Mathematical modeling of drying kinetics of Bhimkol (*Musa balbisiana*) pulp using MATLAB

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Abstract: Bhimkol pulp was dried by spreading in a thin layer in a hot air dryer at 40°C, 50°C and 60°C respectively. The drying kinetics was accomplished by using a general code developed in MATLAB R2014a software. Among the five thin layer drying models used for the analysis, Midilli-Kucuk showed the best-fitted result with the greatest determination coefficient (R^2) value and lowest root mean square error (RMSE) and the sum of square error (SSE) values. For a better generalization of the drying model, master curve technique and generalized drying constant method were also applied. Master curve technique offered the best fit with R^2 value of 0.997 as compared to generalized drying constant method and normal mathematical modeling technique. From the results, it was also observed that the value of drying constant k and effective diffusivity followed a rising trend with increasing temperature. The activation energy of the drying was 39.67 kJ mol⁻¹. **Keywords:** thin layer drying, mathematical modeling, drying kinetics, general code, a master curve

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1 Introduction

In northeastern India, Bhimkol (Musa balbisiana) is a major fruit crop. It is a good source of natural antioxidant and beneficial against the oxidative stress (Mudoi et al., 2011). It is different from other bananas in terms of vivo and vitro characteristic. The ripe fruit is delicious and can be used as baby food. Fresh ripe pulp (FRP) of the fruit antiperoxidative and has antioxidant properties. Availability of vitamin C, potassium, riboflavin, and pyridoxine makes it a good source for processing (Barthakur and Arnold, 1990; Borborah et al., 2016). Besides that, the fruit is full of seeds which make it be processed properly. Due to the high moisture content in banana, spoilage occurs at a faster rate. Drying is one of the processing methods that can improve the shelf life of fruits and vegetables and also can prevent undesirable changes due to minimal microbial activity.

Generally experimental evaluation of the drying

kinetics is done by taking the weight of the sample at a regular interval of time. Average moisture content versus time, drying rate versus time and drying rate versus average moisture content, are three important representations of the drying curve (Coumans, 2000). Majumdar (1987) has reviewed several theories on the mechanism of moisture migration. However, for the drying of food materials, only capillary and liquid diffusion theories are applicable. An appropriate drying model including differential equations of heat and mass transfer in the interior of the product and at its interphase with the drying agent can be used for describing the drying process (Karathanos, 1999; Tzempelikos et al., 2015).

For the development of new designs and improvement of existing drying systems, simulation models play an important role (Aghbashlo et al., 2008). Many mathematical models are available in the literature that has shown the useful result in the design and analysis of heat transfer processes during drying. In simulation models, drying conditions are directly linked to all the parameters (Babalis et al., 2005).

In mathematical modeling of drying, thin layer drying

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equations are important tools. The thin layer drying equations have given sufficient good results. Normally for the solution of thin layer drying models, separate discrete analysis of each model is required. So, it takes a long time for the thin layer drying kinetic analysis of any food material. In this study, a general MATLAB code was developed for the calibration of thin layer drying models. It provides a very precise analysis of batches of drying experiment at different temperatures with the single input of the drying parameters. It was developed in the MATLAB R2014a software, which selects the best model among thin layer drying models and determines the effective diffusivity and activation energy. By using the code with inputs of drying parameters namely sample box weight, initial moisture content, the thickness of Bhimkol pulp bed, time of drying with respect to weight loss of the pulp and number of drying temperatures, results of the mathematical modeling can be obtained. On the basis of the developed code, thin layer drying kinetic analysis of Bhimkol pulp was conducted in the present study.

For obtaining a generalized model of thin layer drying, a master curve technique and generalized drying constant method were also applied. Normally, for the analysis of thin layer drying, mathematical models are used. But those models are just related to drying time and do not include the interaction effect of other related parameters (Khazaei et al., 2008; Doymaz and Pala, 2003). So, in this study master curve technique and generalized drying constant method was implemented including all the drying parameters. A general model was developed to select proper drying conditions with predictable quality retention. Hence, the objectives of the study were to generalize the thin layer drying kinetics of Bhimkol pulp based on master curve technique and the general code was developed in MATLAB R2014a software.

2 Theoretical considerations

For describing single layer drying behavior of any food material, it is important to model the drying process. The drying phenomenon was modeled using thin layer drying models viz. empirical, theoretical and semi-theoretical. The solution of semi-theoretical Fick's second law model describes the diffusion process during falling rate period of drying (Panchariya et al., 2002; Bezbaruah and Hazarika, 2014; Chakraborty et al., 2016). Fick's second law can be represented by Equation (1).

$$\frac{\partial(M_c)}{\partial t} = D_{eff} \frac{\partial^2(M_c)}{\partial x^2} \tag{1}$$

Where, D_{eff} is diffusivity (m² s⁻¹); M_c is moisture content; t is drying time (s) and x is the thickness of drying bed (m).

The initial and boundary conditions may be articulated as follows:

At $t=0, M_C=M_{Ci}$, for 0 < x < 2L

At t > 0, $M_C = M_{Ce}$, for x = 2L (top, evaporating surface)

At
$$t > 0$$
, $\frac{\partial (M_C)}{\partial x} = 0$, for $x = 0$ (non – evaporating

surface the bottom)

Fick's second law can be solved by using several analytical solutions. The general series of a solution of Fick's second law considering slab geometry may represent by Equation (2) (Bezbaruah and Hazarika, 2014).

$$MR = \frac{M_{t} - M_{e}}{M_{i} - M_{e}} = \sum_{n=0}^{\infty} \frac{8}{\pi^{2}} \exp\left[\frac{(2n+1)2\pi^{2}D_{eff}t}{(2L)^{2}}\right]$$
(2)

Where, MR = Moisture ratio; M_t = Moisture content at each regular interval of drying; M_i = Initial moisture content; M_e = Equilibrium moisture content.

Neglecting terms of the higher order for higher drying time, Equation (2) can be resolved as follows

$$MR = \left(\frac{8}{\pi^2}\right) \exp\left(\frac{-\pi^2 D_{eff} t}{(2L)^2}\right)$$
(3)

The gradient of the linear plot of $\ln(MR)$ vs *t* was used to estimate the effective diffusivity. Here, 2*L* is the thickness of Bhimkol pulp bed.

Table 1 shows different thin layer drying models used for the modeling of Bhimkol pulp drying kinetics.

The activation energy was estimated by Arrhenius type equation (Equation (4)) (Lopez et al., 2000; Akpinar et al., 2003).

$$D = D_0 \exp\left(-\frac{E_a}{RT_a}\right) \tag{4}$$

Where, E_a is activation energy (kJ mol⁻¹); T_a is absolute air temperature (K), and D_0 is the pre-exponential factor of the Arrhenius equation (s m² s⁻¹); *R* is universal gas constant (8.3143 J mol⁻¹ K⁻¹).

Table 1Thin layer drying models used for the Bhimkol pulp
drying

Model Name	Туре	References
Newton	$MR = \exp(-kt)$	Mujumdar (1987)
Page	$MR = \exp(-kt^n)$	Diamante and Munro (1993)
Midilli-Kucuk	$MR = a \exp(-kt^n) + bt$	Midilli et al. (2002)
Henderson and Pabis	$MR=a \exp(-kt)$	Zhang and Litchfield (1991)
Logarithmic	$MR = a \exp(-kt) + c$	Karathanos (1999)

3 Generalization of drying model

Generalization of drying model can be achieved by extending the range of operating conditions (Bezbaruah and Hazarika, 2014). In this study, the following two approaches were applied to generalize the drying model based on Bezbaruah and Hazarika (2014).

3.1 Generalization of drying constant

In this approach, a general model was developed as a function of drying temperature and time by describing the dependence of the drying rate constant, k on drying temperature in the form of Arrhenius type model (Equation (5)) (Bezbaruah and Hazarika, 2014). It was then derived using regression analysis:

$$k = k_0 \exp\left(-\frac{E_a}{RT}\right) \tag{5}$$

where K_0 = Pre-exponential factor of Arrhenius equation; E_a is activation energy; R is universal gas constant (8.3143 kJ mol⁻¹ K⁻¹); T is absolute air temperature (K) (Bezbaruah and Hazarika, 2014).

3.2 Technique of master curve

For creating a master curve as a generalized model, different drying temperatures were shifted with respect to a reference temperature. As a result of which, the dimensionless average temperature shift factor a_T was obtained by using Equation (6) (Bezbaruah and Hazarika, 2014).

$$\log(a_{T,i}) = \log t_R - \log t_i \tag{6}$$

Where, t_R = time at a reference temperature and t_i = time at other temperature, corresponding to given *MR*, *i* represents *i*th temperature. At reference temperature, log $(\alpha_{T,i}) = 0$, or, α_T at reference is 1.0. For $t_R = t * \alpha_T$, called temperature reduced time, *t*', the drying curve shifted horizontally by averaged α_{T_1} forms one master curve MR(t') at each of the drying temperatures.

4 Methods and materials

4.1 Sample procurement and moisture content determination

Raw sample of the Bhimkol was procured from the local market of Tezpur, India. Vacuum drying method was implemented for the moisture content determination of Bhimkol pulp (Nguyen and Price, 2007). By using magnesium sulphate desiccant, the sample was kept at 60°C respectively by keeping a relative humidity of 10% for 48 h. For reducing the error, the experiment was conducted in triplicates.

4.2 Pulping of fruit and drying procedure

After initial peeling, the fruits were sliced uniformly. By using hand operated screw type fruit juice extractor, the sliced fruits were crushed and separated from hard and round seeds. For increasing pulp recovery, the roughage of fruit was recycled. By keeping a uniform thickness of 5.0 ± 0.5 mm, the pulp was spread over the drying tray. The drying was performed in a hot air dryer at 40°C, 50°C and 60°C respectively. After setting desired drying conditions, the samples were kept in the dryer. The moisture loss was recorded by using a digital balance of 0.001 g accuracy (Shimadzu UW1020H, made in Japan) at a regular interval. The drying was continued until a constant weight was achieved. The data collected during the experiment was further analyzed to study the drying behavior of the Bhimkol pulp.

5 Development of the general MATLAB code

The code was developed in MATLAB R2014a software. The objective of the code was to make the thin layer drying kinetic analysis shorter and easier. The code was made with the combination of the main program and subprogram. The developed code was as follows:

function[gof, Deff]=function name(s_n, w_n, m₁,t_n,L,T,n)
%%weight of sample=w_n
%%sample box weight=s_n
%%Initial moisture content=m1 (%wb)
%%time=t_n

%%Dimension of dried material=2L
%%Temperature of drying=T
%%No. of drying experiments=n
w=(w1-s1);
u=w(1)*(1-m1);
v=u./w;
mwb=(1-v);
mdb= mwb./(1-mwb);
L=length(mdb);
me=mdb(L);
M1=mdb(1)-me;
M2=mdb-me;
MR1=M2./M1
%%Mathematical modeling of thin layer drying
y=MR1;
x=t1;
[fitresult1 gof1]=model_name (x,y);
model result=fitresult1, gof1

This portion of the program was computed to find out moisture content in wet basis and dry basis and finally to find out moisture ratio for the particular drying experiment.

In next part of the program, mathematical modelling analysis of the thin layer drying was carried out.

%%Mathematical modeling of thin layer drying

y=MR;

x=t;

%% Fit: 'untitled fit 1'.

[xData, yData] = prepareCurveData(x, y);

% Set up fittype and options.

ft = fittype('model equation', 'independent', 'x',
'dependent', 'y');

```
opts = fitoptions( ft );
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opts.Display = 'Off';

opts.Lower = [lower limit upper limit];

opts.StartPoint = [starting value of the coefficients];opts.Upper = [lower limit upper limit];

% Fit model to data.

[fitresult, gof,adjrsquare] = fit(xData, yData, ft, opts) % Plot fit with data.

figure('Name', 'untitled fit 1');

h = plot(fitresult, xData, yData);

legend(h, 'Moisture content(%db) vs. Time', 'untitled

fit 1', 'Location', 'NorthEast'); % Label axes xlabel('Time') ylabel('Moisture content(%db)') gtext('Experimental') gtext('Predicted') grid on

Hence by using the above-mentioned program, solution of the mathematical models was obtained. This program gave value of the coefficients, statistical parameters and plot between predicted and experimental moisture ratio for each drying model.

Next part of the program was computed for finding out diffusivity of the drying process.

%%Diffusivity of the drying

MR(MR<0.001)=[]; m=length (MR);

t1 = t(1:m);

y=log (MR);

Z=polyfit(t1,y,1);

Deff=(4*L^2*Z(1))/(-pi^2);

By using this program, diffusivity was found out. It was executed by taking the dimension of the product before drying. In this program, it was denoted as L. After finding out the best model, diffusivity was determined.

Next part of the program was computed to find out activation energy of the drying experiment.

%%Activation energy D=diffus' L=log(D) T1=(T'+273) X=T1.^(-1) Z=polyfit(X,L,1) Ea=-8.314*Z(1)

6 Results and discussions

6.1 Drying curves

The drying of Bhimkol pulp at different temperatures is shown in Figure 1. The drying rate increased with the rising trend of temperature and consequently moisture ratio decreased at a faster rate. Rapid moisture removal occurred during the initial stage of drying, whereas it slowed down gradually in the next stages.



Figure 1 Moisture ratio vs time plot for Bhimkol at different temperatures

6.2 Selection of best mathematical model

Thin layer drying models as listed in Table 1 were used to analyze the drying behavior of Bhimkol pulp. The best model was selected based on the statistical performance parameters (highest R^2 , lowest SSE and RMSE) (Bezbaruah and Hazarika, 2014). Statistical results of the drying models are shown in Table 2.

From Table 2 it can be observed that Midilli-Kucuk drying model was the best model to predict the drying behavior of wild banana. Bezbaruah and Hazarika, 2014 reported similar findings for turmeric slice. Table 3 illustrates the model parameters of the Midilli-Kucuk model.

 Table 2
 Statistical performance parameters for selected models

Model name			Drying temperature		
			40°C	50°C	60°C
		R^2	0.871	0.837	0.852
Newton	Statistic of fit	RMSE	0.103	0.111	0.104
		SSE	0.485	0.501	0.359
Page	Statistic of fit	R^2	0.953	0.933	0.919
		RMSE	0.063	0.072	0.078
		SSE	0.176	0.207	0.197
Midilli-Kucuk	Statistic of fit	R^2	0.992	0.991	0.988
		RMSE	0.027	0.028	0.032
		SSE	0.032	0.029	0.030
Henderson and Pabis		R^2	0.980	0.986	0.985
	Statistic of fit	RMSE	0.041	0.033	0.033
		SSE	0.076	0.044	0.036
Logarithmic	Statistic of fit	R^2	0.954	0.943	0.941
		RMSE	0.063	0.067	0.068
		SSE	0.172	0.175	0.144

Table 3	Mode	l parameters	for	Midilli-Kucuk model
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Temperature (°C)	20% imc				
	а	Ь	k	п	
40	1.046	-5.876×10^{-05}	0.0586	0.447	
50	1.044	-3.484×10^{-05}	0.0711	0.461	
60	1.042	-2.816×10^{-05}	0.0791	0.489	

Figure 2 shows predicted drying behavior of Bhimkol pulp at different temperatures by using a hot air dryer. The best mathematical model calculated the predicted values, i.e., a Midilli-Kucuk model for this study. From the figure, it can be observed that the drying curve between predicted values vs time and experimental values vs time followed the almost similar type of behavior.



Figure 2 Experimented and Predicted MR of Bhimkol banana at different temperature

6.3 The generalized drying rate constant and drying model parameters

Increase in drying rate constants (k and n) with the increase in drying temperature can be observed from Table 3. Equation (7) shows the Arrhenius equation for the temperature dependent drying rate constant.

$$k = 8.153 \exp\left(\frac{1541}{T_{abs}}\right) \tag{7}$$

A modified model as a function of drying temperature and time (Equation (8)) was obtained by introducing function *k* into the Midilli-Kucuk model. The model had two variables namely drying time and drying temperature. The R^2 value of the developed model was 0.98.

 $MR(t,T)=1.03\exp(-kt^{0.497})+(-2.61\times10^{-5})t$ ($R^2=0.98$) (8) 6.4 Generalized master curve technique based on shift factor

Master curve, (Figure 3) for the temperature reduced time of $t'=t(\alpha_T)$ was obtained with the reference of a central drying temperature (Bezbaruah and Hazarika, 2014). Table 4 shows the values of the temperature shifting factor (α_T) . The equation of α_T as a function of temperature (*T*) is given in Equation (9).



Figure 3 Plot between experimental and predicted drying curves based on master curves technique

Table 4	Temperature shifting factor (α_T) at different
	temperatures

Temperature (°C)	Temperature shifting factor (α_T)
40	0.60
50	1.00
60	1.59

Equation (10) shows the relationship between MR, drying time and drying temperature as obtained by fitting a master curve with Midilli-Kucuk model (Bezbaruah and Hazarika, 2014). The R^2 value of the developed model was 0.997.

$$MR(t',T) = 0.732 \exp(1.650(t')^{n}) + 8.75 \times 10^{-5}t' \qquad (10)$$
$$t' = t.\alpha_T = 9 \times 10^{-5} T_{2.399}$$

From the Equation (8) and (10), it can be observed that generalized model obtained by using master curve gave better fit as compared to the generalized drying constant method in terms of highest R^2 value. Hence, it can be concluded that the master curve technique is a convenient technique for predicting moisture ratio of thin layer drying process as a generalized model. Figure 3 shows a plot between experimental and predicted drying curves, obtained by using the master curve technique and generalized drying rate constant (k) method.

6.5 Effective diffusivity of Bhimkol pulp

The slope of the linear plot for $\ln(MR)$ and time was used to obtain the effective diffusivity. Effect of drying temperature on effective diffusivity is shown in Figure 4. A similar range of effective diffusivity was obtained for a banana as given by Kadam and Dhingra (2011). Here, the increasing effect of drying temperature on effective diffusivity can be observed. The is due to the reason that water molecules move faster at a higher temperature which results in an increased rate of diffusion.



Figure 4 Dependence of effective diffusivity on drying temperatures

6.6 Activation energy of Bhimkol pulp

The Arrhenius type equation (Equation (4)) was used to estimate the activation energy. The activation energy for the drying of Bhimkol pulp was $39.67 \text{ kJ mol}^{-1}$.

7 Conclusion

Thin layer drying of Bhimkol pulp was carried out in a hot air dryer at 40°C, 50°C and 60°C respectively. Thin layer drying kinetics of Bhimkol pulp drying has been accomplished by using general drying code developed in MATLAB R2014a. In the code, five thin layer drying models were used for the analysis. Among which Midilli-Kucuk showed the best-fitted result with greatest R^2 value and lowest RMSE and SSE values. Two approaches viz. generalized drying rate constant and master curve technique by temperature superposition was also used to generalize the Midilli-Kucuk model as a function of drying time and temperature. Master curve technique obtained the best-fitted model with a highest R^2 value of 0.997 as compared to the generalized drying constant method. From the results, it was also observed that the value of drying constant *k* varied between 0.0586 and 0.0791 with time in min indicating high temperature as the driving force for heat and mass transfer. Effective diffusivity of the drying also followed a rising trend with the increasing temperature. The activation energy of the drying was 39.67 kJ mol⁻¹.

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