# Mathematical model of a dynamic process of transporting a bulk material by means of a tube scraping conveyor

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**Abstract:** The results of theoretical studies of simultaneous transporting the components of feed mixtures along the curvilinear paths of tubular conveyors are presented in this article. The mathematical model of a technological process of moving a bulk material (grain) by means of a cable with a connected scraper is proposed. The model is presented as a system of elastic one-dimensional bodies, which are seamlessly moved by a bulk material. Nonlinear differential equations with partial derivatives that describe the dynamics of horizontal and vertical lines of a tube conveyor under the corresponding boundary conditions are deduced. Based on the results, the technique of determining the technological parameters, which ensure the reduction of energy consumption while bulk materials with the given quality of feed mixtures, is proposed.

**Keywords:** mathematical model, cable, scraper, conveyor, bulk medium

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## 1 Introduction

Conveyor transport is widely used to transport cereal products and feed mixes used to feed animals and poultry. Based on the analysis of literature sources and experimental results of studying the processes of bulk materials transportation in closed jackets (Owen and Cleary, 2010; Rohatynskyi et al., 2015; Roberts, 2015), the vast majority of screw conveyors are found to possess the limited functionality; therefore, they can be used only on short paths of material movement. Tube conveyors are considered as reliable and effective means to solve the problem of moving a predetermined amount of feed to a given distance at a set time (Horak, 2003; Loeffler, 2000; Lyashuk et al., 2018; and Masood et al., 2005). Many scientists have studied the methods of improving the operational and functional performance of screw and tubular conveyors, and the ways to protect their drive elements (Hevko et al., 2018a; Baranovsky et al., 2018; Hevko et al., 2018b; Hevko et al., 2018c).

However, the modern tube scraping cable conveyors that move bulk materials in guide tubes of different configurations are characterized by limited functional capabilities, since they carry only transport functions. Therefore, the challenge is to expand the functional capabilities and operational performance of such conveyors. At the same time, oscillatory processes that accompany technological processes cause undesirable phenomena. In particular, significant deformations occur, and the operation durability of knots and mechanisms component parts is ultimately reduced. The cable is the driving element of tube conveyors. It allows, at the design stage, to determine the range of eigenfrequencies, to

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select parameters and operating modes in such a way as to avoid resonant phenomena and to ensure long-term operation of conveyors (Katterfeld and Williams, 2008; Krause and Katterfeld, 2004). Many scientists such as Andronov and Bulanova (1995), Babakov (1965), Blakyer (1969), Grabov (1961), Hu et al. (2010), Kuzio and Sokil (2000), Vasilenko (1980), Zareiforoush et al. (2010a, 2010b, 2010c) have investigated the oscillating systems of a conveyor driving working body. Therefore, the challenge is to develop mathematical models that influence the motion of a working body moving an unevenly distributed mass (bulk material) with a constant velocity component of longitudinal motion.

## 2 Materials and methods

The main characteristics of dynamic systems vary along their length (area, volume) according to a certain law called systems with distributed parameters.

The dependence of oscillations frequency on amplitude, loss of stability, etc. is investigated. Based on the velocity component of oscillations and the influence of kinematic, physic-mechanical, geometric parameters on the dynamic process, the mathematical model of oscillations of the conveyor driving working body was developed. This mathematical model is based on:

1) the principle of one-frequency oscillations in nonlinear systems with many degrees of freedom and distributed parameters (Kurant, 1964);

2) the wave theory of motion (Van der Pol, 1920) adapted for longitudinal moving systems (Stotsko et al., 1999; Stotsko et al., 2000);

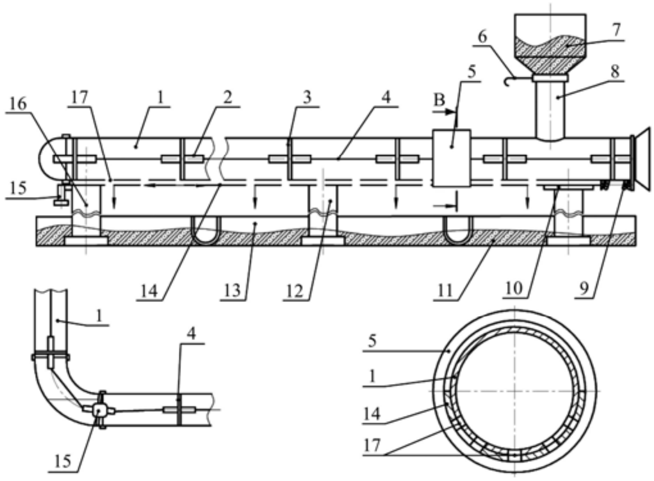
3) the distribution of the basic idea of Van der Pol method to the classes of dynamical systems under consideration.

Their motion is described by equations with partial derivatives under certain boundary conditions. The conveyor working body that moves the bulk material can be considered one-dimensional elastic body, the mass of which varies along the length (Figure 1).

The boundary conditions factor were in:

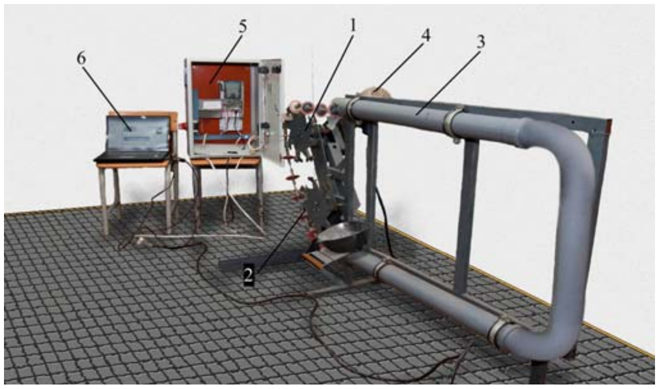
1) the movement of extreme points of the working body parts, which are considered as small quantities;

2) the movement of the extreme right point of the lower part of the working body is consistent with the movement of the lower point of the vertical part of the working body. The movement of the extreme right point of the upper part is consistent with the movement of the upper point of the vertical part of the working body. Differential equations describing the investigated process are presented in Euler variables (Kurant, 1964).




1. U-shaped tube 2. Mounting spacer 3. Round feeding discs 4. Cable 5. Reservoir 6. Regulating gate 7-11. Bulk material 8. Feed magazine 9. Atorage reservoir 10. Frame 12-16. Vertical supports 13. Half round chute 14. Cut-off plate 15. Cam lifter 17. Hole

(a) feed scraping conveyor



1. Drive sprocket 2. Working body 3. Reduction unit 4. Electromotor 5. Frequency convertor of Altivar series 7.1 6. Personal computer

(b) experimental assembly for moving the bulk medium

Figure 1 Model of the process of moving the bulk medium by means of a cable conveyor (Hevko et al., 2010)

For example, when the motion of a bulk material relative to the working body can be neglected, the mass of a bulk material can be considered as distribution by a certain law along the cable. In this case, all the points of the normal cross-sections of the system ‘bulk medium (material) - working body’ possess the same kinematic characteristics (except the corner areas). Based on the above, the object under study can be interpreted as a one-dimensional mechanical system with distributed parameters. Therefore, in the studies (Vasilenko, 1980; Stotsko et al, 1999; Stotsko et al., 2000), a bulk material in technological operations can be considered as an elastic body with some averaged integral characteristics. The total mass of the working body and bulk material is considered as the mass of the length unit of a specified conditional one-dimensional body (bulk material and working body).

To study the mathematical model of the dynamic process, some restrictions were imposed:

- the mass of a working body of a tube scraper conveyor is inextricably linked to a bulk material. It is considered a slowly variable function. The law of mass distribution in Lagrange's variables is assumed to be described by the function $m(x) = m_0 + m_1 \cos\left(\dfrac{\pi}{2l}x + \varphi_0\right)$, $m_1 << m_0$, $\varphi_0$ are constants;

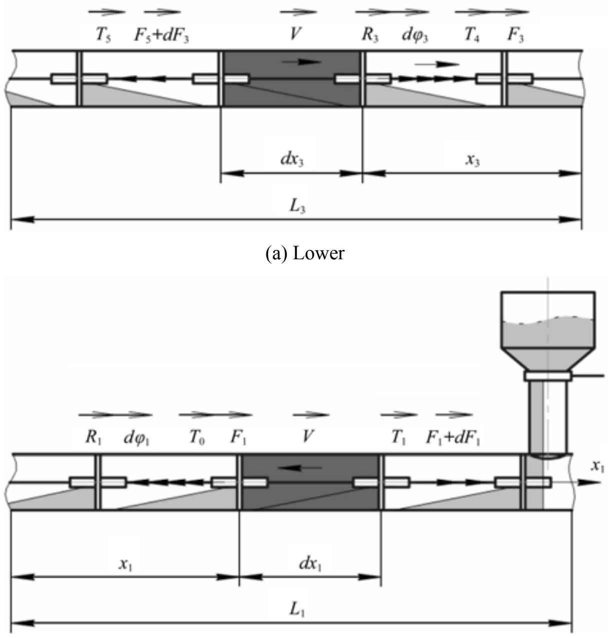
- the maximum value of the resistance strength is small in comparison with the linear component of the restoring force, i.e. $\max R_i\left(\dfrac{\partial u_i(x_i,t)}{\partial t}\right) << \max EA\dfrac{\partial^2 u_i(x_i,t)}{\partial x_i^2}$;

- the working body material possesses an expressed nonlinearly elastic properties, that is $k$<<1;

- the linear movements of the extreme right (vertical points) are minor.

One-dimensional system with distributed parameters at its longitudinal oscillations is determined by a function that describes the deformation of the conditional elastic material line at an arbitrary time. To determine the given function, three areas are conditionally considered: two horizontals (upper and lower) and one vertical. To obtain a differential equation that describes the dynamic process of the lower (upper) branch of a working body, the forces acting on the conditionally separated element of the length $\Delta$($dx_1$ or $dx_3$) should be applied (Figure 2).

In Figure 2, the following notations are introduced: $\overrightarrow{T_0}$ ($\overrightarrow{T_5}$) - tension force acting on the left part of the selected element; $\vec{T}_1$ ($\vec{T}_4$) - tension force acting on the right part of the selected element; $\vec{R}_1$ ($\vec{R}_3$) - the strength of the resistance, the nature of which is determined by the mechanism of interaction of the cable with the bulk medium and a tube; $\vec{F}_1$ and $\vec{F}_1 + d\vec{F}_1$, ($\vec{F}_3$ and $\vec{F}_3 + d\vec{F}_3$) - forces of elasticity caused by elastic deformation of the conditional element; $m_1$ ($m_3$) - the weight of the specified elements of the cable and the bulk medium.




(b) Upper

Figure 2 Distribution of forces acting on the conditionally selected element of the horizontal part of a conveyor branch

Based on the principle of D' Alembert, the 'dynamic equilibrium' equation of selected elements (lower and upper branches) of the working body are deduced:

$$d\vec{\Phi}_1 + \vec{T}_0 + \vec{T}_1 + \vec{R}_1 + d\vec{F}_1 = 0, \quad d\vec{\Phi}_3 + T_4 + \vec{T}_5 + \vec{R}_3 + d\vec{F}_3 = 0 \tag{1}$$

where, $d\vec{\Phi}_1$ and $d\vec{\Phi}_3$ - the inertia forces of the specified conditionally selected elements.

The horizontal displacements of geometric points that coincide with the middle of selected elements at an arbitrary time $t$ are denoted as $u_1(x_1,t)$ and $u_3(x_3,t)$; consequently, the total derivatives with respect to time are expressed by means of the local ones in the form (Chen et al., 2004, 2009; Malinovskiy, 2001).

$$\frac{du_1(x_1,t)}{dt} = \frac{\partial u_1(x_1,t)}{\partial t} + V\frac{\partial u_1(x_1,t)}{\partial x} \tag{2}$$

$$\frac{du_3(x_3,t)}{dt} = \frac{\partial u_3(x_3,t)}{\partial t} + V\frac{\partial u_3(x_3,t)}{\partial x} \tag{3}$$

$$\frac{d^2u_1(x_1,t)}{dt^2} = V^2\frac{\partial^2 u_1(x_1,t)}{\partial x_1^2} + 2V\frac{\partial^2 u_1(x_1,t)}{\partial x_1 \partial t} + \frac{\partial^2 u_1(x_1,t)}{\partial t^2} \tag{4}$$

$$\frac{d^2u_3(x_3,t)}{dt^2} = V^2\frac{\partial^2 u_3(x_1,t)}{\partial x_3^2} + 2V\frac{\partial^2 u_3(x_3,t)}{\partial x_3 \partial t} + \frac{\partial^2 u_3(x_3,t)}{\partial t^2} \tag{5}$$

The Equations (2)-(5) factor in the same steel constituents of the longitudinal velocity $V$ of the upper and lower parts of conveyor working body branches.

In addition, the mass of the working body with a bulk material varies along the length $m_1 = m_1(\tilde{x}) = m_1(x_1 - Vt)$, $m_3 = m_3(\tilde{x}) = m_3(x_3 - Vt)$, and the strength of the resistance, which depends on the velocity $R_1 = R_1\left(\frac{\partial u_1(x_1,t)}{\partial t}\right)$, $R_3 = R_3\left(\frac{\partial u_3(x_3,t)}{\partial t}\right)$.

The elastic properties of the working body material that satisfy the nonlinear law of elasticity (Malinovskiy, 2001) can be represented as $\sigma = E\cdot(1+k\cdot E\cdot\varepsilon^2)\cdot\varepsilon$, where $\sigma$ and $\varepsilon$ - the stresses and relative deformations of the cable elements of the working body ($\frac{\partial u_1(x_1,t)}{\partial x_1}$ or $\frac{\partial u_3(x_3,t)}{\partial x_3}$); $E$ - the modulus of elasticity; $k$ is expressed by the modulus of volume compression $K$, the shear modulus $G$ and the constant of the working body cable material $\gamma$, which dependence $k = -\frac{2}{3}\cdot\frac{K}{3K+G}\cdot\frac{\gamma}{G}$.

The elastic characteristics of the working body cable substantially differ from the elastic characteristics of the material; and the graphic dependences of the modulus of elasticity of the working body cable differ from the relative deformation (Hevko et al., 2010).

Based on the principle of D' Alembert, the differential equations describing the longitudinal oscillations of the lower and upper conveyor branches are deduced as follows:

$$m_1(x_1 - Vt)\left(V^2\frac{\partial^2 u_1(x_1,t)}{\partial x_1^2} + 2V\frac{\partial^2 u_1(x_1,t)}{\partial x_1 \partial t} + \frac{\partial^2 u_1(x_1,t)}{\partial t^2}\right)$$
$$= EA\left(1 + kE\left(\frac{\partial^2 u_1(x_1,t)}{\partial x_1^2}\right)^2\right)\frac{\partial^2 u_1(x_1,t)}{\partial x_1^2} - R_1\left(\frac{\partial u_1(x_1,t)}{\partial t}\right) \quad (6)$$

$$m_3(x_3 - Vt)\left(V^2\frac{\partial^2 u_3(x_3,t)}{\partial x_3^2} + 2V\frac{\partial^2 u_3(x_1,t)}{\partial x_1 \partial t} + \frac{\partial^2 u_3(x_3,t)}{\partial t^2}\right)$$
$$= EA\left(1 + kE\left(\frac{\partial^2 u_3(x_3,t)}{\partial x_3^2}\right)^2\right)\frac{\partial^2 u_3(x_3,t)}{\partial x_3^2} - R_3\left(\frac{\partial u_3(x_3,t)}{\partial t}\right) \quad (7)$$

where, $A$ is the cross-sectional area of the cable; and it is considered to be a constant value.

The operating forces, boundary and initial conditions determined the dynamic process of systems with distributed parameters (Babakov, 1965). For the horizontal displacement of geometric points that coincide with the extreme left ends of the working body of the conveyor horizontal branches are small. The mathematical model of the dynamic process of the conveyor horizontal branches is reduced to the integration of Equations (6) and (7) under the boundary conditions is written in the form:

$$u_1(x_1,t)_{|x_1=0} = 0\,,\ u_3(x_3,t)_{|x_3=0} = -l(1-\cos\alpha)\,,$$
$$u_1(x_1,t)_{|x_1=L1} = l(1-\cos\alpha)\,,\ u_3(x_3,t)_{|x_3=L3} = 0 \quad (8)$$

where, $\alpha$ - the inclination angle of the working body to the horizon at the angular points, which can be approximated by means of the trigonometric equation $l\sin\alpha = \frac{D}{2}\left(1-\cos\frac{V}{l}t\right)$.

$l$ - the distance between adjacent scrapers; $D$ - the inner diameter of the conveyor tube.

The distribution of forces acting on an arbitrary and conventionally separated element of this branch of the length Δ ($dx_3$)) is shown in Figure 3.

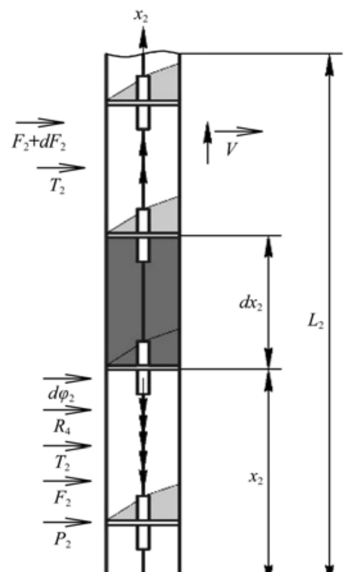



Figure 3 Analytical model of distributing the forces acting on the conventionally separated element of the vertical part of a scraper transporter branch

Compared with the forces acting on the horizontal branches of the conveyor working body, we can conclude that the conveyor vertical branch is loaded with an additional variable force $\vec{P}_2$.

The indicated force $\vec{P}_2$ is found from the ratio

$$P_2 = P_2(\tilde{x}_2) = g\int_0^{\tilde{x}_3} m_2(x)dx \quad (9)$$

where, $\tilde{x}_2$ - Lagrange coordinate of the cut part of the cable.

For its vertical component, the differential equation of its longitudinal oscillations has the form

$$m_2\left(x_2 - Vt\right)\left(V^2\frac{\partial^2 u_2(x_2,t)}{\partial x_2^2} + 2V\frac{\partial^2 u_2(x_1,t)}{\partial x_2 \partial t} + \frac{\partial^2 u_2(x_2,t)}{\partial t^2}\right)$$
$$= EA\left(1 + kE^2\left(\frac{\partial^2 u_2(x_2,t)}{\partial x_2^2}\right)^2\right)\frac{\partial^2 u_2(x_2,t)}{\partial x_2^2} - R_1\left(\frac{\partial u_2(x_2,t)}{\partial t}\right)$$
$$-g\int_0^{\tilde{x}_3} m_2(x)dx \tag{10}$$

and boundary conditions

$$u_2(x_2,t)_{|x_2=0} = -l(1-\cos\alpha),\quad u_2(x_2,t)_{|x_2=L} = -l(1-\cos\alpha) \tag{11}$$

The above allows to introduce a small parameter in the differential Equations (2), (3), (10) and to submit them in the form

$$\frac{\partial^2 u_i(x_i,t)}{\partial t^2} + 2V\frac{\partial^2 u_i(x_i,t)}{\partial t \partial x_i} - \left(\frac{EA}{m_0} - V^2\right)\frac{\partial^2 u_i(x_i,t)}{\partial x_i^2}$$
$$= \mu f_i\left(u_i, \frac{\partial u_i(x_i,t)}{\partial t}, \frac{\partial^2 u_i(x_i,t)}{\partial x_i^2}, \vartheta\right) \tag{12}$$

where, $\mu = \dfrac{k}{m_0}$ – small parameter, $\vartheta = \dfrac{\pi}{l}Vt - \vartheta_0$, functions $f_i\left(u_i, \dfrac{\partial u_i(x_i,t)}{\partial t}, \dfrac{\partial^2 u_i(x_i,t)}{\partial x_i^2}, \vartheta\right)$ assume a form:

$$f_1(u_1,...,\vartheta) = \frac{1}{m_0}EA\left(k\left(\frac{\partial^2 u_1 x_1,t}{\partial x_1^2}\right)^2\right)\frac{\partial^2 u_1(x_1,t)}{\partial x_1^2} -$$
$$\frac{1}{km_0}R_1\left(\frac{\partial u_1(x_1,t)}{\partial t}\right) - \frac{m_1}{m_0}S(u_1(x,t))\cos\left(\frac{\pi}{2l}(x_1 - Vt) + \varphi_0\right),$$
$$f_2(u_2,...,\vartheta) = \frac{1}{m_0}EA\left(k\left(\frac{\partial^2 u_2\left(x_2,t\right)}{\partial x_2^2}\right)^2\right)\frac{\partial^2 u_2(x_1,t)}{\partial x_2^2} -$$
$$\frac{1}{km_0}R_1\left(\frac{\partial u_1 x_1,t}{\partial t}\right) - \frac{m_1}{m_0}S\left(u_2(x_2,t)\right)\cos\left(\frac{\pi}{2l}(x_2 - Vt) + \varphi_0\right),$$
$$f_3(u_3,...,\vartheta) = \frac{1}{m_0}EA\left(k\left(\frac{\partial^2 u_2(x_2,t)}{\partial x_2^2}\right)^2\right)\frac{\partial^2 u_2(x_1,t)}{\partial x_2^2} -$$
$$\frac{1}{km_0}R_1\left(\frac{\partial u_1(x_2,t)}{\partial t}\right) - \frac{m_1}{m_0}S\left(u_3(x_3,t)\right)\cos\left(\frac{\pi}{2l}(x_2 - Vt) + \varphi_0\right)$$
$$++\frac{m}{k}g\int_0^{x_3-Vt} m_1(x)dx,\ S\left(u(x,t)\right) = \frac{\partial^2 u(x,t)}{\partial t^2} + 2V\frac{\partial^2 u(x,t)}{\partial t \partial x}$$

## 3 Results

The obtained nonlinear differential equations differ only in boundary conditions and right-hand sides, which are proportional to a small parameter $\mu$. They are called systems with low nonlinearity. To study these systems effectively, the technique based on perturbation methods (Cole, 1972; Nayfe, 1976; Proskuriakov, 1977) should be applied, particularly asymptotic methods of nonlinear mechanics (Mitropolskiy and Moseyenkov, 1976) or their modification (Mitropol'skii, 1966; Mykhailov et al., 1984). The reason for their application is the explicit existence of a solution of the corresponding undisturbed ($\mu$=0) boundary problems, that is, the solution of the equation.

$$\frac{\partial^2 u(x,t)}{\partial t^2} + 2V\frac{\partial^2 u(x,t)}{\partial t \partial x} - \left(\frac{EA}{m_0} - V^2\right)\frac{\partial^2 u(x,t)}{\partial x^2} = 0 \tag{13}$$

under homogeneous boundary conditions

$$u(x,t)_{|x=0} = 0,\quad u(x,t)_{|x=L} = 0 \tag{14}$$

To find the influence of nonlinear forces on nonlinear oscillations of scraper conveyor branches under conditions of small perturbations of boundary conditions, the following equation is deduced

$$u_i(x_i,t)_{|x_i=0} = \mu g_{i0}(u(x_i,t), \frac{\partial u(x_i,t)}{\partial x_i}, \frac{\partial u(x_i,t)}{\partial t})_{|x_i=0},$$
$$u_i(x_i,t)_{|x_i=L} = \mu g_{iL}(u(x_i,t), \frac{\partial u(x_i,t)}{\partial x_i}, \frac{\partial u(x_i,t)}{\partial t})_{|x_i=L} \tag{15}$$

The right-side parts of the above ratios are consistent with the right-side parts of the boundary conditions (2-5, 10) as follows:

$$g_{10}(u(x_1,t), \frac{\partial u(x_1,t)}{\partial x_1}, \frac{\partial u(x_1,t)}{\partial t})_{|x_1=0} = 0,$$
$$g_{1L}(u(x_1,t), \frac{\partial u(x_1,t)}{\partial x_1}, \frac{\partial u(x_1,t)}{\partial t})_{|x_1=L} = l(1-\cos\alpha),$$
$$g_{2L}(u(x_2,t), \frac{\partial u(x_2,t)}{\partial x_2}, \frac{\partial u(x_2,t)}{\partial t})_{|x_2=0} = -l(1-\cos\alpha),$$
$$g_{2L}(u(x_2,t), \frac{\partial u(x_2,t)}{\partial x_2}, \frac{\partial u(x_2,t)}{\partial t})_{|x_2=L} = -l(1-\cos\alpha),$$
$$g_{3L}(u(x_3,t), \frac{\partial u(x_3,t)}{\partial x_3}, \frac{\partial u(x_3,t)}{\partial t})_{|x_3=0} = -l(1-\cos\alpha),$$
$$g_{3L}(u(x_3,t), \frac{\partial u(x_3,t)}{\partial x_3}, \frac{\partial u(x_3,t)}{\partial t})_{|x_3=L} = 0.$$

Thus, the problem is to reduce the integration of Equation (10) under nonuniform boundary conditions (15).

$$u_i(x_i,t) = v_i(x_i,t) + \mu w_i(x_i,t) \tag{16}$$

If functions $w_i(x_i, t)$ and $v_i(x_i, t)$ are solutions of differential equations, then

$$\frac{\partial^2 w_i(x_i,t)}{\partial x_i^2} = 0 \tag{17}$$

and

$$\begin{gathered}\frac{\partial^2 v_i(x_i,t)}{\partial t^2} + 2V\frac{\partial^2 v_i(x_i,t)}{\partial t \partial x_i} - \left(\frac{EA}{m_0} - V^2\right)\frac{\partial^2 v_i(x_i,t)}{\partial x_i^2} \\ = \mu F_i\left(v_i(x_i,t), \frac{\partial v_i(x_i,t)}{\partial t}, \frac{\partial^2 v_i(x_i,t)}{\partial x_i^2}, \vartheta\right)\end{gathered} \tag{18}$$

and satisfy the boundary conditions

$$\begin{gathered} w_i(x_i,t)_{|x_i=0} = \mu g_{i0}(v(x_i,t), \frac{\partial v(x_i,t)}{\partial x_i}, \frac{\partial v(x_i,t)}{\partial t})_{|x_i=0}, \\ w_i(x_i,t)_{|x_i=L} = \mu g_{iL}(v(x_i,t), \frac{\partial v(x_i,t)}{\partial x_i}, \frac{\partial v(x_i,t)}{\partial t})_{|x_i=L} \end{gathered} \tag{19}$$

and

$$v(x,t)_{|x=0} = 0, \quad v(x,t)_{|x=L} = 0 \tag{20}$$

the obtained functions $u_i(x_i,t)$ will satisfy in the first approximation the problems with uniform boundary conditions.

Considering $\frac{D}{2l} << 1$, the first distinguished boundary allows to replace $\sin\alpha$ by the magnitude of the inclination angle of the working body at the angular points, that is $\alpha$. Keeping the specified accuracy order in the boundary conditions, the functions $w_i(x_i,t)$ are presented in the form

$$\begin{gathered} w_1(x_1,t) = \left(\frac{D}{2l}\right)^2 x_1 \cos^2\frac{V}{l}t, \\ w_2(x_2,t) = -\left(\frac{D}{2l}\right)^2 (1+x_2)\cos^2\frac{V}{l}t \\ w_3(x_3,t) = \left(\frac{D}{2l}\right)^2 \cos^2\frac{V}{l}t \end{gathered} \tag{21}$$

Based on the above, the right-side parts of the Equations (12) assume the form

$$\begin{gathered}
\overline{f}_1(a,x,\psi,\vartheta,\overline{\vartheta}) = \frac{a^3}{m_0}EA(k[-\mathrm{K}\sin(\mathrm{K}x_1+\psi)+\mathrm{H}\sin(\mathrm{H}x_1-\psi)]^2)([-\mathrm{K}^2\cos(\mathrm{K}x_1+\psi)+\mathrm{H}^2\cos(\mathrm{H}x_1-\psi)) - \\
\frac{k_1}{km_0}(a\Omega(-\sin(\mathrm{K}x_1+\psi)-\sin(\mathrm{H}x_1-\psi)) - \left(\frac{DV}{2l^2}\right)^2 x_1\cos 2\frac{V}{l}t - \left(\frac{D}{2l}\right)^2\frac{V}{l}\sin 2\frac{V}{l}t - + \frac{m_1}{m_0}a\Omega\left[\Omega+2V(\mathrm{K}-\mathrm{H})\right] \\
(\cos(\mathrm{K}x_1+\psi)-\cos(Hx_1-\psi))\cos\left(\frac{\pi}{2l}(x_1-Vt)+\varphi_0\right), \\
\overline{f}_3(a,x,\psi,\vartheta,\overline{\vartheta}) = \frac{a^3}{m_0}EA(k[-\mathrm{K}\sin(\mathrm{K}x_3+\psi)+\mathrm{H}\sin(\mathrm{H}x_3-\psi)]^2)([-\mathrm{K}^2\cos(\mathrm{K}x_3+\psi)+\mathrm{H}^2\cos(\mathrm{H}x_3-\psi)) - \\
-\frac{k_1}{km_0}\left(a\Omega(-\sin(\mathrm{K}x_3+\psi)-\sin(\mathrm{H}x_3-\psi))\right) + \left(\frac{DV}{2l^2}\right)^2(1+x_3)\cos 2\frac{V}{l}t + \left(\frac{D}{2l}\right)^2\sin 2\frac{V}{l}t + \\
+\frac{m_1}{m_0}a\Omega\left[\Omega+2V(\mathrm{K}-\mathrm{H})\right](\cos(\mathrm{K}x_3+\psi)-\cos(Hx_1-\psi))\cos\left(\frac{\pi}{2l}(x_3-Vt)+\phi_0\right), \\
\overline{f}_2(a,x_2,\psi,\vartheta,\overline{\vartheta}) = \frac{a^3}{m_0}EA(k[-\mathrm{K}\sin(\mathrm{K}x_1+\psi)+\mathrm{H}\sin(\mathrm{H}x_1-\psi)]^2)([-\mathrm{K}^2\cos(\mathrm{K}x_1+\psi)+\mathrm{H}^2\cos(\mathrm{H}x_1-\psi)) \\
-\frac{k_1}{km_0}(a\Omega[-\sin(\mathrm{K}x_1+\psi)-\sin(\mathrm{H}x_1-\psi)) + \left(\frac{DV}{2l^2}\right)^2(1+x_2)\cos 2\frac{V}{l}t + \left(\frac{D}{2l}\right)^2\sin 2\frac{V}{l}t + \frac{m_1}{m_0}a\Omega\left[\Omega+2V(\mathrm{K}-\mathrm{H})\right] \\
(\cos(\mathrm{K}x_2+\psi)-\cos(\mathrm{H}x_2-\psi))\cos\left(\frac{\pi}{2l}(x_2-Vt)+\varphi_0\right) + g(x_2-Vt) + \frac{m_1}{m_0}g\frac{2l}{\pi}\sin\left(\frac{\pi}{2l}(x_2-Vt)+\varphi_0\right)
\end{gathered} \tag{22}$$

The amplitude-frequency characteristic of the dynamic process of the working body is also determined. In the case of a uniform distribution of the bulk medium between the scrapers is determined by differential equations:

$$\frac{da}{dt} = \frac{-\mu k_1\Omega(2\Omega+V(H-X))}{2[(\Omega+VK)^2+(\Omega-VH)^2]}a, \quad \frac{d\phi}{dt} =$$

$$\frac{\mu EA\{\omega(\mathrm{K}^4+4\mathrm{K}^2\mathrm{H}^2+\mathrm{H}^4)+V(\mathrm{K}^5+2\mathrm{K}^3\mathrm{H}^2-2\mathrm{K}^2\mathrm{H}^3-\mathrm{H}^5)\}}{4m_0k\pi[(\Omega+V\mathrm{K})^2+(\Omega-V\mathrm{H})^2]}a^2 \tag{23}$$

In Figure 4 and Figure 5, the time dependences of the amplitude and frequency of oscillations of the working body at different movement velocities are presented.

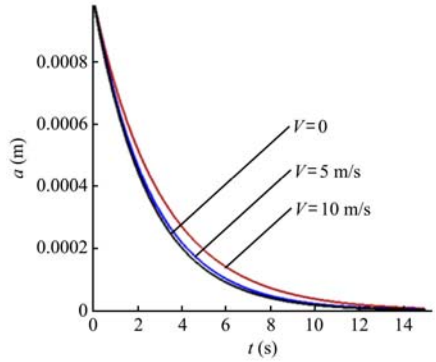
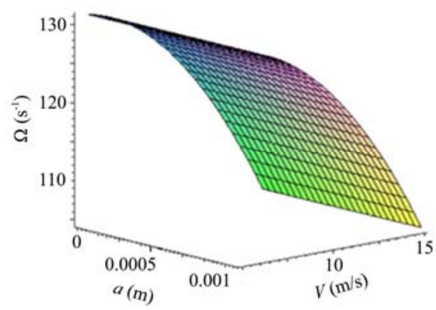
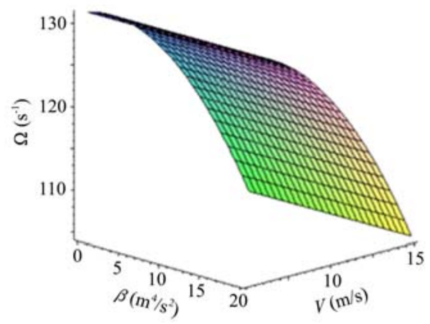


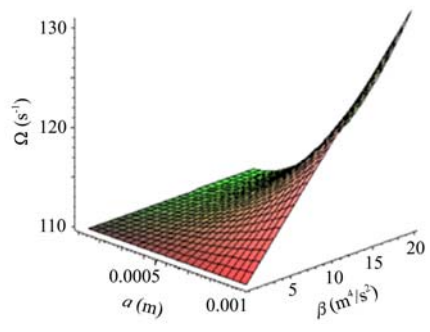

Figure 4 Changes in time of the oscillations amplitude of the working body at different velocity values of its longitude movement




(a)




(b)




(c)

Figure 5 Influence of the amplitude, velocity rate, and the parameters $\beta = \frac{kEA}{m_0}$ on the frequency of natural oscillations of a tube scrape conveyor system

## 4 Conclusion

The mathematical models of a technological process of moving the bulk medium with the help of a cable scraping working organ in the form of a system of elastic one-dimensional bodies that seamlessly move the bulk material is proposed. Adequate mathematical models of the dynamic process are constructed, which are nonlinear differential equations with partial derivativ. These equations describe the dynamics of the horizontal, vertical rows of the scraper working body and the corresponding boundary conditions. The presented graphic dependences prove that for the larger velocity values of the cable scraper working body in the tube conveyor, the damping of the amplitude is smaller; and the velocity of amplitude longitudinal motion effects the frequency of oscillations of the working body. We found that for determining the technological parameters, which ensure the reduction of energy consumption while bulk materials with the given quality of feed mixtures, is proposed the maximum output for corn is 3,975 kg $h^{-1}$, which is 10%-15% higher than for wheat.

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