ₁	0.1433	b ₄	-0.0310	SSE	171.1032	
	b ₂	-2.5961	b ₅	0.2582	R ²	0.9982	
	b ₃	14.9172	b ₆	-0.2872	RMSE	1.6351	
$\rho_b=1.7\text{ g cm}^{-3}$	b ₁	0.8033	b ₄	-0.0938	SSE	114.8261	
	b ₂	-6.4710	b ₅	0.6484	R ²	0.9988	
	b ₃	16.1698	b ₆	-0.7928	RMSE	1.2056	
$D = (c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b)t^{0.5} + (c_7 + c_8\theta_i^2 + c_9\theta_i + c_{10}\rho_b^2 + c_{11}\rho_b + c_{12}\theta_i\rho_b)t$							
c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	SSE	859
0.0025	0.4531	-2.0502	-17.3064	37.4817	-1.0609	R ²	0.997
c ₇	c ₈	c ₉	c ₁₀	c ₁₁	c ₁₂	RMSE	2.093
24.8681	-0.0662	-0.1049	8.052	-28.6684	0.3423		

Table 10 Coefficients of the enhanced Horten model

$D = (a_1\rho_b^2 + a_2\rho_b + a_3)t + (a_4\rho_b^2 + a_5\rho_b + a_6)\left(1 - e^{(a_7\rho_b^2 + a_8\rho_b + a_9)t}\right)$								
$\theta_i=1.50\%$	a ₁	14.6061	a ₄	-276.399	a ₇	5.1099	SSE	745.0985
	a ₂	-52.2383	a ₅	930.2453	a ₈	-15.7969	R ²	0.9944
	a ₃	46.9259	a ₆	-740.501	a ₉	11.9961	RMSE	3.2395
$\theta_i=3.44\%$	a ₁	17.3904	a ₄	-807.604	a ₇	-4.4219	SSE	135.7989
	a ₂	-59.3844	a ₅	2519.409	a ₈	14.6356	R ²	0.99845
	a ₃	51.2079	a ₆	-1934.6	a ₉	-12.2313	RMSE	1.4567
$\theta_i=4.95\%$	a ₁	6.9236	a ₄	-164.319	a ₇	-3.9346	SSE	99.8561
	a ₂	-25.0706	a ₅	478.7877	a ₈	13.1257	R ²	0.9976
	a ₃	22.909	a ₆	-319.147	a ₉	-11.007	RMSE	1.2795
$D = (b_1\theta_i^2 + b_2\theta_i + b_3)t + (b_4\theta_i^2 + b_5\theta_i + b_6)\left(1 - e^{(b_7\theta_i^2 + b_8\theta_i + b_9)t}\right)$								
$\rho_b=1.49 \text{ g cm}^{-3}$	b ₁	-0.0497	b ₄	1.3569	b ₇	0.0163	SSE	159.2712
	b ₂	0.148	b ₅	-9.4378	b ₈	-0.1023	R ²	0.9984
	b ₃	1.4154	b ₆	42.6019	b ₉	-0.0819	RMSE	1.7335
$\rho_b=1.6 \text{ g cm}^{-3}$	b ₁	-0.0304	b ₄	0.9705	b ₇	-0.0005	SSE	247.4316
	b ₂	0.1327	b ₅	-10.3517	b ₈	0.0395	R ²	0.9973
	b ₃	0.6084	b ₆	53.6744	b ₉	-0.2602	RMSE	1.9663
$\rho_b=1.7 \text{ g cm}^{-3}$	b ₁	-0.068	b ₄	5.2198	b ₇	0.0176	SSE	572.1051
	b ₂	0.4279	b ₅	-40.1238	b ₈	-0.1082	R ²	0.9942
	b ₃	-0.1543	b ₆	90.6209	b ₉	0.0312	RMSE	2.6911
$D = (c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b)t + (c_7 + c_8\theta_i^2 + c_9\theta_i + c_{10}\rho_b^2 + c_{11}\rho_b + c_{12}\theta_i\rho_b)\left(1 - e^{(c_{13}+c_{14}\theta_i^2+c_{15}\theta_i+c_{16}\rho_b^2+c_{17}\rho_b+c_{18}\theta_i\rho_b)t}\right)$								
c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	SSE	2019.666	
23.9014	-0.0379	-0.8757	5.3551	-23.1287	0.6595			
c ₇	c ₈	c ₉	c ₁₀	c ₁₁	c ₁₂	R ²	0.9931	
711.2986	1.3824	26.8900	286.4277	-869.055	-25.3916			
c ₁₃	c ₁₄	c ₁₅	c ₁₆	c ₁₇	c ₁₈	RMSE	3.2101	
-0.2819	-0.0029	0.0105	0.0973	-0.0886	0.0076			

Table 11 Coefficients of the enhanced SCS Model

$D = (a_1\rho_b^2 + a_2\rho_b + a_3)t^{(a_4\rho_b^2+a_5\rho_b+a_6)} + 0.6985$							
$\theta_i = 1.50\%$	a ₁	-194.409	a ₄	4.2032	SSE	197.6622	
	a ₂	619.936	a ₅	-14.2233	R ²	0.9985	
	a ₃	-483.394	a ₆	12.5119	RMSE	1.6685	
$\theta_i = 3.44\%$	a ₁	-119.843	a ₄	7.7521	SSE	125.7951	
	a ₂	356.7239	a ₅	-24.3312	R ²	0.9986	
	a ₃	-257.721	a ₆	19.6862	RMSE	1.402	
$\theta_i = 4.95\%$	a ₁	79.2249	a ₄	-1.8558	SSE	41.7205	
	a ₂	-278.309	a ₅	6.141	R ²	0.999	
	a ₃	247.1019	a ₆	-4.4619	RMSE	0.8270	
$D = (b_1\theta_i^2 + b_2\theta_i + b_3)t^{(b_4\theta_i^2+b_5\theta_i+b_6)} + 0.6985$							
$\rho_b=1.49 \text{ g cm}^{-3}$	b ₁	0.2831	b ₄	-0.0142	SSE	95.873	
	b ₂	-1.8487	b ₅	0.0652	R ²	0.9990	
	b ₃	10.6361	b ₆	0.5908	RMSE	1.345	
$\rho_b=1.6 \text{ g cm}^{-3}$	b ₁	0.3141	b ₄	-0.0074	SSE	150.8373	
	b ₂	-3.9699	b ₅	0.0801	R ²	0.9984	
	b ₃	16.5065	b ₆	0.4023	RMSE	1.5352	
$\rho_b=1.7 \text{ g cm}^{-3}$	b ₁	1.0546	b ₄	-0.0602	SSE	104.5999	
	b ₂	-8.412	b ₅	0.4261	R ²	0.999	
	b ₃	18.755	b ₆	-0.0209	RMSE	1.1507	
$D = (c_1 + c_2\theta_i^2 + c_3\theta_i + c_4\rho_b^2 + c_5\rho_b + c_6\theta_i\rho_b)t^{(c_7+c_8\theta_i^2+c_9\theta_i+c_{10}\rho_b^2+c_{11}\rho_b+c_{12}\theta_i\rho_b)} + 0.6985$							
c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	SSE	1034.335
57.8912	0.6472	4.9045	21.3048	-60.3742	-6.4172	R ²	0.9965
c ₇	c ₈	c ₉	c ₁₀	c ₁₁	c ₁₂	RMSE	2.2972
1.4837	-0.0263	-0.2498	-0.2777	-0.2771	0.2710		

This approach's purpose was to reduce the number of equations. Instead of using nine equations for each case of bulk density and moisture content, it was reduced to six equations and then to one comprehensive equation for the six models in terms of both moisture content and bulk density.

A trial was done to propose a new form of infiltration equation that combines the six infiltration models in the following form:

$$D = a + be^{ct} + dt + ft^g \quad (14)$$

Each coefficient of this equation is a function of bulk density and initial moisture content as follows:

$$\text{coefficient} = a_1 + a_2\theta_i^2 + a_3\theta_i + a_4\rho_b^2 + a_5\rho_b \quad (15)$$

Therefore, this proposed equation containing 30 parameters whose values were found using the Solver tool. The results of the statistical criteria SSE, R^2 , and RMSE for this equation were 439.67, 0.9985, and 1.498, respectively. It is noteworthy that this equation exhibits a lower (SSE) compared to the six developed equations presented in the tables above. As shown in Table 12.

$$D = (a_1 + a_2\theta_i^2 + a_3\theta_i + a_4\rho_b^2 + a_5\rho_b) + (a_6 + a_7\theta_i^2 + a_8\theta_i + a_9\rho_b^2 + a_{10}\rho_b)e^{(a_{11} + a_{12}\theta_i^2 + a_{13}\theta_i + a_{14}\rho_b^2 + a_{15}\rho_b)t} + (a_{16} + a_{17}\theta_i^2 + a_{18}\theta_i + a_{19}\rho_b^2 + a_{20}\rho_b)t + (a_{21} + a_{22}\theta_i^2 + a_{23}\theta_i + a_{24}\rho_b^2 + a_{25}\rho_b)t^{(a_{26} + a_{27}\theta_i^2 + a_{28}\theta_i + a_{29}\rho_b^2 + a_{30}\rho_b)} \quad (16)$$

Table 12 Comprehensive equation coefficients for all models

a1	a2	a3	a4	a5		
1.33795	4.512594	-33.3168	3.490031	-0.00181	SSE	439.6711
a6	a7	a8	a9	a10		
-6.96133	-4.17205	29.44038	0.683108	1.063223	R ²	0.998497
a11	a12	a13	a14	a15		
0.078369	0.000105	-0.00363	-0.04257	0.016854	RMSE	1.497738
a16	a17	a18	a19	a20		
3.288643	-0.02105	0.012149	-3.24992	3.285794		
a21	a22	a23	a24	a25		
18.98644	0.222284	-1.70546	-12.7803	16.4254		
a26	a27	a28	a29	a30		
-1.11919	-0.01127	0.09176	0.235649	0.574538		

Due to the difficulty and impossibility of monitoring the progress of the wetting front within the soil profile during the irrigation process, an equation was modeled through which the depth of the wetting front can be estimated from knowledge of the cumulative infiltration depth, as shown below:

$$Z = at^b D^c \rho_b^d \theta_i^e \quad (17)$$

Where,

a, b, c, d, e = equation constants;

t = time;

D = cumulative infiltration depth;

Z = wetting front depth;

ρ_b = bulk density;

θ_i = moisture content.

Using the Solver tool, the values of the equation coefficients were found, and these coefficients achieved good values for the three statistical criteria, as shown in Table 13. It is worth noting that this equation is also useful in field infiltration measurements, which typically allow only infiltration to be estimated, while the advance of the wetting front remains invisible.

Table 13 Wetting front depth equation coefficients

Equation constants		Fitting evaluation parameters	
a	72.62459	SSE	43391.32
b	0.471467	R ²	0.990388
c	0.155651	RMSE	14.88
d	-2.88228		
e	0.214072		

4 Conclusion

Infiltration is one of the fundamental processes in the water cycle. It is important in many aspects of water resources engineering, such as the design and scheduling of irrigation systems. In this study, nine laboratory experiments were conducted to investigate the influences of both bulk density and initial moisture content on cumulative infiltration and wetting front depth. Sandy loam soil was used and divided into three parts, each with three different values of moisture content, and three values of bulk density were applied to each part.

Six mathematical infiltration models (Kostiakov, Modified Kostiakov, Revised Modified Kostiakov, Philip, Horten, and Soil Conservation Services (SCS)) were adopted to estimate cumulative infiltration and the using the Solver tool built into Excel to calculate the parameter values of these models. The performance of the models was evaluated using three statistical criteria: the sum of squared errors (SSE), the coefficient of determination (R^2), and the root mean square error (RMSE). The model coefficients were developed as quadratic functions of both bulk density and moisture content, and these relationships were then combined into a comprehensive equation that includes the six models, with the effects of both bulk density and moisture content. Given the difficulty of measuring the depth of the wetting front directly within the soil, an equation was also developed to estimate the depth of the wetting front from knowledge of the cumulative infiltration depth.

This study confirmed an inverse relationship between bulk density and both cumulative infiltration and wetting front depth, both decreasing with increasing bulk density. The cumulative infiltration had the highest decrease of 56.9%, while the lowest decrease was 30.6%. In terms of the wetting front, the highest decrease was 46%, whereas the lowest decrease was 17.5%. Moisture content had an inverse relationship with cumulative infiltration, where the maximum reduction reached 37.5% and the minimum was 14.2%. Conversely, wetting front depth is directly

relationship with moisture content, increasing as the initial soil moisture content increases. The greatest increase in the wetting front was 33.6%, and the smallest increase was 2.6%. The revised modified Kostiakov model outperformed other models in terms of performance based on statistical criteria. It achieved the lowest value for the sum of squared errors (SSE) of 152.49, and the smallest value for the root mean square error (RMSE) of 0.882. It also recorded the highest value for the coefficient of determination (R^2) of 0.99996, which indicates a significant concordance between the observed and estimated values. Both the comprehensive and improved equations for estimating the depth of the wetting front from the cumulative infiltration demonstrated high statistical indices and are therefore recommended for use in assessing the wetting front depth and the cumulative infiltration in various sandy loam soil conditions.

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